Issues in Bayes Nets and Influence Diagrams: With Applications in Accounting and Auditing

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1. Introduction

Recently substantial research in artificial intelligence (AI) has focused on Bayes nets and the closely related influence diagrams. However, there has been little work in the accounting and auditing communities in this area of AI research.

Bayes nets and influence diagrams have some advantages over deterministic AI approaches. These approaches employ probabilities to model decision making. Probabilities are a powerful approach for solving non-deterministic problems.

In addition, Bayes nets and influence diagrams offer at least two advantages over other probabilistic approaches, such as expert systems. First, rather than heuristic and approximate solutions, both Bayes nets and influence diagrams provide normative solutions that are consistent with probability theory. Second, unlike expert systems, influence diagrams allow the user the ability to compute the cost of information. Thus, there are important reasons to study the use of Bayes nets and influence diagrams in accounting and auditing.

1.1 CONTRIBUTIONS OF THIS PAPER

This paper has three primary contributions. First, it presents a brief summary of Bayes nets and influence diagrams. Second, it discusses the formulation of auditing problems as influence diagrams, in order to take advantage of some of the capabilities of influence diagrams, such as cost of information.

Third, this paper investigates adopting Bayes nets and influence diagrams in order to solve generic problems in auditing and accounting. Some of those issues include, representation of reliability of audit evidence, and probabilities as qualitative assessments. In some cases, extensions are made to Bayes nets and influence diagrams in order to accommodate the needs of the auditing context.

1.2 THIS PAPER

This paper proceeds as follows. Section 2 provides background information on Bayes nets and influence diagrams. Section 2 also investigates some example problems in auditing and how those problems can be addressed by Bayes nets and influence diagrams.

Sections 3 through 7 focus on adopting Bayes nets and influence diagrams to meet the generic needs of accounting and auditing systems. Section 3 investigates how Bayes nets and influence diagrams can account for the fineness of information. Those results are then extended to an application of increasing the fineness in an influence diagram and Bayes net. In particular, section 4 analyzes the impact of accounting for the reliability of information. Section 5 addresses the issue of representing probabilities as qualitative assessments. Section 6 examines the possible use of side constraints on the probability networks. Section 7 analyzes some of the issues of concern with utility and costs of information. Section 8 provides a brief summary of the paper.

2. Background: Bayes Nets and Influence Diagrams with Applications in Accounting and Auditing

Bayes nets and influence diagrams are ways of describing the dependencies among different variables, and possibly decisions. They are used to specify probabilistic interdependencies and states of information.

Bayes nets and influence diagrams can be used to represent any decision problem that can be represented as an acyclic graph. For example, they can represent any problem that can be represented as a decision tree (Howard and Matheson [1981]).

Bayes nets and influence diagrams are composed of nodes and arcs. The arcs indicate influence and direction of influence. The nodes are the states being impacted by the influences. They are represented

probabilistically using conditional probability. If there is no arc between a pair of nodes then that can indicate conditional independence between the two nodes.

Influence diagrams have up to four types of nodes: deterministic, chance, decision and value. Bayes nets are influence diagrams that have no decision or value nodes.

Deterministic nodes are deterministic in occurrence. A deterministic arc from node n_1 and n_2, indicates that $Pr(n_2|n_1)=1$.

Chance nodes generally are treated as discrete probabilities. Associated with each chance node n_c are a set of nodes that can influence that node, a set of states associated with those influences and probabilities of those states.

Decision nodes generally are used to indicate a discrete set of decisions that can be made. Value nodes are used to capture the cost or value associated with different sets of outcomes.

