

# A MULTIPLE GOAL APPROACH TO THE CHOICE OF PENSION FUND MANAGEMENT

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## I. INTRODUCTION

The purpose of this paper is to address a problem faced by the financial and personnel departments of many firms. This paper presents a goal programming model to help firms choose an investment manager for the firm's pension funds.<sup>1</sup>

The Employee Retirement Income Security Act (ERISA) of 1974 made corporations legally responsible for their pension funds. The law directly impacts over 500 billion dollars in pension fund money of employees covered under private pension funds.

The choice of how to invest this substantial amount of pension fund monies is a complex decision process. Accordingly, few firms directly

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manage their own funds. Instead, many firms turn to investment managers. The choice of the investment manager is a complex decision that must reflect multiple goals. These goals may include choosing a firm that has a high return on investment history over the last year, choosing a firm that will invest only in "blue chip" stocks, or choosing a firm in close proximity.

This first section has briefly reviewed the problem of selecting pension fund managers. Section II reviews the previous research in pension fund choice models. Section III develops the goals and constraints of pension fund management. Section IV summarizes the goal programming model. Section V discusses the support and use of the model.

## II. PREVIOUS RESEARCH

### A. Models of Pension Fund Management

There have been few models developed to aid in the choice of a pension fund manager. A survey by Lin [7] revealed no goal programming models dealing with pension fund management or the choice of pension fund policies.

A closely related topic is how individual pension fund managers choose their investments. Farrell [4, 5] describes the investment strategy of a large pension fund management firm. Ahlers [1] discusses an information system used by a large bank to aid the pension fund manager in the choice of investments.

### B. Goal Programming

Goal programming (GP) is an extension of mathematical programming that enables the user to develop models that more readily meet the varied demands of the user. GP provides the ability to model multiple objectives. It is often an attempt by the user to extend linear programming models to include more realistic objectives. GP allows an objective function that is composed of nonhomogeneous units of measure. An example of this technique is available in Burbridge, Koch, and Lawrence [2]. A general survey is available in Charnes and Cooper [3].

## III. GOALS AND CONSTRAINTS OF PENSION FUND MANAGEMENT

The choice of a pension fund manager is made based on a set of multiple goals. These goals derive from four fundamental areas:

Return on Investment  
 Cost of the Manager  
 Security of the Investment  
 Investment Strategy of the Manager

### A. Return on Investment

Generally, the firm and the employees would prefer an investment manager who can provide a larger return on investment. However, there are multiple measures of return on investment. Generally, investment managers provide statistics that reflect the gross return on investment. This is a measure of the rate of return of the portfolio, excluding the cost of the investment manager. However, this measure is, in part, a function of the market and not just the investment manager. An alternative is the ranking of the investment manager relative to other managers.

A third measure of return is the volatility of the returns. Firms normally desire their pension funds to provide low volatility of returns to ensure an ability to meet pension fund requirements. However, there is the classic risk (volatility) return trade-off.

Each of these goals can be based on information over the preceding one, two, . . . , ten years. In the model developed below, we have chosen to use a time frame of one, three, and five years.

### B. Cost of the Manager

The cost of the manager is an important concern to the firm. In general, all other things equal, the firm would prefer an investment manager whose services were less expensive than other investment managers.

Investment managers' fees normally consist of two components. There is usually a fixed charge for each year and a variable charge based on the amount of the investment. Thus the firm wishes to minimize the sum of these costs.

### C. Security of the Investment

The security of the pension fund is of utmost importance. Security reflects the assurance of continuity of investment and conformance with the firm's investment objectives.

It is difficult to measure security. This paper uses two approaches to operationalize this concept. First, a subjective measure of security can be made by giving each firm a "security number" between 1 and 8, with 8 being the most secure. Second, geographical proximity may lead to a

greater control over the investment. As a result, the geographic closeness of the firm may provide a feeling of security. However, proximity may lead to a disfunctional choice of investment managers, since it may be important to choose a firm in one of the major geographic capital markets, e.g., New York.

#### D. Investment Strategy of the Manager

Investment strategies proposed by individual investment firms are highly variant. Investment strategies can include the used of common stock, real estate, futures markets, options, bonds, guaranteed insurance contracts, and international investments.

