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Normative Target-Based Decision Making

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This paper relates normative expected-utility decision making to target-based decision making, and introduces a new quantity, the aspiration equivalent. We show that using the aspiration equivalent as a target provides a new method for choosing between lotteries that is consistent with expected-utility maximization. Furthermore, we show that the aspiration-equivalent target provides a win–win situation for executive–manager delegation. This result furnishes a new link between normative decision analysis and target-based decision making. Copyright © 2005 John Wiley & Sons, Ltd.

INTRODUCTION: THE PATH NOT TAKEN

The original version of Von Neumann and Morgenstern's utility theory assessed the utility values for arbitrary prizes or prospects using a probabilistic indifference argument and therefore assigned utility values that ranged from zero to one (Von Neumann and Morgenstern, 1944). Using Von Neumann and Morgenstern's interpretation of utility values, the expected utility of a lottery has an intuitive meaning. Any complex lottery can be reduced into an equivalent lottery with only two outcomes, the best and worst. The expected utility of a lottery is the probability of getting the best prospect in the equivalent two-outcome lottery.

When the Von Neumann and Morgenstern utility was applied to lotteries with monetary outcomes, the notion of the certain (or certainty) equivalent was introduced as the utility inverse of the expected utility. The certain equivalent is the amount of money for sure a decision maker would regard as indifferent to receiving the uncertain lottery. Maximizing the certain equivalent yielded new insights into the decision situations, such as the decision maker's willingness to pay to substitute one decision alternative for another and the calculation of the values of information and control.

For our discussion, it is useful to think of the certain equivalent of a lottery as replacing the cumulative distribution of the lottery with a step cumulative distribution that jumps from zero to one at some specific value of x. When this value is set so that the expected utility of the step distribution function is equal to the expected utility of the original lottery, we call this value the certain equivalent and designate it as $\tilde{x}$. Over a half-century has passed and many important applications and results have been made along this path.

There is, however, a parallel path of development that has not been explored. Assuming that the original utility function is normalized to range from zero to one (as is the case with Von Neumann and Morgenstern's initial suggestion), we can replace the utility function by a step utility function, and then set the value where the step occurs so that the expected utility with the new step utility function is equal to the original

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expected utility. We call this new value the aspiration equivalent and designate it as $\hat{x}$. Traditionally, the step utility function defines an aspiration level (in this case it is the aspiration equivalent), which divides the range of outcomes into two intervals: satisfactory if the outcome is above the aspiration equivalent and unsatisfactory if the outcome is below it. We will show that the expected utility of a lottery is equal to the probability that the outcome of the lottery exceeds the aspiration equivalent.

Thus calculating the aspiration equivalent for each alternative and selecting the alternative with the highest probability of exceeding its aspiration equivalent provides a new method for choosing between lotteries that is equivalent to maximizing the expected utility. We also provide a functional relation between the certain equivalent and the aspiration equivalent.

The idea of an aspiration utility function relates naturally to target-based settings where satisfaction is measured by exceeding a target or goal. Target-based approaches produce an incentive for a manager to choose the alternative that maximizes the probability of meeting his or her target. This chosen alternative is often different from the one that maximizes the corporation's expected utility. We will show that when using the aspiration equivalent of each alternative as a target, a manager that maximizes the probability of meeting his target does indeed maximize the corporation's expected utility. This result provides a win–win situation for executive–manager delegation and furnishes a link between expected-utility decision making and target-based decision making.

REVIEW OF EXPECTED-UTILITY DECISION MAKING AND TARGET-BASED INCENTIVES

Review of Expected-Utility Decision Making

If a decision maker follows the axioms of decision analysis, then any complex lottery can be reduced to an equivalent lottery with only two outcomes, the best and worst. Furthermore, the probability of getting the best outcome in this equivalent lottery is equal to the expected utility of the lottery. This standard derivation is illustrated graphically in Figure 1 below.

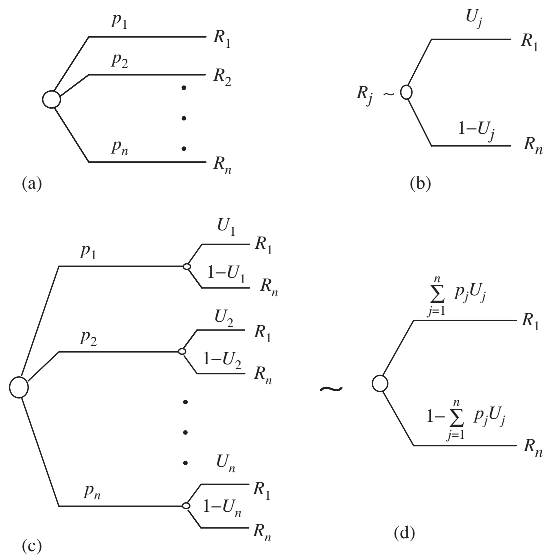

Figure 1. Interpretation of expected utility of a lottery as the probability of the best outcome of an equivalent lottery containing only two outcomes; the best and the worst.