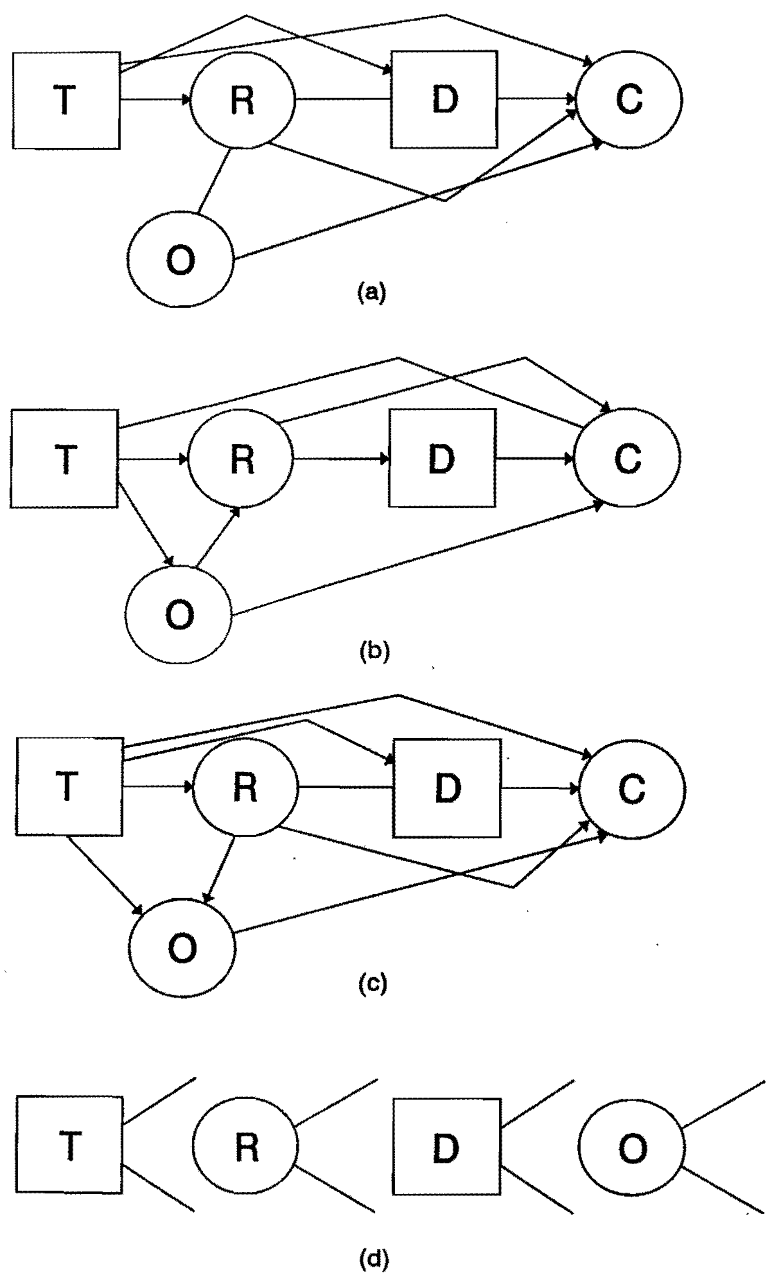
2.1 GENERAL AUDITING EXAMPLE [1]

Consider the example of a general audit test. The auditor can select among various tests T at different costs. Then the outcome of those tests is observed for its results, R. Based on the outcome of the tests, the auditor will make an audit decision D (for example, do additional audit tests), and then, receive some value or incur some cost C, that depends on the state of the audit (i.e., the underlying process), O. Figure 1a shows the influence diagram associated with this process. The arrows show that the test results R depend on the test selected T and the state of the decision O. The decision D is chosen knowing the test selected T and its results R. The value of C depends on the decision alternative chosen D, on the test selected T (as a result of the cost of the test), on the outcome O, and on the test results R. This last influence allows for the possibility that the value may depend directly on the results of the test, for example if the test is destructive. The outcome O does not depend on any other variable, so that $Pr(O|T,.)=Pr(O|.)$. This assumption is based on the belief that the firm being audited does not change the nature of the audit, based on the tests selected.

Influence diagram software allows the user to develop networks. Then the software is used to change the influence diagram into a decision tree for solution evaluation purposes.

At this stage, this influence diagram is a decision network, but not a decision tree network because node O is a predecessor of node D, but

FIGURE 1
General Auditing Example



not a direct predecessor. To create a decision tree network, we must reverse the arrow connecting node O to node R. The first step of this reversal is to ensure that both nodes have the same information state. This can be done by adding an arc from T to O. Then the arc from O to R can be reversed. As a result of these operations, the influence diagram can be redrawn as a tree, seen in Figure 1b.