The investment strategy of the firm chosen can be evaluated in two primary ways. First, a subjective measure of the strategy can be developed as was done with security. Second, a goal can be established based on a minimizing the deviation from a desired level of investment in fixed income securities. This goal is not pursued in this paper.

#### E. Constraints on the Choice of Pension Fund Management

The user can implement an implicit constraint on the choice of pension fund management by not including a firm in database supporting the model. The user may wish to make additional explicit constraints by, e.g., allowing only a portion of the firm's yearly pension fund investment to go to a given investment manager or requiring a minimum amount go to particular investment managers.

### IV. MODEL

The model developed in this paper is a fixed charge model (analogous to, e.g., the warehouse selection model, [6]). The discussion of the model lists the parameters and variables and a description of their meaning. Then the discussion focuses on formalizing the objectives and constraints.

#### A. Subscripts, Parameters, Goals, and Variables

##### 1. Subscripts

- $i$  firm offering the investment management,  $i = 1, \dots, n$ ,
- $j$  type of investment strategy,
- $t$  year  $t$ ; equals 1, 3, or 5.

#### 2. Parameters

- Port total amount of the portfolio to be invested,
- $B_{ij}$  minimum investment for which a fixed fee is charged,
- $L_{ij}$  lower bound of investment allowed in firm  $i$  of type  $j$ ,
- $U_{ij}$  upper bound of investment allowed in firm  $i$  of type  $j$ ,
- $P_{ijt}$  percent rate of return from firm  $i$  of type  $j$  based on assumption of  $t$ ,
- $R_{ijt}$  ranking of rate of return of firm  $i$  of type  $j$  based on assumption of  $t$ ,
- $V_{ijt}$  volatility measure of the rate of return of firm  $i$  of type  $j$  based on assumption of  $t$ ,
- $C_{ij}$  cost per dollar of investment of investing with firm  $i$  with strategy of type  $j$ ,
- $F_i$  fixed cost of investing with firm  $i$ ,
- $S_i$  security of having firm  $i$  as investment manager,
- $D_i$  proximity of investment manager  $i$  from the firm,
- $I_{ij}$  investment strategy congruence of investing with firm  $i$  and strategy  $j$ ,

#### 3. Goals

- $G_i$  goal  $i$ ,  $i = 1, 2, \dots, 6$ ,
- $G_i^-$  the amount by which goal  $i$  is underachieved,
- $G_i^+$  the amount by which goal  $i$  is overachieved,
- $P$  return on investment goal,  $P > P_{ijt}$ , for all  $i, j, t$ ,
- $R$  ranking goal,  $R \leq R_{ijt}$ , for all  $i, j, t$ ,
- $V$  volatility goal,  $V \leq V_{ijt}$ , for all  $i, j, t$ ,
- $S$  security goal,  $S \geq S_i$ , for all  $i, j, t$ .

#### 4. Variables

- $w_{ij}$  fraction of portfolio invested with firm  $i$  of type  $j \geq 0$ ,
- $x_{ij}$  total amount of portfolio (Port) times  $w_{ij}$ ,
- $y_{ij}$  = 1 if  $w_{ij} > 0$  for some  $j$ ; = 0 if  $w_{ij} = 0$  for all  $j$ .

#### B. Goal Constraints

Associated with each of the goals in a goal programming model is a goal constraint. The purpose of the constraint is to summarize in a single variable the measurement of each goal.

##### 1. Return on Investment

The return on investment goals are based on using the model, given that a value has been established for  $t$ . As discussed above, it is assumed

that  $t$  is either 1, 3, or 5 years. There are three goals related to return on investment.

The first goal is maximizing the rate of return on investment. Alternatively, the goal is to minimize the amount by which we underachieve the profit goal. That is, minimize  $G_1^-$ , where

$$G_1^- = P - \sum_{ij} P_{ijt} \times w_{ij}$$

The second goal is to maximize the ranking of the investment manager's rate of return. This goal can be formulated to minimize  $G_2^+$ , where

$$G_2^+ = \sum_{ij} \left( \frac{\text{Port} \times R_{ijt}}{n} \right) \times y_{ij} - R \times \text{Port}$$