Figure 1(a) shows a discrete lottery with prospects $R_1, \ldots, R_n$ and corresponding probabilities $p_1, \ldots, p_n$. For convenience, let us assume that R_1 is the best prospect and that R_n is the worst. Figure 1(b) shows an example of a Von Neumann and Morgenstern utility assessment, U_j, assigned to prospect R_j in terms of the best and worst prospects. Figure 1(c) shows the substitutions made for each of the prospects in terms of their Von Neumann and Morgenstern utilities, and Figure 1(d) shows the equivalent lottery after multiplying the probabilities in Figure 1(a) by the Von Neumann and Morgenstern utilities. Note that the probability of the best prospect in Figure 1(d) is equal to the expected utility of the original lottery, $\sum_{j=1}^{n} p_j U_j$.

Figure 1(d) illustrates that any lottery can be reduced into an equivalent lottery with only two outcomes, the best and the worst, by using Von Neumann and Morgenstern utility assessments and the appropriate substitutions. Furthermore, the expected utility of a lottery is equal to the probability of attaining the best outcome in the equivalent lottery. Normative expected-utility decision making now suggests choosing the lottery that has the highest expected utility.

One common form of utility functions that is used often in the literature, and we will use later in this paper, is the exponential utility function,

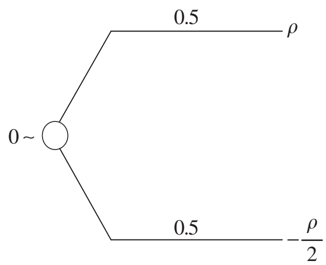

Figure 2. Risk tolerance assessment for an exponential utility function.

which can be normalized to range from zero to one on an interval $[a,b]$ as

$$U(x) = \frac{e^{-\gamma a} - e^{-\gamma x}}{e^{-\gamma a} - e^{-\gamma b}}, \quad a \leqslant x \leqslant b, \tag{1}$$

where γ is known as the risk-aversion coefficient (Pratt, 1964; Arrow, 1965). The reciprocal of the risk-aversion coefficient is known as the risk tolerance, ρ. When the risk-aversion coefficient goes to zero (risk tolerance $\rightarrow \infty$), the decision maker values deals at their expected value and is said to be risk neutral . In general, the value of the risk tolerance needs to be assessed from the decision maker in order to calculate his/her expected utility for each alternative.

For an exponential utility function, it suffices to assess the decision maker's certain equivalent for any single uncertain lottery in order to determine his risk attitude. An approximate value for the risk tolerance is equal to the value of ρ (Figure 2 above) that makes the decision maker indifferent between doing nothing or playing a binary lottery where he receives \$$\rho$ with a probability 0.5 and loses \$$(\rho/2)$ with probability 0.5.

Having discussed normative approaches to decision making and the steps needed to determine the optimal decision alternative, let us now consider a typical target setting situation and discuss the implications an organization's incentive structure has on the decision making process.

Target Setting in Organizations: A Motivating Example

In corporations, executives sometimes ask managers to meet fixed targets. A behavioral rationale for setting targets is a translation from the executive world of strategic decision making to an operational world of managing with targets, using tools such as management by objectives (MBO) and balanced scorecards.

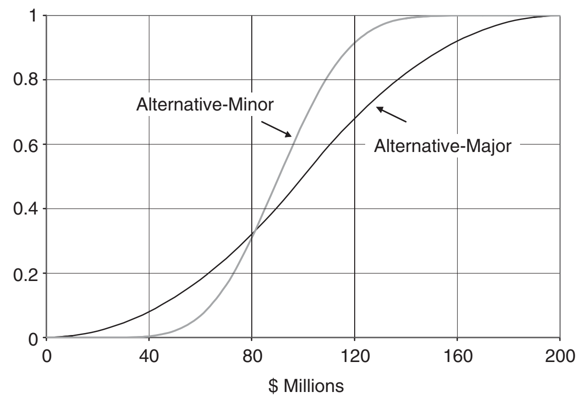

Figure 3. Two design alternatives: Major upgrades and Minor upgrades.

Let us consider a typical executive–manager delegation problem, where the manager is rewarded only by meeting (or exceeding) his target. For example, the manager may receive a bonus or get promoted if the performance of his project exceeds a given fixed target, and this target is set without consideration given to the lotteries that the manager is facing. This incentive scheme leads the manager to give priority to the actions that maximize the probability of meeting his given target and, as a consequence, may lead the manager to make choices that do not maximize the executive's expected utility.

In order to demonstrate this further, let us consider the following example, where a project manager of an automobile manufacturing company is facing two design alternatives: (1) 'Major'—designing a new innovative vehicle with major upgrades or (2) 'Minor'—performing minor upgrades to the previous year's vehicle. In this example, we assume the Major alternative has a triangular probability density function that is symmetric on the domain [\$0, \$200M] and the Minor alternative has a probability density function given by a scaled Beta(10,12) on the interval [\$0, \$200M]. Figure 3 shows the cumulative distributions for the two design alternatives.

