A MATHEMATICAL PROGRAMMING APPROACH TO THE HOSPITAL CASH MANAGEMENT PROBLEM AND EXTENSIONS

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Abstract
The purpose of this paper is to examine hospital cash management and to suggest a method, Goal Programming, which could result in better utilization of a hospital's cash resources. As cash management is a critical part of a hospital's approach to its management, one would assume several goals must be satisfied. These goals include the multiple objectives of the hospital and additional concerns such as organizational values and environmental constraints. This paper presents a Goal Programming formulation of the hospital cash management problem. Multiple goals with individual weighted priorities are organized in a hierarchical structure which are considered in the solution process. In addition, sources of revenue and timing of disbursements are addressed.

INTRODUCTION
The advent of high interest rates, soaring medical costs and the increasing amounts and types of government transfer payments have directed hospital financial administrators into a position of accountability. This has resulted in the increased awareness and management of the direct cost of patient services, revenues from services, the long-term capital budget and short-term sources and uses of cash.

Hospitals derive revenue from four groups:
Table I: HEM[10].
- Public - includes Medicare, Medicaid, and federal, state and local support.
- Private Insurance - includes Blue Cross and other commercial insurance carriers.
- Private Pay - includes patient payments without insurance coverage.
- Philanthropy - includes gifts and contributions.

Medicare and Medicaid have moved hospitals to direct costing of all services as reimbursement to the hospital is derived from the direct cost of the resource. This has led to accurate record keeping for all sources of revenue and for good information about all sources and uses of cash.

The revenue that a hospital receives is a function of the type of payee; public, private insurance, private pay, and their respective percentage and overall utilization of hospital services.

It can be assumed that short-term influences will only affect revenues positively and that long-term losses in market share are not the concern of the cash management problem. Revenue derived from public and private insurance are assumed to represent certain revenues with an uncertain payment schedule. Private pay patient revenue represents uncertain revenue with an uncertain payment schedule.

Negative cash flow is experienced by the hospital from the time the patient receives services until the hospital's bank is credited funds that it is due. To reduce this time, Electronic Fund Transfer Systems (EFTS) are being installed between private insurance companies and hospitals. This has the capability of reducing the time between patient admittance and receipt of funds from the insurance company, from thirty to sixty days to a few minutes. Medicare has arranged with compliant hospitals to reimburse for patient days while the patient is still in the hospital. Private pay patients are encouraged to use bank credit cards and deposits. Hospitals are currently one of the largest grantees of credit to the noncorporate person. This credit extended results in losses and bad receivables unless the hospital grants credit as would any other financial institution.

Daily hospital operations are dependent on the cash that is received from its revenue sources for payment of payroll, accounts payable and other payables. As noted in Orger [7] "the need for
cash arises from the lack of synchronization between cash inflows and outflows and the difficulty of accurately predicting the sum of these flows. Consequently, an adequate amount of cash should be maintained to perform regular transactions and to meet unexpected requirements."

Hospitals generally maintain bank accounts, own short-term securities and maintain open lines of credit with its banking sources to either invest cash overages or to borrow from if a cash shortage exists. Short-term negative cash flows are not to be confused with consistent negative flows which can lead to a decreasing credit rating, high interest borrowing and embarrassing publicity. As a result, a hospital must arrange its affairs so that enough cash is maintained to meet short-term requirements. High interest rates and their costs and returns to the hospital are reason enough not to keep too much or too little cash in liquid form. Idle cash can be invested in high yielding securities or can be used to pay off costly short-term debts.

A major effort in formulating the cash management problem as a linear programming model was made in Orgler [7]. This paper provides an analysis of his model and then extends his model to the cash management application for hospitals. The paper also provides the hospital administrator with the capability of obtaining multiple goals. The resultant model is referred to as a hospital goal programming model of the cash management problem.

A different emphasis in the analysis of the cash management problem has been explored by a number of other authors. An analysis of the accounts receivable management problem is contained in Levy [5]. The lock box problem is discussed in Stancill [9] and McAdams [6]. Finally, different linear programming models of bank services, including lock boxes and accounts receivable are discussed in Calman [2] and Pogue, Faucett and Bussard [8]. A qualitative non-quantitative approach to the hospital cash management problem is included in Frank [11].