The reversals require that the original probability assessments be changed. In particular, Pr(R|T,O) and Pr(O|T), must be changed to Pr(R|T) and Pr(O|T,R). This is done by integrating or summing Pr(R|T,O) over O (depending on whether O is a continuous or discrete variable). In addition, Bayes Theorem can be used so that

Pr(O|T,R) = Pr(R|T,O) Pr(O|T)/Pr(R|T). These changes are summarized in Figures 1c-1d.

2.2 UTILITY AND COST TRADE-OFFS— THE VALUE OF INFORMATION

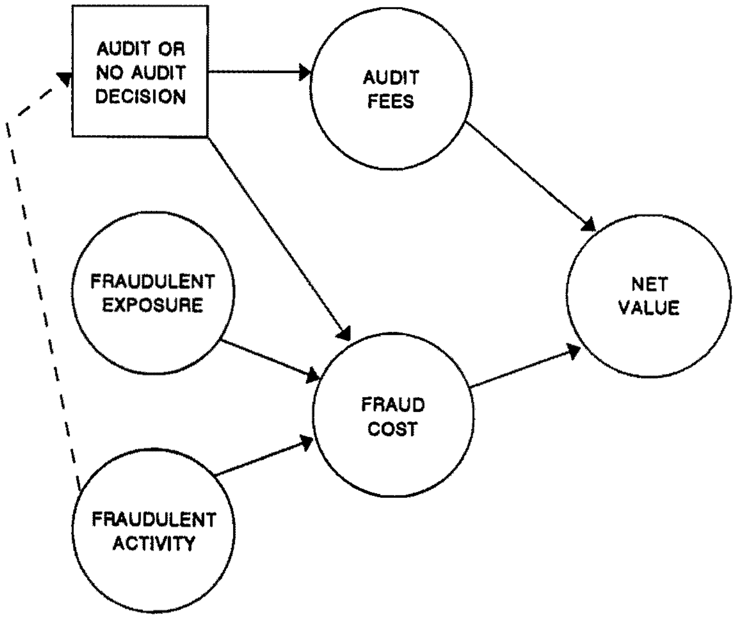
Since influence diagrams have value nodes, they also have the ability to determine the value of information. This is done, basically in the same way, that the classic Bayesian investigation of cost of perfect information is done (Raiffa and Schlaifer [1972]).

Including utility in the value nodes, allows the value of perfect information can be assessed by the system. In order to calculate the value of perfect information for a chance node we draw an influence arc from the chance node to the decision node under consideration. The result is compared to the same network without the added arc.

As an example, consider the case of an auditor concerned about performing an audit.[2] Associated with each audit are the audit fees, net of costs, and the potential for fraud, leading to additional costs to the auditor. Fraud cost occurs only with fraudulent activity. The audit firm may assume that there is an additional cost to the firm, if the firm is exposed to a fraudulent audit situation. This discussion is summarized in Figure 2.

The auditor can either perform the audit or not perform the audit. Assume that the expected value to the auditor of performing the audit, accounting for the impact of fraud, is $22,000 dollars. If we are able to add an arc from the fraud activity to the audit decision problem, then we know the degree of fraud when we make the audit engagement decision. In that situation, if the expected value was, say $30,000, then the value of perfect information would be $8,000.

