A GOAL PROGRAMMING MODEL FOR THE CASH MANAGEMENT PROBLEM

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ABSTRACT

Most of the models developed for the cash management problem have focused upon the optimization of a single objective. This approach is unnecessarily limiting. These models ignore multiple objectives of business and additional concerns such as organizational values and environmental constraints. These additional concerns can greatly influence the actual decision process.

This paper presents a goal programming formulation for a cash management problem in which multiple goals are considered during the solution process. Priorities are given to each goal so that a hierarchical goal structure is included in this optimization.

THE CASH MANAGEMENT PROBLEM

The daily operations of an organization are highly dependent on the liquid asset: cash. Business firms hold large quantities of cash. As a result, efficient management of cash is highly beneficial to virtually any business. As noted in Orgler[4] "the need for cash arises from the lack of synchronization between cash inflows and outflows and the difficulty of accurately predicting some of these flows. Consequently, an adequate amount of cash should be maintained to perform regular transactions and to meet unexpected requirements".

Occasionally a shortage of cash can develop, in which case, the organization can borrow from an open line of credit or the organization can sell marketable securities. However, a continual shortage of cash may also lead to a decreasing credit rating, high interest borrowing, or worse, insolvency. As a result, it is important that an organization keep a large enough cash balance to meet these requirements. It is also important to not keep too much cash on hand. Idle cash can be used to pay off debts or it can be invested in income producing assets.

A major effort in formulating the cash management problem, as a linear programming model, was made in Orgler [4].

GOAL PROGRAMMING

Goal Programming is an extension of mathematical programming that enables the user to develop models that "satisfice". It is often an attempt by the user to extend linear programming models to include more realistic multiple objectives and constraints. Further, a goal programming model can have an objective function that is composed of nonhomogeneous units of measure. An example of this technique is available in Burbridge, Koch, and Lawrence [1]. General surveys of goal programming are available in Charnes and Cooper [3].

A LINEAR PROGRAMMING MODEL FOR THE CASH MANAGEMENT PROBLEM: SUBSCRIPT, VARIABLE AND PARAMETER DEFINITION

In [4 , pp. 99-100], Orgler presents a summary of a linear programming model of the cash management problem. For completeness that model is summarized. For more detail, the reader is referred to the original model formulation.

Throughout the paper, the following subscripts will be used:

- g Indicator of the period in which purchases or borrowing occurs.
- h Indicator of the different types of payments and sources of short terms funds.
- i Maturity period indicator.
- j Time period indicator.
- m Indicator of borrowing source.

The following variables will be used in the model:

x_{hgj} The amount which is scheduled to be paid at time j for a purchase made in day g, of the type h.

w_{hg} The amount borrowed in period g of type h.

z_{ij} The amount of security sales in time j maturing in time i.

b_{jm} The cash balance in time j at source m.

y_{it} The amount of investment in securities in time y maturing in time $i \geq = t+1$.

The following paramaters are used in the model:

C_{hgj} Net revenue coefficient from payment x_{hgj}.

D_{ij} Net revenue coefficient from investments in securities y_{ij}.

E_{ij} Cost coefficient of security sales z_{ij}.

F_{hg} Cost coefficient of borrowing, w_{hg}.

u,v Number of different payment types of which the first s types apply to accounts payable, and the remaining u-s types correspond to repayments on loans borrowed within the framework of the model. Remaining types of loans not subject to early repayment are efined as u+1, ..., v.

T Time horizon of model.

L_{hg} Amount of liability which becomes outstanding in period g of type h.

a_{hgj} Technical coefficient of payment x_{hgj}, expressing the effect of timing on the payment size.

K_h Amount available for borrowing of type h.

e_{ij} Technical coefficient of security sales z_{ij}, calculated from the yield on the security since the yield and the price are directly related $e_{ij} = 1 + E_{ij}$.

S_i Amount of a security from the initial portfolio maturing in period i.

M_{jm} Minimum balance in time j at bank m.