The third goal is to minimize the volatility of the rate of return of the investment manager. This goal assumes a volatility measure that is additive. This paper assumes a volatility measure based on the standard deviation. This goal can be formulated to minimize  $G_3^+$

$$G_3^+ = \sum_{ij} \left( \frac{V_{ijt}}{n^2} \right) \times y_{ij} - V$$

### 2. Cost of the Manager

The goal on the cost of the manager is to minimize the variable and fixed costs associated with the investment manager. This goal can be formulated as follows

$$G_4 = \sum_{ij} C_{ij}(x_{ij} - B_{ij}) + \sum_i F_i \times y_i$$

### 3. Security of the Investment

The security of the investment can be measured using two goals. The first goal uses a subjective measure of security offered by the investment manager. This goal can be formulated by minimizing the variance from a security measure goal as follows

$$G_5^- = \sum_i \text{Port} \times S - \left( \frac{\text{Port} \times S_i}{n} \right) \times y_i$$

The second goal uses a proximity measure as the basis for the security of the investment. In particular, this security goal can be expressed as minimizing  $G_6$ , where

$$G_6 = \sum_i D_i \times y_i$$

### 4. Investment Strategy

No goals have been included to reflect the investment strategy; however, this goal can be measured in a manner similar to the other goals.

### C. Objective Function

The objective function will reflect each of the goals discussed above. Associated with each objective is a weighting factor  $P_i$  that reflects the importance of each goal to the firm. The objective function takes the following form:

$$\text{Min } z = P_1 \times G_1^- + P_2 \times G_2^+ + P_3 \times G_3^+ + P_4 \times G_4 + P_5 \times G_5^- + P_6 \times G_6$$

### D. Model Constraints

In addition to the objective function and the goal constraints, the model has a number of feasibility constraints. If the user has any additional constraints on the choice of investment managers, then those constraints can also be added.

Constraints (1) and (2) ensure that the fractions of the portfolio distributed to the investment firms equal one. Constraints (3) and (4) imply that the sum of the fractions are a lower bound on the variable denoting that a firm is chosen ( $y_i$ ). Constraints (5) and (6) indicate that the total portfolio times the portion of the portfolio allocated to an investment firm gives the amount that must lie within prespecified bounds.

$$(1) \quad \sum_{ij} w_{ij} = 1$$

$$(2) \quad 1 \geq w_{ij} \geq 0$$

$$(3) \quad y_i \geq \sum_{ij} w_{ij} \geq 0$$

$$(4) \quad y_i = 0 \text{ or } 1$$

$$(5) \quad x_{ij} = \text{Port} \times w_{ij}$$

$$(6) \quad L_{ij} \leq x_{ij} \leq U_{ij}$$

## V. SUPPORT AND USE OF THE MODEL

The previous sections developed and defined a model for the choice of pension fund managers. The model was developed so that the user could

determine a solution for  $t = 1, 3, 5$ , given an entire set of parameters. This indicates the importance of the development and maintenance of an appropriate set of parameters. It also indicates the importance of analyzing the model's output.

#### A. Model Support

The parameters of this model are based on publicly available and audited data and internally generated measures of the different investment management firms. The output is a function of the data supplied to the model. Thus it is important to establish and periodically update the database that supports the model. It is also important to provide data on a reasonably large set of firms so that there exists a legitimate choice problem.

#### B. Use of the Model

Mathematical programming algorithms provide an "optimal" solution for the formulated problem. However, that solution is dependent on the specified formulation and parameters. The formulation of the problem is only a representation of the "actual" problem. There may be variables that are not included in the model, but are part of the firm's concern, or the model parameters are treated as deterministic in the model, but they are stochastic.

For example, the use of three historical sets of return data ( $t = 1, 3$ , and  $5$ ) may generate three different solutions. The user can use these solutions only as a basis to respond to the choice of investment managers.

These limitations in the model indicate that the user of the model should not take the model output as an optimal solution. Instead, the model can be used to aid the decision process and as the basis for "What If?" questions and sensitivity analysis (e.g., Meyer [8]).

### VI. CONCLUSION

This paper has developed a goal programming model to aid the user in the choice of a pension fund manager. The paper suggests that the model be used as an aid to the decision process and as a basis for asking "What If?" questions. A future paper will deal with the optimal choice of pension policies.

### NOTE

1. The model developed in this paper reflects the authors' interaction with corporate clients and prior experience as decision makers for a firm's investment strategy, asset allocation, and investment manager selection.

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