FIGURE 2
Example Influence Diagram for Fraud



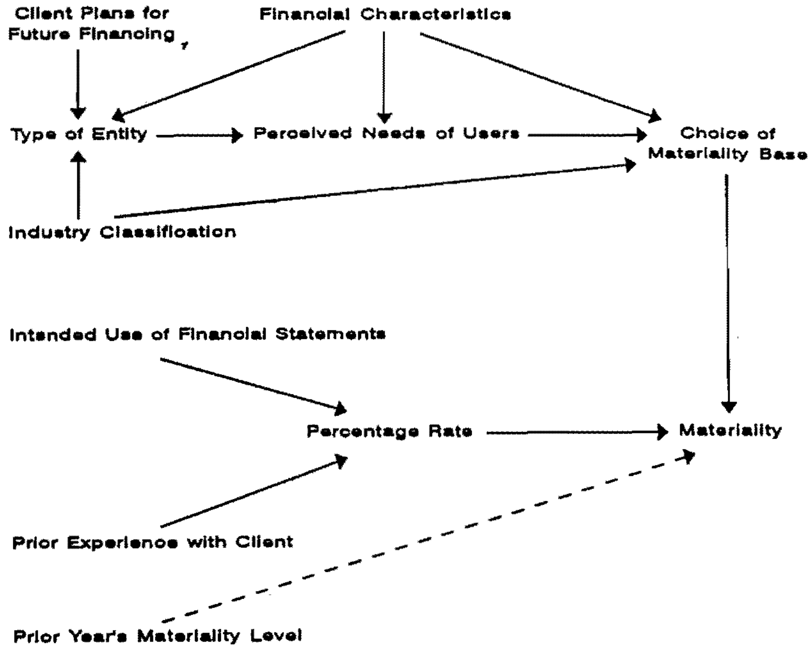
2.3 AUDITPLANNER AS AN INFLUENCE DIAGRAM

Although there has been little explicit or direct use of influence diagrams or Bayes Nets, there have been a number of systems that employed diagrams that capture the notion of "influence." For example, in the development of AUDITPLANNER, Steinbart [1987] developed what might be referred to as an influence diagram, without the probabilities (Figure 3). In addition, he supplied a number of rules, that were used to build a decision tree (abstracted from Steinbart [1987] for purposes of this paper in Figure 4). That decision tree can be structured as an influence diagram or Bayes net using the reverse of the approach above. This is characteristic of many accounting and auditing problems that can be structured as influence diagrams.

2.4 LIMITATIONS

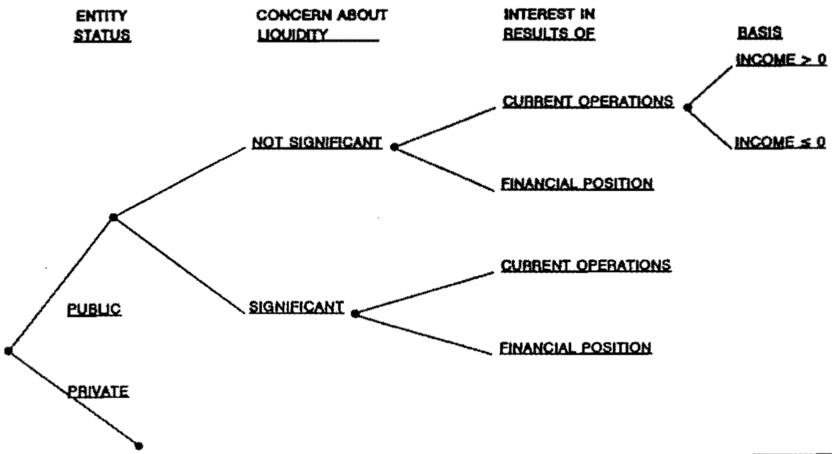
There are some limitations of Bayes nets and influence diagrams that potentially limit the successful implementation in accounting and

FIGURE 3
AUDITPLANNER's judgment model



Source: Steinbart [1987]

FIGURE 4
Audit Expert Systems as Influence Diagram



auditing applications. First, an advantage of expert systems is the ability to access an explanation for solutions proposed by the system. Unfortunately, there is little in the way of "explanation" available from either a Bayes net or influence diagram.

Second, either a Bayes net or an influence diagram requires substantial amounts of information, such as probabilities. Gathering that data must be cost beneficial or otherwise an alternative approach would be used. Thus, in some cases we would expect that a substantial payoff must be under consideration. As a result, it is easy to understand why legal applications have been proposed as a source of applications. Edwards [1991] does a comprehensive analysis of a legal case using Bayes nets. With the substantial base of litigation facing public accounting firms, this set of tools may prove quite useful.