M Total number of banks.

t_j Number of days in period j.

A Average daily minimum over T days.

B_o Cash balance at the beginning of the model's first period.

B_i Cash balance at the end of period i.

N_j Net amount of fixed cash flows in period j, i.e., other receipts less other payments.

D_{ij} Increase in the value of the investment y_{ij}.

B_x Upper limit on the balance of accounts payable.

k_h Prespecified time period that applies to payment type h.

THE LINEAR PROGRAMMING MODEL

The model presented in Orgler [4, pp. 99-100] makes two assumptions, a finite horizon and the daily periods are aggregated into longer unequal intervals. The model can be expressed in the following manner.

A. OBJECTIVE FUNCTION

$$(1)\quad \text{Max } Z = \sum_{j=G}^{T} \sum_{g=-k_h+1}^{T} \sum^{u} C_{h,g,j} x_{h,g,j} + \sum_{i=j+1}^{T+1} \sum_{j=1}^{T} (D_{i,j} y_{i,j} - E_{i,j} z_{i,j}) - \sum_{h=s+1}^{v} \sum_{g=1}^{T} F_{h,g} w_{h,g}$$

where G=1 for $g \leq -1$ and G=g for $G \geq 1$.

B. CONSTRAINTS

a. Payments

$$\sum_{j=g}^{g+k_h} a_{h,g,j} x_{h,g,j} = L_{h,g} \quad \text{for } g = -k_h + 1, \ldots, T - k_h \text{ and } h = 1, \ldots, s.$$

$$\sum_{j=g}^{T} a_{h,g,j} x_{h,g,j} \leq L_{h,g} \quad \text{for } g = T - k_h + 1, \ldots, T \text{ and } h = 1, \ldots, s.$$

$$(4)\quad \sum_{j=g}^{T} a_{h,g,j} x_{h,g,j} - w_{h,g} \leq 0 \quad \text{for } g = 1, \ldots, T \text{ and } h = s + 1, \ldots, u.$$

b. Financing

$$(5)\quad \sum_{g=1}^{T} w_{h,g} \leq K_h \quad \text{for } h = s + 1, \ldots, v.$$

c. Security Sales

$$(6)\quad \sum_{j=1}^{i-1} e_{i,j} z_{i,j} \leq S_i \quad \text{for } i = 2. \ldots, T + 1.$$

d. Minimal Cash Balance
Absolute minimum:

$$(7)\quad b_{j,m} \geq M_{j,m} \quad \text{for } j = 1, \ldots, T \text{ and } m = 1, \ldots, M.$$

Average daily minimum:

$$(8)\quad \sum_{j=1}^{T} b_{j,m} t_j \geq A \sum_{j=1}^{T} t_j.$$

e. Cash Flows
First period:

$$(9)\quad \sum_{g=-k_h+1}^{1} \sum_{h=1}^{u} x_{h,g,1} + \sum_{i=z}^{T+1} (y_{i,1} - z_{i,1}) - \sum_{L=s+1}^{v} w_{h,1} + b_1 = B_o + N_1 + S_1.$$

All other periods:

$$(10)\quad \sum_{g=-k_h+t}^{t} \sum_{h=1}^{u} x_{h,g,t} + \sum_{i=t+1}^{T+1} (y_{i,t} - z_{i,t}) - \sum_{h=s+t}^{v} w_{h,t} - \sum_{j=1}^{t-1} (d_{t,j} y_{t,j} - e_{t,j} z_{t,j}) - b_{t-1} + b_t = N_t + S_t \quad \text{for } t = 2, \ldots, T.$$

f. Termination
Accounts Payable

$$(11)\quad -\sum_{j=G}^{T} \sum_{g=T-k_h+1}^{T} \sum_{h=1}^{p} a_{h,g,j} x_{h,g,j} \leq B_x - \sum_{g=T-k_h+1}^{T} \sum_{h=1}^{p} L_{h,g}.$$