The Major alternative has the higher expected value of the two alternatives, which is equal to \$100M (by symmetry), while the Minor alternative has an expected value equal to the mean of the

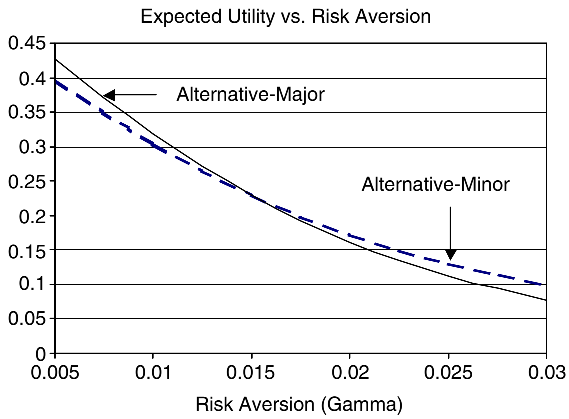

Figure 4. Sensitivity of expected utility with the risk-aversion coefficient.

scaled Beta(10,12) distribution given by $[10/(10+12)]200 = \$90.9M$. As a result, a risk-neutral organization (or decision maker) would choose the Major design alternative because it has the highest expected value.

As the risk-aversion increases, however, the Minor design alternative becomes optimal. Figure 4 shows the normalized expected utility of each design alternative for different values of the risk-aversion coefficient, γ, assuming the company has an exponential utility function.

Now, if the project manager were given a fixed target by the organization, this incentive scheme would lead him to choose the project that maximizes his probability of meeting this target and achieving success regardless of his own risk-aversion coefficient or that of the organization. To illustrate this point, observe that Figure 3 shows the Major alternative stochastically dominates the Minor alternative in the region above $82 million, while the Minor alternative stochastically dominates in the region below $82 million. Thus if the manager were given a fixed target and if the target were set at any value below the cross-over point of $82M, he would choose Minor (as it gives him a higher probability of meeting his target), and if the target were set higher than $82M he would choose Major.

This situation highlights a typical scenario in practice, where a target set by an organization may not be aligned with the expected-utility decision making process. This example also illustrates how a fixed target that is set independently of the lotteries may induce the manager to choose sub-optimal alternatives, which do not maximize the company's expected utility. Therefore, in a normative scheme we should set the target in a way that depends on the lottery or project that is under consideration.

We now ask the question 'Is there an analytical procedure for setting targets in organizations that is compatible with expected-utility maximization?' In order for this procedure to be normatively optimal, such targets must depend on the alternative (lottery) being considered as well as the organization's utility function to incorporate their risk-aversion. In this paper, we will explore how targets can be set to motivate the manager to maximize the organization's expected utility while simultaneously maximizing the probability of meeting his own target. In our work, we will assume that both the executive and the manager agree on the probability distributions for each alternative, or equivalently, that the executive trusts the manager's probability assessments.

THE ASPIRATION EQUIVALENT

Assuming a continuous and increasing utility function, it is common to define the certain equivalent by replacing a given cumulative probability function with a step cumulative probability function yielding the same expected utility. The certain equivalent is the point at which this step occurs. In general, the value of the certain equivalent depends on both the utility function and probability distribution that have been specified.

Assuming a continuous and increasing cumulative probability function, we define a new quantity, the aspiration equivalent, by replacing the utility function with a step utility function yielding the same expected utility. The aspiration equivalent is the point at which this step occurs. Like the certain equivalent, in general the value of the aspiration equivalent depends on both the utility function and probability distribution that have been specified. A step utility function is shown in Figure 5, and is known as an aspiration utility function.

The aspiration utility function appears often in the literature of descriptive and behavioral decision making. The point at which its step occurs is traditionally known as the aspiration level. Simon

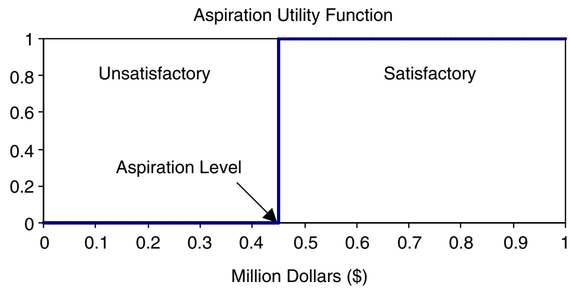

Figure 5. Aspiration utility function.

(1955) defined the aspiration level as a 'satisfactory alternative' and suggested that individuals could simplify decision problems by having binary goals: satisfactory if the outcome is above the aspiration level or unsatisfactory if it is below it.

> However, there are certain dynamic considerations, having a good psychological foundation that we should introduce at this point. Let us consider, instead of a single static choice situation, a sequence of such situations. The *aspiration level*, which defines a satisfactory alternative, may change from point to point in this sequence of trials. [Italics Simon's]

Simon suggested that the aspiration level could change with changing 'choice situations' or with different lotteries faced by the individual. In the following discussion, we develop a mathematical formulation to define a special aspiration level, which we define as the aspiration equivalent, and show how to adapt it based on changes in the lottery that the individual is facing.