2.5 COMMERCIALLY AVAILABLE SOFTWARE

The availability of commercial software to solve influence diagram problems is important since it indicates that the computational costs of using Bayes nets and influence diagrams will be tractable. In the future, we might expect some of that software to accommodate some of the issues discussed later in this paper.

Using the work of Schachter [1986, 1988a] at least two different systems have been developed for different computational environments. INDIA has been developed for an MS/DOS environment. DAVID (Schachter [1988b] has been developed for the Macintosh.

HUGIN, based on Lauritzen and Spiegelhalter [1988] is available for Unix-based machines. Another program, ERGO, for Bayes nets and not influence diagrams, is just emerging from a beta test (Edwards [1991]).

3. Fineness and Distribution of Weights of Evidence

This section introduces the notion of fineness and discusses some of its implications. The concept of fineness can be extended to other issues of direct concern to accounting and auditing systems, including reliability.

3.1 FINENESS

Marschak and Radner [1972, p. 53] note that ". . . given two information structures, n_1 and n_2, that n_1 is as fine as n_2 if n_1 is a subpartition of n_2; that is every set in n_1 is contained in n_2. (Thus, n_1 tells us all

that n_2 can tell and possibly more besides.)If n_1 and n_2 are distinct, and n_1 is as fine as n_2, then we shall say that n_1 is finer than n_2."

The notion of fineness can be used to assess two different influence diagrams of the same process. One influence diagram, G_1, is finer than G_2 if every set in G_2 is also in G_1.

3.2 DISTRIBUTION OF PROBABILITY

Fineness can be used to assign probability to different sets of events. For example, suppose x is assigned the probability P(x) in some influence diagram G_1. If G_2 is a finer influence diagram with respect to the event x, then that indicates that x (or x given z) is partitioned further, with respect to some other set of events, say y and y'. In any case, the same amount of probability is spread across the same partitions in each of G_1 and G_2. As a result, in G_2, we would see the probability spread

$$Pr(x) = Pr(x,y) + Pr(x,y') \text{ or}$$

$$Pr(x|z) = [Pr\ (x,y,z) + Pr\ (x,y',z)]/Pr(z).$$

The increased fineness in the first equation can be seen in the following diagrams. In order to accommodate increased fineness these subgraphs would be a part of G_2

```
x_<--------y        x'<--------y
/\                  /\
  \                   \
   \                   \
    y'                  y'
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Fineness can be used to capture any increased set of events. In the next section, the categorization of audit evidence, E is made finer using the issue of reliability.

4. Reliability

Another critical issue is the reliability of the audit evidence and the impact of that reliability on audit decisions. In some cases, some evidence may be more reliable than in other cases. For example, as discussed in Bamber [1983] audit workpapers may be less than perfectly reliable. The ability of different audit team members can vary, thus, impacting reliability.

Since such source reliability issues exist, they need to be accounted for in the modeling of audit decisions, and representations of those decisions. One approach to capturing the reliability in audit situations is the source reliability approach of Schum and DuCharme [1971]. If x is the probability of an event and x# is the report of that event, the Pr(x#|x) is used to represent reliability.

Reliability can be used to build greater fineness into the networks. Accordingly, reliability can be used to increase the representations of x and x' to the following.

```
x#<--------x                    x#'<--------x
/\                              /\
  \                               \
   \                               \
    x'                              x'
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5. Representing Probabilities as Qualitative Assessments

Some of the influence diagram software promulgates a categorical match between probabilities used and computed by the system and various descriptors. This capability facilitates the user interaction with Bayes nets and influence diagrams. For example, in one system, Heckerman et al. [1990], use five categories: "absent" (0); "rare"(0-2); "present" (3-15); "many" (16-50) and "striking" (>50).

Medical systems are not the only area of application that employ such category explanations. For example, KPMG [1989, p. 11-52] promulgates a scale that relates quantitative probability estimates and qualitative terminology in the audit process.