g. Nonnegitivity Constraints

$$(12)\quad x_{h,g,j} \geq 0 \quad \text{for all h,g, and j}$$

$$y_{i,j} \geq 0 \quad \text{for all i and j}$$

$y_{i,j} \geq 0$ for all i and j

$z_{i,j} \geq 0$ for all i and j

$w_{h,g} \geq 0$ for all h and g

Constraint (4) is based on the assumption that loans borrowed within the scope of the model are due beyond the horizon, i.e., $g+k_h \geq T$. If certain type of loan have to be paid within the horizon, i.e., $g+k_h \leq T$, the inequality sign of the constraint simply changes to an equality and $j=g, \ldots, g+k_h$ for these types.

EXPLAINATION OF THE LINEAR PROGRAMMING MODEL

The objective function (1) is a sum of the revenues from payments and investments in securities and the costs of security sales and borrowing over all periods in the model.

The constraints can be viewed as follows:

a) Payments
The equations (2) and (3), refer to the constraints required to pay the accounts payable. The constraint (4) ensures the payments of the loans borrowed within the model.

b) Financing
The constraint reflects limitations on each type of borrowing.

c) Security Sales
Sales of the securities are limited to those available in the portfolio.

d) Minimal Cash Balance
Constraint (7) reflects the minimum balances desired and constraint (8) reflects the average daily minimum balance.

e) Cash Flows
Constraints (9) and (10) equate the net cash flows and the changes in the cash balance.

f) Termination Constraints
Termination constraints are required in the last period of the model as an adjustment for its truncation at the horizon and to avoid excessive buildups in current liabilities and depletion of certain assets at the end of the planning period.

GOAL PROGRAMMING MODEL: THE CONSTRAINTS

This goal programming model of the cash management problem has been developed to optimize for more than one goal. This requires the establishment of a heirarchy of importance between these potenttially incompatable goals. In this paper, it is assumed that management is able to elicit an ordinal ranking of the goals or priorities, in terms of their desirability. This ordinal ranking results in their preference level. Each of these priorities is reflected in a constraint or set of constraints.

In this problem the following priorities were considered (by preference level):

1. Maximize the total revenues from the cash.
2. Maximize the satisfaction of the banking sources.
3. Minimize the costs of security sales.
4. Minimize the costs of borrowing.
5. Minimize the total amount of borrowing.
6. Minimize the total amount of security sales.

The first of the goal constraints is concerned with the maximization of total revenues from cash. The constraint is of the following form:

$$(13) \quad \sum_{j=G}^{T} \sum_{g=-k_h+1}^{T} \sum_{h=1}^{u} C_{h,g,j} x_{h,g,j} + \sum_{i=j+1}^{T+1} \sum_{j=1}^{T} D_{i,j} y_{i,j} + Q^-_{TR} - Q^+_{TR} = TR,$$

where,

TR is the total revenues budgeted,

Q^-_{TR} is the under-attainment of the budget level of total revenues, and

Q^+_{TR} is the over-attainment of the budget level of total revenues.

The second set of goal constaints is concerned with the maximization of the satisfaction of the banking sources. This goal takes the following form for each bank, and for each j:

$$(14) \quad b_{jm} + Q^-_{TS_{jm}} - Q^+_{TS_{jm}} = TS_{jm}$$

where,

TS_{jm} is the level desired in bank m at time j,

$Q^-_{TS_{jm}}$ is the under-attainment of the minimum balance at bank m at time j and

$Q^+_{TS_{jm}}$ is the over-attainment of the minimum balance at bank m, at time j.

The third type of goal constraint is concerned with the minimization of the net cost of security sales. This constraint has the following form:

$$(15) \quad \sum_{i=j+1}^{T-1} \sum_{j=1}^{T} E_{i,j} z_{i,j} + Q^-_{SS} - Q^+_{SS} = SS$$

where,

SS is the budgeted net cost of security sales,

Q^-_{SS} is the under-attainment of the budget level of the net cost of security sales and

Q^+_{SS} is the over-attainment of the budget of the cost of security sales.