Calculating the Aspiration Equivalent

The expected utility of a continuous lottery is defined as

$$U(\tilde{x}) \triangleq \text{expected utility} = \int_{-\infty}^{\infty} U(x)\,\mathrm{d}F(x) = \int_{-\infty}^{\infty} U(x)f(x)\,\mathrm{d}x, \quad (2)$$

where $F(x)$ is the cumulative probability function of the lottery, $f(x)$ is the probability density function, $U(x)$ is the decision-maker's utility function that is normalized to range from zero to one, and $\tilde{x}$ is the certain equivalent of the lottery, which is unique if $U(x)$ is a continuous increasing function at $\tilde{x}$. From (2) we can write an expression for the certain equivalent as

$$\tilde{x} = U^{-1}\left(\int_{-\infty}^{\infty} U(x)\,\mathrm{d}F(x)\right) = U^{-1}\left(\int_{-\infty}^{\infty} U(x)f(x)\,\mathrm{d}x\right). \quad (3)$$

Using the rule of integration by parts we can express the expected utility as

$$\begin{aligned}\text{Expected utility} &= \int_{-\infty}^{\infty} U(x)\,\mathrm{d}F(x)\\ &= U(x)F(x)|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} F(x)\,\mathrm{d}U(x)\\ &= 1 - \int_{-\infty}^{\infty} F(x)\,\mathrm{d}U(x)\\ &\triangleq 1 - \text{expected disutility.} \quad (4)\end{aligned}$$

We define the last term on the right-hand side as the expected disutility, and mathematically define the aspiration equivalent, $\hat{x}$, in terms of the expected disutility by

$$F(\hat{x}) \triangleq \int_{-\infty}^{\infty} F(x)\,\mathrm{d}U(x) = \int_{-\infty}^{\infty} F(x)u(x)\,\mathrm{d}x, \quad (5)$$

where $u(x)$ is the decision-maker's utility density function. A utility density function, $u(x)$, is defined as the derivative of a normalized utility function, $U(x)$, (Abbas, 2002). Note that a utility density function is non-negative (due to the non-decreasing property of utility functions) and integrates to unity. The utility density function is thus a parallel to the probability density function and simplifies many mathematical derivations.

The integrals in Equation (5) are similar to those of the expected utility of Equation (2), except that the roles of F and U reversed. The aspiration equivalent is unique if $F(x)$ is a continuous increasing function at $\hat{x}$. From (5), we can write an expression for the aspiration equivalent as

$$\hat{x} \triangleq F^{-1}\left(\int_{-\infty}^{\infty} F(x)\,\mathrm{d}U(x)\right) = F^{-1}\left(\int_{-\infty}^{\infty} F(x)u(x)\,\mathrm{d}x\right). \quad (6)$$

Combining Equations (2), (4) and (5), for any normalized utility function we have

Expected utility + expected disutility = 1,

$$U(\tilde{x}) + F(\hat{x}) = 1. \quad (7)$$

Equation (7) presents a fundamental identity that relates the certain equivalent and aspiration equivalent, and (as we shall see) provides several new interpretations. Recall that the expected utility of a lottery is the probability of getting the best prospect in the equivalent two-outcome lottery (Figure 1). From Equation (7) we note that the sum of expected utility and expected disutility is equal to one. Therefore, the expected disutility of Equation (5) is the probability of getting the worst prospect in that same equivalent two-outcome lottery. The problem of choosing the lottery that has the highest expected utility is thus equivalent to the problem of choosing the lottery that has the lowest expected disutility. While much of the literature has focused on the interpretation of the expected utility in terms of the probability of the best vs. worst outcome in an equivalent two-outcome lottery, we now focus on another interpretation of the expected utility in terms of the newly defined aspiration equivalent and the decision-maker's lottery. Let us first re-arrange Equation (7) as follows:

$$\text{Expected utility} = U(\tilde{x}) = 1 - F(\hat{x}) = G(\hat{x}), \qquad (8)$$

where $G(x) \triangleq 1 - F(x)$ is the excess distribution function.

From Equation (8) we have a new interpretation for the expected utility of a lottery in terms of the aspiration equivalent. The expected utility of a lottery is the probability that the outcome of the lottery exceeds its aspiration equivalent.

This result provides us with a new target-based method for choosing between lotteries; we choose the lottery that has the highest probability of meeting its aspiration equivalent.

The aspiration equivalent is the point that divides the x-axis of the cumulative probability distribution into two portions, the probability of the portion to the right is numerically equal to the expected utility of the lottery, and the probability of the portion to the left is the expected disutility. This is shown in Figure 6.

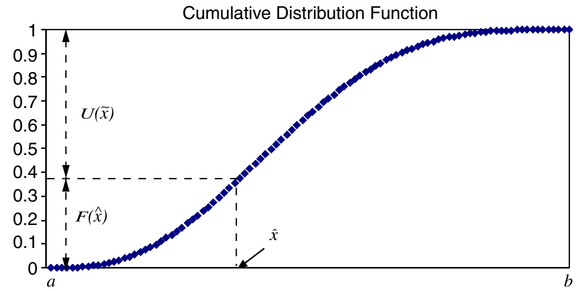

Figure 6. Interpreting expected utility as the probability of exceeding the aspiration equivalent.

The Aspiration Equivalent as a Target

To use the aspiration equivalent concept as a target in organizations, a separate aspiration equivalent would be calculated for each alternative and assigned as a target if this alternative is choosen. If the manager is rewarded by exceeding his target, then he will pick the alternative with the highest probability of meeting its target. Because the target is the aspiration equivalent, the manager's chosen alternative will thus correspond to the alternative that maximizes the company's expected utility.