> The planned assessment level of internal control risk is a matter of judgment, depending on the individual circumstances of the client. However, in order to allow a mathematical representation of the audit risk model, these assessments of high, moderate, low or very low have to be given numerical values. KMPG has assigned the following probabilities to prevent or detect an aggregate error equal to or larger than tolerable error to these assessments:

Qualitative	Quantitative
High	More than 40%
Moderate	Between 20% and 40%
Low	Less than 20%
Very Low	Less than 10%

Thus, the capability to capture qualitative estimates as quantitative factors can be important in the use of influence diagrams in accounting and auditing situations.

5.1 IMPACT ON BAYES NETS AND INFLUENCE DIAGRAMS IN ACCOUNTING AND AUDITING

In auditing (e.g., Bamber [1983]) the primary activity has been with the following formulation.

$$Pr(E|H)=Pr(E\#|H,E)Pr(E|H) + Pr(E\#|H,E')Pr(E'|H).$$

In this formulation, it is assumed that knowledge of the hypotheses does not influence the report of the evidence, and thus, Pr(E#|H,E) is set equal to Pr(E#|E) and referred to as the reliability of the report. Thus, reliability of audit information can be built into the Bayes nets and influence diagrams.

5.2 RANKING OF OUTPUT

In addition to increasing the fineness of the Bayes nets and influence diagrams, reliability can play an important role in the user interface. Many Bayes nets and influence diagram systems limit the list of paths through the networks in their user interface, to say the ten with the highest likelihood ratio of P(E#|H)/P(E#|H') (e.g., Heckerman et al. [1990]) or only those over a certain cut-off point for the likelihood ratio.

Such an approach can force the users into suboptimal choices, since if reliability is considered, the ordering may change and the probability associated with different diagnoses may change (O'Leary [1991]). There are two distinct cases: different diagnoses and same evidence; and different diagnoses and different evidence. In the first case it can be shown that in under very general conditions, if the same evidence impacts each hypothesis the same then reliability has no impact. However, in the second case, in general, if different evidence impacts different hypotheses, then the reliability will have an impact on order. Thus, ranking using likelihood ratios generally can bias the user.

6. Constraints on Probabilities

In a discussion of the well-known mining expert system "Prospector," Konolige [1979] introduced the notion of constraints on probabilities. Konolige [1979] contended that when soliciting probability information from experts two problems were encountered. First, ". . . the parameters specified are usually incomplete," i.e., the expert would only be able to provide certain marginal or conditional probabilities. Second, some of the probability estimates might be inconsistent. Thus, Konolige [1979] proposed constraints on the probabilities as a basis of assigning probabilities that the expert had been unable to assign and to test the consistency of the probabilities. A number of approaches were proposed including linear equalities of the nature where one conditional probability is greater than another and entropy based assignment of probabilities. These approaches are summarized in Konolige [1979], with some extensions in O'Leary [1990].

6.1 CONSTRAINTS IN AUDIT ENVIRONMENT

The auditing context also generates constraints on probabilities. In the example, above, "high" was stated to be "more than 40%," etc. These institutional constraints can be accommodated by introducing constraints on the probabilities. Thus, one constraint might be $p_i \geq .4$. Such constraints can be particularly helpful with ensuring that policies are maintained.

6.2 CONSTRAINTS: EXTENSION TO INFLUENCE DIAGRAMS

In general Bayes nets and influence diagrams do not accommodate constraints on the probabilities. However, there are at least two reasons for using constraints on probabilities in influence diagrams, in general. First, as with expert systems, influence diagrams and Bayes nets require the estimation of probabilities. Thus, the probability estimates must be consistent. As in expert systems, such constraints can be used to ensure consistency.

Second, one of the problems with Bayes nets and influence diagrams is that there may be a large number of probabilities to estimate. In fact, probabilistic expert systems typically employ heuristic devices so that the number of probabilities that require estimation is minimized. Thus, if constraints can be used to develop other probabilities, the constraints can minimize the extent of estimation of probabilities.

The general solution of networks, subject to such constraints has been addressed. McBride [1987] refers to such constraints as "side constraints." Thus, we might expect that influence diagrams or Bayes nets be extended to accommodate such constraints.