HOSPITAL INTERFACE WITH OTHER ORGANIZATIONS

Hospitals interface with a number of other organizations including, e.g., CPA (Certified Public Accountant) firms. In particular, the CPA firm (or some state or federal agency) provides a financial accounting audit. As part of that audit, if the hospital is handling its cash in a disfunctional manner, it is likely that an audit comment reflecting this situation will be made in the audit report. The model in this paper can be used to limit this potential audit concern.

Municipal hospitals face an additional concern. They often have joint costs incurred by the hospital for the city, by the city for the hospital, or both. Substantial reimbursement (and thus cash resources) can be at stake from state, federal, private insurance, etc. sources in terms of indirect cost reimbursement.

GOAL PROGRAMMING

Goal Programming is an extension of mathematical programming that enables the user to develop models that more readily meet the varied demands of the user. It is often an attempt by the user to extend linear programming models to include more realistic multiple objectives and constraints. Further, a goal programming model can have an objective function that is composed of nonhomogeneous units of measure. An example of this technique is available in Burbridge, Koch, and Lawrence [1]. General surveys of goal programming are available in Charmes and Cooper [3] and Lee [4].

A LINEAR PROGRAMMING MODEL FOR THE CASH MANAGEMENT PROBLEM: SUBSCRIPT, VARIABLE AND PARAMETER DEFINITION

In [7], pp. 99-100, Orgler presents a summary of a linear programming model of the cash management problem. For completeness that model is summarized. For more detail the reader is referred to the original model formulation.

Throughout the paper, the following subscripts will be used:

- \( g \) Indicator of the period in which purchases or borrowing occurs.
- \( h \) Indicator of the different types of payments and sources of short-term funds.
- \( i \) Maturity period indicator.
- \( j \) Time period indicator.
- \( m \) Indicator of borrowing source.

The following variables will be used in the model:

- \( x_{hgj} \) The amount which is scheduled to be paid at time \( j \) for a purchase made in day \( g \), of the type \( h \).
- \( w_{hg} \) The amount borrowed in period \( g \) of type \( h \).
\( z_{ij} \) The amount of security sales in time \( j \) maturing in time \( i \).

\( b_{jm} \) The cash balance in time \( j \) at source \( m \).

\( y_{it} \) The amount of investment in securities in time \( y \) maturing in time \( i \) \( t+1 \).

The following parameters are used in the model:

- \( C_{hj} \) Net revenue coefficient from payment \( x_{ghj} \).
  \( \text{(This includes the impact of float).} \)

- \( D_{ij} \) Net revenue coefficient from investments in securities \( y_{ij} \).

- \( E_{ij} \) Cost coefficient of security sales \( z_{ij} \).

- \( F_{hg} \) Cost coefficient of borrowing, \( w_{hg} \).

- \( u, v \) Number of different payment types of which the first \( s \) types apply to accounts payable, and the remaining \( u-s \) types correspond to repayments on loans borrowed within the framework of the model. Remaining types of loans not subject to early repayment are defined as \( u+1, \ldots, v \).

- \( T \) Time horizon of model.

- \( L_{hg} \) Amount of liability which becomes outstanding in period \( g \) of type \( h \).

- \( a_{hj} \) Technical coefficient of payment \( x_{hj} \), expressing the effect of timing on the payment size.

- \( K_h \) Amount available for borrowing of type \( h \).

- \( e_{ij} \) Technical coefficient of security sales \( z_{ij} \), calculated from the yield on the security since the yield and the price are directly related \( e_{ij} = 1 + E_{ij} \).

- \( S_i \) Amount of a security from the initial portfolio maturing in period \( i \).

- \( M_jm \) Minimum balance in time \( j \) at bank \( m \).

- \( M \) Total number of banks.

- \( t_j \) Number of days in period \( j \).

- \( A \) Average daily minimum over \( T \) days.

- \( B_0 \) Cash balance at the beginning of the model's first period.

- \( B_i \) Cash balance at the end of period \( i \).