We note here that setting different targets for different alternatives (or projects) in an organization does in fact make intuitive sense. The rationale is that different alternatives (or projects) generally yield different returns. Consider for example an oil company that has to choose between drilling oil wells in previously developed fields and drilling new exploratory wells. Each drilling alternative will have different probabilities of achieving a various profit levels and, as a consequence it should have a different target. To the manager, the actual amount of personal reward for succeeding would be the same in each case, only the probability of achieving the reward would vary with the chosen alternative. Since the manager's probability of achieving the reward is also the executive's expected utility, choosing the alternative that maximizes the manager's probability of success also maximizes the executive's expected utility.

Using the aspiration equivalent as a target in delegation thus provides a win–win situation for both executives and managers. It achieves the highest expected utility for the executive and simultaneously the highest probability of exceeding the target for the manager. Furthermore, we shall show that if the manager discovers a lottery which has a higher expected utility than any of his current projects, then it is in his best interest to bring it forward to the executive, since it will result in a target for which he has a higher probability of exceeding.

All we need for the manager to choose this way is that the incentives be aligned with this scheme. If the manager's reward for achieving the target is sufficiently high, compared to other economic or social incentives related to his choice, then he has the incentive to choose the alternative with the highest probability of meeting its target. Note that this choice is independent of the manager's own risk attitude because the amount of his reward for succeeding is constant across alternatives.

Numerical Example for Aspiration Equivalent Calculation

Now we go back to the automobile manufacturer example and show a direct numerical calculation of the aspiration equivalent and the certain equivalent for the 'Major' upgrade design alternative. We consider a decision maker with an exponential utility function having a risk-aversion coefficient of $\gamma = 3 \times 10^{-8}$ over the domain [\$0, \$200M], which has a symmetric, triangular probability density function over the same domain, as shown in Figure 7 below.

The equation for the normalized utility function is given as

$$U(x) = \frac{1 - e^{-\gamma x}}{1 - e^{-200\gamma}}, \quad 0 \leqslant x \leqslant 200. \tag{9}$$

By direct integration, we can calculate the certain equivalent and aspiration equivalent of the decision maker for the given lottery and given risk-aversion coefficient.

$$\begin{aligned}\text{Expected utility} &= \int_0^{100} \left(\frac{x}{100}\right) \frac{1 - e^{-\gamma x}}{1 - e^{-200\gamma}}\, dx \\ &\quad + \int_{100}^{200} \frac{(200 - x)}{100} \frac{1 - e^{-\gamma x}}{1 - e^{-200\gamma}}\, dx \\ &= 0.899. \end{aligned} \tag{10}$$

In Figure 8, we have superimposed the excess distribution function, $G(x)$, and the utility function, $U(x)$. If we draw a horizontal line at the value of the expected utility, it intersects $G(x)$ at the aspiration equivalent and $U(x)$ at the certain equivalent. This simple construction shows graphically how these two quantities are related.

In the following example we will discuss the sensitivity of the aspiration equivalent to the risk-aversion coefficient.

Example—sensitivity to risk-aversion coefficient. Now we discuss the variation of both certain equivalent and aspiration equivalent with the risk attitude of the decision maker. Using an exponential utility function, Figure 9 shows a sensitivity analysis of both the certain equivalent and the aspiration equivalent to the risk-aversion coefficient, γ, using the triangular lottery of the previous example.

Observe from Figure 9 that for the Major lottery both the aspiration equivalent and the certain equivalent decrease with increasing risk-aversion (as γ increases). To generalize this result for other lotteries, we now explore the asymptotic behavior of the aspiration equivalent using an exponential utility density function over a domain $[a,b]$, which can be written as

$$u(x) = \frac{\gamma e^{-\gamma x}}{e^{-\gamma a} - e^{-\gamma b}}. \tag{11}$$

Using Equations (5) and (11), for a given lottery, $F(x)$, and aspiration equivalent, $\hat{x}$, we have

$$\begin{aligned} F(\hat{x}) &\triangleq \int_a^b u(x) F(x)\, dx \\ &= \int_a^b \frac{\gamma e^{-\gamma x}}{e^{-\gamma a} - e^{-\gamma b}} F(x)\, dx. \end{aligned} \tag{12}$$

As $\gamma \to +\infty$,

$$u(x) = \frac{\gamma e^{-\gamma x}}{e^{-\gamma a} - e^{-\gamma b}} \to \delta(x - a), \tag{13}$$

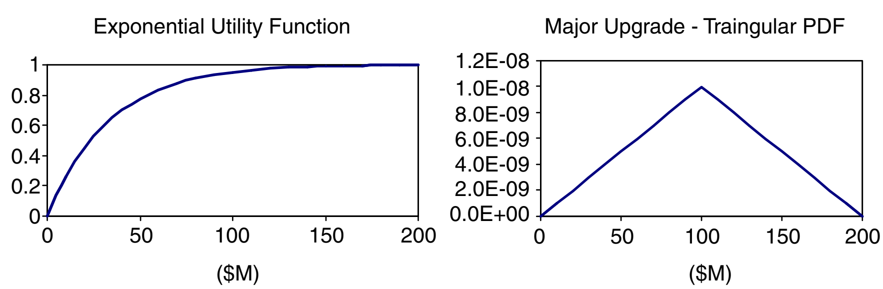

Figure 7. Exponential utility and triangular probability for Major upgrade alternative.