7. Utility/Costs and Cost of Information

Unlike other forms of knowledge representation, including Bayes nets, influence diagrams can be used to assess the cost of missing information and the cost of gathering more information. Typically this is done through the capture of the utility of the decision maker (e.g., Heckerman et al. [1990]). Thus, we would expect that influence diagrams would be used in accounting and auditing contexts that include either cost or utility. However, there are at least two major issues that must be addressed in the application to auditing settings: determination of decision maker and unit of utility.

7.1 DETERMINATION OF THE DECISION MAKER

Perhaps the most fundamental issue in auditing applications is the determination of the decision maker. In order for the appropriate costs and benefits to be captured it must be done from a given perspective.

The choice of cost and benefits may be either a single or multiple agent problem (O'Leary [1992]). In the case of audit applications, there are individual, team and company, biases and points of view that can enter into utility determination, depending on which decision maker is modeled. If auditing firms and accountants do adopt influence diagrams, in order to maintain a single agent perspective, it may be beneficial if, say, firm policy were used to develop the appropriate costs and benefits. Otherwise individual fears, greed, and other issues, may lead decision makers using influence diagrams to make suboptimal decisions.

Influence diagrams and Bayes nets do not handle multiple agent problems easily. The development of consensus networks is controversial. The most successful implementations have been in domains where expertise or knowledge can be either "averaged" or is relatively noncontroversial.

7.2 UNIT OF UTILITY

The other primary issue is the unit of utility. That issue has been a difficult one for decision analysts, in general.

The development of Pathfinder by Heckerman et al. [1990], employed a version of Howard's [1980] "worth-numeraire." In that model, "utilities are associated with major misdiagnoses are measured in terms of life-and-death gambles, whereas utilities associated with minor diagnoses are measured in terms of dollars."

The limitation of this approach is that it basically assumes that the major and minor problems do not interact. However, there are many situations where relatively minor events signal major events. For example, very small (and apparently insignificant) differences between sets of accounting numbers have signaled a wide range of events, including adding errors and major frauds. Thus, so-called minor diagnoses may signal life-and-death gambles.

8.3 THE HOWARD MODEL IN AN AUDIT SETTING

The Howard model has not been tested in an accounting or auditing setting. However, there appears to be substantial opportunity for such a model. For example, we would probably identify the savings and loan crisis as containing some major audit diagnoses or misdiagnoses, that may have benefited from analysis using influence diagrams.

We can speculate as to how this approach might be applied. First, some division between minor and major diagnoses would have to be developed. Major diagnoses might refer to diagnoses on which the future of the firm, or the individual's job depended. Minor diagnoses probably would include individual elements of the internal control evaluations or even the control evaluations. Major diagnoses would probably be associated with material frauds, and minor diagnoses with immaterial deviations.

Second, the subset of minor diagnoses that related to major diagnoses would be identified. Third, differential amounts associated with each of the diagnosis or events would be developed.

8. Summary

There has been substantial use of expert systems and other tools of artificial intelligence in accounting and auditing. One tool that has received little attention is influence diagrams. This paper reviewed the influence and Bayes nets. Then the paper investigated the use of these tools in accounting and auditing problems. Bayes nets and influence diagrams can be used in virtually any decision situation where the decision can be represented as an acyclic network. They offer a number of advantages over other tools such as expert systems, including exact as opposed to heuristic solutions and the ability to determine the cost of information.

The paper also developed a number of generic issues of concern to auditors and accountants. The influence diagrams were formulated to accommodate finer representations of information. This lead to detailed analysis of reliability of information.

Then two useful features of influence diagrams were discussed. It was found that system capabilities that allow capture of qualitative assessments as probabilities in auditing system can be beneficial. In addition, it was found that constraints on probabilities can provide a useful basis for the ensuring the probabilities are correct.

Finally, the impact of being able to estimate cost of information in influence diagrams was developed. Two of the primary concerns include the basis of utility (e.g., money) and who provides the utility estimates in accounting and auditing situations.

ENDNOTES

1. This example is an extrapolation of an example in Howard and Matheson [1981, pp. 744-746]
2. This example is an extrapolation of an example in Howard and Matheson [1981, pp. 747-750]

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