- \( N_j \) Net amount of fixed cash flows in period \( j \), i.e., other receipts less other payments.
  \( N_j = N_{1j} + N_{2j} + N_{3j} + N_{4j} \), where \( N_{1j} \) are revenues from private insurance, \( N_{2j} \) are the revenues from Medicare, \( N_{3j} \) are the revenues from self pays, and \( N_{4j} \) are the revenues from other sources.

- \( D_{ij} \) Increase in the value of the investment \( y_{ij} \).

- \( B_X \) Upper limit on the balance of accounts payable.

- \( k_n \) Prespecified time period that applies to payment type \( h \). This can reflect late payments as required.

**THE LINEAR PROGRAMMING MODEL**

The model presented in Orgler 7, pp. 99-100 makes two assumptions, a finite horizon and the daily periods are aggregated into longer unequal intervals. The model can be expressed in the following manner.

**A. OBJECTIVE FUNCTION**

\[
\text{Maximize} \quad Z = \sum_{j=G}^{T} \sum_{h=1}^{H} \sum_{g=1}^{G} c_{h,s,j} x_{h,s,j} + \sum_{i=1}^{T} \sum_{j=1}^{v} \sum_{h=1}^{H} F_{h,g} \delta_{h,g} \delta_{h,g} + \sum_{h=1}^{H} \sum_{g=1}^{G} h_{s+1} g_{k} h_{s, h, g} + \sum_{i=1}^{T} \sum_{j=1}^{v} \sum_{h=1}^{H} \sum_{g=1}^{G} h_{s+1} g_{k} h_{s, h, g}
\]

\[
T \to T \quad u \quad g=-k + 1 \quad h=1 \quad 8 \quad j=1 \quad x_{h,s,j} + \quad h=1 \quad 8 \quad j=1 \quad T+1 \quad T \quad \sum_{i=1}^{T} \sum_{j=1}^{v} \sum_{h=1}^{H} \sum_{g=1}^{G} h_{s+1} g_{k} h_{s, h, g} + \sum_{i=1}^{T} \sum_{j=1}^{v} \sum_{h=1}^{H} \sum_{g=1}^{G} h_{s+1} g_{k} h_{s, h, g}
\]

where \( G=1 \) for \( g = 1 \) and \( G=g \) for \( G = 1 \).

**B. CONSTRAINTS**

**a. Payments**

\[
\sum_{j=G}^{T} a_{h,s,j} x_{h,s,j} = L_{h,s} \quad \text{for } g = -k + 1, \quad \ldots, \quad T - k_h \quad \text{and } h = 1, \ldots, s.
\]
f. Termination

Accounts Payable

(11) \[ T \sum_{j=0}^{T-k_1} \sum_{h=0}^{T} \sum_{g=0}^{p} x_{h,g,j} \leq B \]

\[ j=0 \quad g=T-k+1 \quad h=1 \]

\[ T \quad p \quad \sum_{h=1}^{T} \sum_{g=0}^{p} h_{g} \]

\[ g=T-k+1 \quad h=1 \]

\[ \sum_{g=0}^{p} g \]

Nonnegativity Constraints

(12) \[ x_{h,g,j} \geq 0 \quad \text{for all } h, g, \text{ and } j \]

\[ y_{i,j} \geq 0 \quad \text{for all } i \text{ and } j \]

\[ z_{i,j} \geq 0 \quad \text{for all } i \text{ and } j \]

\[ \omega_{h,g} \geq 0 \quad \text{for all } h \text{ and } g \]

Constraint (4) is based on the assumption that loans borrowed within the scope of the model are due beyond the horizon, i.e., \( g \geq k \geq T \). If certain types of loan have to be paid within the horizon, i.e., \( g + k \leq T \), the inequality sign of the constraint simply changes to an equality and \( j = g \) for these types.

EXPLANATION OF THE LINEAR PROGRAMMING MODEL

The objective function (1) is a sum of the revenues from payments and investments in securities and the costs of security sales and borrowing over all periods in the model.