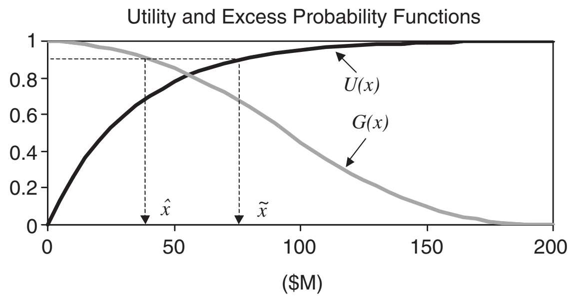

Figure 8. Certain equivalent and aspiration equivalent calculation.

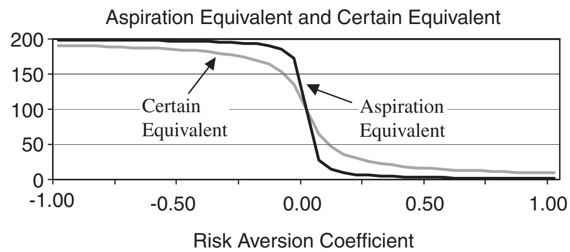

Figure 9. Sensitivity analysis of the certain equivalent and the aspiration equivalent to the risk-aversion coefficient.

where $\delta(x-a)$ is an impulse function at $x=a$, thus

$$\lim_{\gamma\to\infty} F(\hat{x}) = \int_a^b \delta(x-a)F(x)\,\mathrm{d}x = F(a) = 0 \Rightarrow \hat{x} \to a. \quad (14)$$

Similarly as $\gamma \to -\infty$,

$$u(x) = \frac{\gamma \mathrm{e}^{-\gamma x}}{\mathrm{e}^{-\gamma a} - \mathrm{e}^{-\gamma b}} \to \delta(x-b), \quad (15)$$

where $\delta(x-b)$ is an impulse function at $x=b$, and

$$\lim_{\gamma\to-\infty} F(\hat{x}) = \int_a^b \delta(x-b)F(x)\,\mathrm{d}x = F(b) = 1 \Rightarrow \hat{x} \to b. \quad (16)$$

From the previous results, we can see that if the decision maker is facing a lottery and her aspiration equivalent is set close to the lower bound, she is effectively acting with risk-averse behavior. On the other hand, if a decision maker is facing a lottery and her aspiration equivalent is set close to the upper bound of the lottery, she is effectively acting with risk-seeking behavior. This result is in harmony with the discussion of shifts of reference points in Kahneman and Tversky's (1979) prospect theory:

> There are situations in which gains and losses are coded relative to an expectation or *aspiration level* that differs from the status quo. For example an unexpected tax withdrawal from a monthly paycheck is experienced as a loss, not as a reduced gain. Similarly, an entrepreneur who is weathering a slump with greater success than his competitors may interpret a small loss as a gain, relative to the larger loss he had reason to expect...a person who has not made peace with his losses is likely to accept gambles that would be unacceptable otherwise. The well known observation that the tendency to bet on long shots increases in the course of the betting day *provides some support for the hypothesis that a failure to adapt to losses or to attain an expected gain induces risk seeking*. [Italics Kahneman and Tversky's]

Kahneman and Tversky show that high aspirations, that are difficult to realize, induce risk-seeking behavior. In our formulation, setting high aspirations is synonymous with risk-seeking behavior.

We can see also from Figure 9 that when the risk-aversion coefficient, γ, approaches zero, both certain equivalent and aspiration equivalent converge to the expected value of the lottery (this is a general result for any symmetric lottery). To illustrate this result further, recall that when the risk-aversion coefficient is equal to zero, the decision maker is acting with a linear (risk neutral) utility function and thus has a certain equivalent equal to the expected value of the lottery. When the utility function is linear, its derivative, the utility density function, is uniform, and we have

$$F(\hat{x}) = \int_a^b u(x)F(x)\,\mathrm{d}x = \int_a^b \frac{1}{b-a} F(x)\,\mathrm{d}x = \frac{b-\bar{x}}{b-a}, \quad (17)$$

where $\bar{x}$ is the expected value of the lottery. If the lottery is symmetric on this interval, then $\bar{x} = \frac{1}{2}(a+b)$ and (17) reduces to

$$F(\hat{x}) = \tfrac{1}{2}. \quad (18)$$

Re-arranging (18), shows that for a symmetric lottery

$$\hat{x} = F^{-1}\left(\tfrac{1}{2}\right) = x_M = \bar{x}, \quad (19)$$

where x_M is the median of the lottery, which is also the mean because of symmetry. Thus both the certain equivalent and aspiration equivalent are equal to the expected value of a symmetric lottery when the decision maker is risk neutral.