The fourth type of constraint is concerned with the minimization of the cost of borrowing. This constraint has the following form:

$$(16) \quad \sum_{h=s+1}^{v} \sum_{g=1}^{T} F_{hg} w_{hg} + Q^-_{TB} - Q^+_{TB} = TB$$

where,

TB is total borrowing cost budgeted,

Q^-_{TB} is the under-attainment of the budget level of total borrowing cost and

Q^+_{TB} is the over-attainment of the budget level of total borrowing.

The fifth set of goal constraints is concerned with minimizing the total amount of borrowing. This goal takes the following form:

$$(17) \quad \sum_{h=s+1}^{v} \sum_{g=1}^{T} w_{hg} + Q^-_{AB} - Q^+_{AB} = AB$$

where,

AB is the amount budgeted for borrowing,

Q^-_{AB} is the under-attainment of the budgeted amount borrowed and

Q^+_{AB} is the over-attainment of the budgeted amount borrowed.

The sixth set of goal constraints considered here, is concerned with the minimization of the total amount of security sales. This constraint takes the following form:

$$(18) \quad \sum_{i=2}^{T+1} \sum_{j=1}^{i-1} z_{ij} + Q^-_{AS} - Q^+_{AS} = AS$$

AS is the total amount of budgeted security sales,

Q^-_{AS} is the under-attainment of budgeted security sales and

Q^+_{AS} is the over-attainment of budgeted security sales.

The final set of goal contraints that needs to be developed are the non-negativity constraints on the "deviational" variables. In particular,

$$(19) \quad Q^-_{TR} \geq 0, \; Q^-_{TS_{jm}} \geq 0, \; Q^-_{SS} \geq 0,$$

$$Q^+_{TR} \geq 0, \; Q^+_{TS_{jm}} \geq 0, \; Q^+_{SS} \geq 0,$$

$$Q^-_{TB} \geq 0, \; Q^-_{AB} \geq 0, \; Q^-_{AS} \geq 0,$$

$$Q^+_{TB} \geq 0, \; Q^+_{AB} \geq 0, \; Q^+_{AS} \geq 0.$$

The total set of goal programming constraints for the cash management problem is a combination of the cash management model constraints and these goal programming constraints. In particular, the model consists of the cash management model constraints, (2), (3), (4), (8), (9), (10), (11), and (12) and the goal programming constraints (13)-(19).

GOAL PROGRAMMING MODEL: THE OBECTIVE FUNCTION

In order to achieve the priorities according to their stated level of importance, the deviations from the goal j will be ranked according to the preemptive priority factor, P_j for each goal j.

The preemptive priority factors for this model will take the following form:

P_1: Preemptive priority factor for the maximimzation of the over-attainment of budgeted total revenues (Q^+_{TR}).

P_2: Preemptive priority factor for the maximimzation of the over-attainment of the minimum balance at bank m, at time j ($Q^+_{TS_{jm}}$).

P_3: Preemptive priority factor for the maximinzation of the under-attainment of the budget level of the net cost of security sales (Q^-_{SS}).

P_4: Preemptive priority factor for the maximimization of the under-attainment of the budget level for total cost of borrowing (Q^-_{TB}).

P_5: Preemptive priority factor for the maximization of the under-attainment of the budget level for the total amount of borrowing (Q^-_{AB}).

P_6: Preemptive priority factor for the maximization of the under-attainment of the budget level for the number of securities sold (Q^-_{AS}).

Thus, the objective function can be stated as,

$$(20) \quad \text{Max } Z = P_1 Q^+_{TR} + P_2 \sum_{j=1}^{T} \sum_{m=1}^{M} Q^+_{TS_{jm}} + P_3 Q^-_{SS} + P_4 Q^-_{TB} + P_5 Q^-_{AB} + P_6 Q^-_{AS}$$

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