The constraints can be viewed as follows:

a) Payments

The equations (2) and (3), refer to the constraints required to pay the accounts payable. The constraint (4) ensures the payments of the loans borrowed within the model.

b) Financing

The constraint reflects limitations on each type of borrowing.

c) Security Sales

Sales of the securities are limited to those available in the portfolio.

d) Minimal Cash Balance

Constraint (7) reflects the minimum balances desired and constraint (8) reflects the average daily minimum balance.
e) Cash Flows
   Constraints (9) and (10) equate the net cash flows and the changes in the cash balance.

f) Termination Constraints
   Termination constraints are required in the last period of the model as an adjustment for its truncation at the horizon and to avoid excessive buildups in current liabilities and depletion of certain assets at the end of the planning period.

GOAL PROGRAMMING MODEL: THE CONSTRAINTS

This goal programming model of the cash management problem has been developed to optimize for more than one goal. This requires the establishment of a hierarchy of importance between these potentially incompatible goals. In this paper, it is assumed that hospital management is able to elicit an ordinal ranking of the goals or priorities, in terms of their desirability. This ordinal ranking results in their preference level. Each of these priorities is reflected in a constraint or set of constraints.

In this problem the following priorities were considered (by preference level):

1. Maximize the total revenues from the cash.
2. Maximize the satisfaction of the banking sources.
3. Minimize the costs of security sales.
4. Minimize the costs of borrowing.
5. Minimize the total amount of borrowing.
6. Minimize the total amount of security sales.

The first of the goal constraints is concerned with the maximization of total revenues from cash. The constraint is of the following form:

\[
\begin{align*}
\sum_{j=1}^{T} \sum_{h=1}^{G} & \sum_{i=1}^{D} \sum_{j=1}^{n} C_{ijh} + d_{ij} + f_{ijh} + \frac{\sum_{j=1}^{T} Q_{TR}^- - Q_{TR}^+}{TR} = TR \\
\end{align*}
\]

where,

- \(TR\) is the total revenues budgeted,
- \(Q_{TR}^-\) is the under-attainment of the budget level of total revenues, and
- \(Q_{TR}^+\) is the over-attainment of the budget level of total revenues.

The second set of goal constraints is concerned with the maximization of the satisfaction of the banking sources. This goal takes the following form for each bank, and for each \(j\) and \(m\):

\[
\begin{align*}
\sum_{j=1}^{T} \sum_{h=1}^{G} & \sum_{i=1}^{D} \sum_{j=1}^{n} (5)\; TS_{jm}^- + TS_{jm}^+ = TS_{jm} \\
\end{align*}
\]

where,

- \(TS_{jm}\) is the level desired in bank \(m\) at time \(j\),
- \(TS_{jm}^-\) is the under-attainment of the minimum balance at bank \(m\) at time \(j\), and
- \(TS_{jm}^+\) is the over-attainment of the minimum balance at bank \(m\) at time \(j\).

The third type of goal constraint is concerned with the minimization of the net cost of security sales. This constraint has the following form:

\[
\begin{align*}
\sum_{j=1}^{T} \sum_{h=1}^{G} & \sum_{i=1}^{D} \sum_{j=1}^{n} Q_{SS}^- + Q_{SS}^+ = SS \\
\end{align*}
\]

where,

- \(SS\) is the budgeted net cost of security sales,
- \(Q_{SS}^-\) is the under-attainment of the budget level of the net cost of security sales and
- \(Q_{SS}^+\) is the over-attainment of the budget level of the cost of security sales.

The fourth type of constraint is concerned with the minimization of the cost of borrowing. This constraint has the following form:

\[
\begin{align*}
\sum_{j=1}^{T} \sum_{h=1}^{G} & \sum_{i=1}^{D} \sum_{j=1}^{n} F_{hb} + h_{gb} + Q_{TB}^- + Q_{TB}^+ = TB \\
\end{align*}
\]

where,

- \(TB\) is total borrowing cost budgeted,
- \(Q_{TB}^-\) is the under-attainment of the budget level of total borrowing cost and
- \(Q_{TB}^+\) is the over-attainment of the budget level of total borrowing.