From the results of Figure 9 we also see that, for a given lottery, there is a one-to-one correspondence between the aspiration equivalent and the risk-aversion coefficient that is used for its calculation. Therefore we can use this relationship in the inverse direction. For a given lottery and given target, there exists an exponential utility function that provides the same expected utility (and aspiration equivalent) thus determining a unique 'effective risk-aversion coefficient'. To derive this effective risk-aversion coefficient mathematically for any lottery and any aspiration equivalent, we refer to the expected disutility of Equation (5) and note

$$F(\hat{x}) = \int_a^b u(x)F(x)\,\mathrm{d}x = \int_a^b \frac{\gamma \mathrm{e}^{-\gamma x}}{\mathrm{e}^{-\gamma a} - \mathrm{e}^{-\gamma b}} F(x)\,\mathrm{d}x. \qquad (20)$$

Using Equation (20) defined on specified interval from a to b, we can now solve for the effective risk-aversion coefficient, $\gamma_{eff} = \gamma(F, \hat{x})$, for any given lottery, F, and aspiration equivalent, $\hat{x}$. This result allows us to derive risk-aversion coefficients from targets.

Setting Normative Targets

We now return again to the automobile example of Figure 3. We have shown the triangular distribution for the 'Major' alternative and the calculation of its aspiration equivalent. The distribution for the 'Minor' alternative is a scaled Beta(10,12). Table 1 shows comparisons of these two lotteries for organizations having various risk tolerances.

The third and forth rows in Table 1 show that a company with risk tolerance of \$66.5M is indifferent between the two alternatives, Major and Minor (as both have the same certain equivalent and the same probability of 0.77 for exceeding their aspiration-equivalent target). However, the aspiration-equivalent target is \$75.0M, for the Minor alternative and only \$67.7M for the Major alternative. This situation shows that even with indifference between the two alternatives, and equality of certain equivalents, the targets themselves may very depending on the shape of the distribution for the chosen alternative.

The last two rows of Table 1 show that as the company gets more risk averse (with a risk tolerance of \$30M) the aspiration-equivalent targets become easier to achieve for both alternatives. The probability of exceeding the aspiration equivalent of the Minor alternative is now 0.94 which is higher than that of the Major alternative of 0.91. Therefore the Minor alternative has a higher expected utility, even though its target is set higher. Choosing Minor maximizes the executive's expected utility (0.94) while simultaneously increasing the manager's probability of achieving his target.

On the other hand with a risk tolerance of \$500M (as shown in the first two rows of Table 1) the aspiration equivalent targets become more difficult to achieve for both alternatives. Now the probability of exceeding the aspiration equivalent of the Major alternative is 0.54 and is higher than that of the Minor alternative of 0.5. Therefore, the Major alternative now has the highest expected utility, and again its target is higher.

In each of these last two cases, the best alternative provides the executive with the highest expected utility while providing the manager with the highest probability of meeting his target. This

Table 1. Aspiration Equivalents for Different Values of Risk Tolerance

Risk tolerance (\$ M)	Alternative	Aspiration-equivalent target (\$ M)	Prob. of exceeding target (expected utility)	Certain equivalent (\$ M)
500	Major	95.7	0.54	98.3
500	Minor	90.5	0.50	90.5
66.5	Major	67.7	0.77	87.7
66.5	Minor	75.0	0.77	87.7
30	Major	40.6	0.91	74.5
30	Minor	58.8	0.94	83.9

target setting approach thus allows organizations the advantages of normative decision making while, at the same time, using their more preferred target-based approach.

Adjusting Aspirations to a New Forecast

The probability distributions for the projects that a company faces may change throughout the course of the projects. The company may then wish to revise its targets based on this updated situation. For example, a financial manager may be given a target to achieve a specific return on his portfolio, but the stock market drops making it more difficult to achieve his original target. Alternatively, there may be other situations where new opportunities are discovered that are best for the organization. In a normative target-based setting, the organization should adjust the target and, simultaneously, motivate managers to explore new opportunities using their new target structures. We now show that using the aspiration equivalent as a target provides a convenient and transparent method to update targets in organizations and also to motivate managers to report new opportunities if they discover them.

Let us refer back to the automobile manufacturer and assume the company has a risk tolerance of \$500M. From Table 1, we see that this corresponds to an aspiration equivalent for Major at \$95.7M and for Minor at \$90.5M. The manager chooses Major and has a probability of 0.54 of exceeding it.

Now suppose the project manager discovers new modification upgrades that will enhance the market share and profit for the Major alternative. He assesses the forecast for this new alternative, Major-Revised, as a scaled Beta(6,3). In this setting, the aspiration equivalent for the new forecast using the same risk tolerance of \$500M is \$118M. Direct calculations show that the manager will have a 0.7 chance of meeting this new target if he chooses Major-Revised (Figure 10). Thus if the manager reports the new alternative and works on it, he will have a new project and new target which he has a higher probability of exceeding (his probability of personal reward has increased by 0.16 from 0.54 to 0.7). At the same time, the company's expected utility increases by this new alternative. This example illustrates the win–win situation in aspiration-equivalent target setting because the Major-Revised alternative is better for the company and better for the project manager. Thus it is in the project manager's best interest to discover new alternatives that provide higher expected utility for the company, since they will result in new targets that he has a higher probability of exceeding.