The fifth set of goal constraints is concerned with minimizing the total amount of borrowing.
\[ \sum_{h=st+1}^{T} \sum_{g=1}^{W} w + Q - Q = AB \]

where,

- AB is the amount budgeted for borrowing,
- Q is the under-attainment of the budgeted AB amount borrowed and
- + Q is the over-attainment of the budgeted AB amount borrowed.

The sixth set of goal constraints considered here, is concerned with the minimization of the total amount of security sales. This constraint takes the following form:

\[ \sum_{i=1}^{T+1} \sum_{j=1}^{I} z + Q - Q = AS \]

AS is the total amount of budgeted security sales.

- Q is the under-attainment of budgeted security AS sales and
- + Q is the over-attainment of budgeted security AS sales.

The final set of goal constraints that needs to be developed are the non-negativity constraints on the "deviational" variables. In particular,

\[ Q_{TR} \geq 0, Q_{TS} \geq 0, Q_{SS} \geq 0, \]

\[ Q_{j} \geq 0, Q_{jm} \geq 0, Q_{jm} \geq 0, \]

\[ Q_{j} \geq 0, Q_{TR} \geq 0, Q_{TS} \geq 0, Q_{SS} \geq 0, \]

\[ Q_{j} \geq 0, Q_{AB} \geq 0, Q_{AS} \geq 0, \]

\[ Q_{j} \geq 0, Q_{AB} \geq 0, Q_{AS} \geq 0, \]

\[ Q_{AB} \geq 0, Q_{AS} \geq 0, \]

The total set of goal programming constraints for the cash management problem is a combination of the cash management model constraints and these goal programming constraints. In particular, the model consists of the cash management model constraints, (2), (3), (4), (8), (9), (10), (11), and (12) and the goal programming constraints (13) - (19).

**Goal Programming Model: The Objective Function**

In order to achieve the priorities according to their stated level of importance, the deviations from the goal j will be ranked according to the preemptive priority factor, \( P \) for each goal j.

The preemptive priority factors for this model will take the following form:

- \( P_{1} \): Preemptive priority factor for the maximization of the over-attainment of budgeted total revenues (\( Q_{TR}^{+} \)).
- \( P_{2} \): Preemptive priority factor for the maximization of the over-attainment of the minimum balance at bank j, at time \( j \) (\( Q_{TS}^{+} \)).
- \( P_{3} \): Preemptive priority factor for the maximization of the under-attainment of the budget level of the net cost of security sales (\( Q_{SS}^{-} \)).
- \( P_{4} \): Preemptive priority factor for the maximization of the under-attainment of the budget level for total cost of borrowing (\( Q_{TB}^{-} \)).
- \( P_{5} \): Preemptive priority factor for the maximization of the under-attainment of the budget level for the total amount of borrowing (\( Q_{AB}^{-} \)).
- \( P_{6} \): Preemptive priority factor for the maximization of the under-attainment of the budget level for the number of security sold (\( Q_{AS}^{-} \)).

Thus, the objective function can be stated as,

\[ \text{Max } Z = P_{1} Q_{TR}^{+} + P_{2} Q_{TS}^{+} + P_{3} Q_{SS}^{-} + P_{4} Q_{TB}^{-} + P_{5} Q_{AB}^{-} + P_{6} Q_{AS}^{-} \]
INTRODUCTION OF RISK INTO THE MODEL

The values of the parameters of the model are unfortunately not known with certainty. Many of the parameters will result from forecasts. Most frequently the values used will be the expected values of the parameters.

Risk can be added to the model by placing the model in a chance constraint format or by simulation. Simulation can be accomplished by using various parameter values and resolving the model numerous times. Risk can also be considered indirectly by traditional sensitivity analysis.

CONCLUSION

In this paper, the hospital cash management issues were discussed and the cash management problem has been formulated as a goal programming problem. This formulation will allow hospital administrators to integrate multiple goals into the solution of the cash management problem.

The model presented here can be extended to include chance constrained goal constraints.

A future paper can provide a goal programming formulation for the problems discussed in Calman [2] and Pogue, Fawcett, and Bussard [8] which will further analyze the integration of lock boxes and other services, into cash management.

REFERENCES


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