The utility function should be defined and normalized for a domain broad enough to include

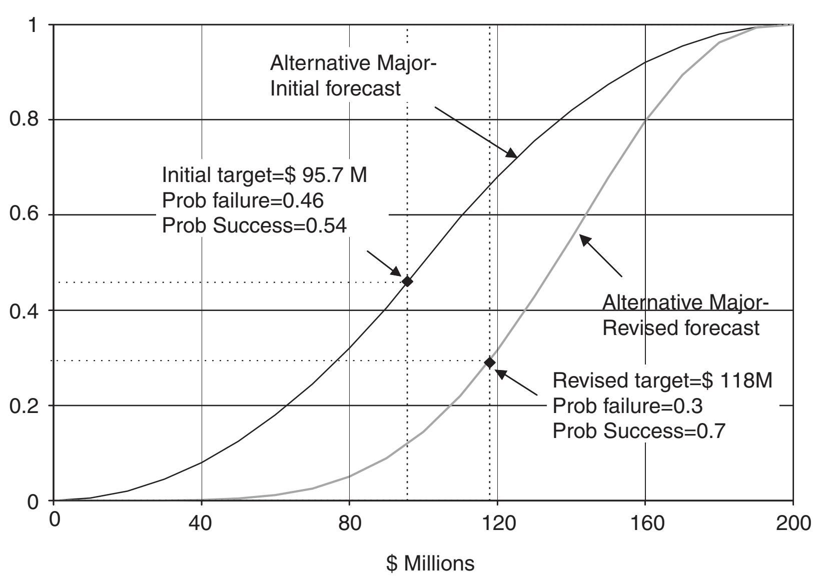

Figure 10. Adjusting aspirations in the face of a new forecast.

all lotteries under consideration, and updates of these lotteries if new information arrives, so that a consistent set of normative targets can be established in all instances. Expanding the domain may change the targets somewhat, but they will still remain a consistent, normatively correct set.

COMPARISON OF DIFFERENT INCENTIVE MECHANISMS

We have discussed how the aspiration equivalent can be used as a target, where the manager chooses the lottery that has the highest probability of meeting its own target. We have also shown how fixed targets that are set without consideration of the lotteries and the utility function may lead managers to choose sub-optimal alternatives. Let us now compare several possible choices of targets might be reasonably proposed: (a) fixed fractiles of each distribution, (b) the certain equivalents, (c) simulated random targets; and (d) the aspiration equivalents. Our comparison will be based on two main criteria for the target: (i) that it can be used to choose normatively between lotteries (and is thus consistent with expected-utility decision making); and (ii) that when used for delegation from an executive to a manager, who maximizes his probability of exceeding his target, it maximizes the executive's expected utility.

The first criterion implicitly requires the target to depend on the executive's utility function; otherwise it cannot be used to choose between lotteries. Fractiles of a lottery fail this basic criterion since they do not include information about the decision-maker's utility function.

The certain equivalent, on the other hand, incorporates information about the lottery and the decision-maker's utility function. Expected utility theory suggests that a decision maker should choose the lottery that maximizes his certain equivalent. Now we consider the use of the certain equivalent in target setting from an executive to a manager who maximizes his probability of meeting this target. We observe that the lottery with the highest probability of meeting its certain equivalent is necessarily the lottery that maximizes the executive's expected utility. There are many examples to illustrate this fact. We provide one simple example for a risk-neutral organization below.

Alternative 1—is a scaled Beta(3,2) distribution on the domain [\$0M, \$200M]. It has a certain equivalent equal to its mean of \$120M and a 52.5% chance of exceeding the certain-equivalent target.

Alternative 2—is a scaled Beta(120,40) distribution on the same domain. This alternative has a certain equivalent of \$150M but only a 51% chance of meeting the certain-equivalent target.

A risk-neutral organization would prefer the second alternative as it has a higher certain equivalent, while a manager who maximizes his probability of meeting his target would choose the first alternative as it gives him a higher probability of meeting his target. The certain equivalent target thus fails the second criterion in normative target-based settings.

Castagnoli and LiCalzi (1996) and Bordley and LiCalzi (2000) interpret a normalized utility function as a probability distribution of a hypothetical lottery that is independent of the lotteries being faced by the decision maker. Using this interesting interpretation, we might consider providing a simulated random target to a manager. The random target, T, is generated as a single sample from a random-number generator that uses the executive's utility function, $U(x)$, as a sampling probability distribution. The value of the target is revealed to the manager only after the results of his project are complete. With this target-based method, the probability of exceeding the simulated random target if the manager faces an independent lottery, $F(x)$, is equal to

$$\begin{aligned}\Pr(x > T) &= 1 - \int_{-\infty}^{\infty} F(T)\, \mathrm{d}U(T)\\ &= \int_{-\infty}^{\infty} U(x)\, \mathrm{d}F(x)\\ &= \text{expected utility.}\end{aligned} \tag{21}$$

Thus if the utility function is used as a sampling distribution to select a random target revealed only after the performance is known, the probability of exceeding this random target is equal to its expected utility. If there are several alternatives the same sampling distribution should be used in each case because it represents the common utility function. This method will induce the manager to follow the executive's utility function, but the target level of performance required to gain a personal reward remains uncertain until after execution of the project. It would likely be difficult to motivate managers who will not know their actual target until after the project is over.

Now we consider the aspiration equivalent as a target. The aspiration equivalent incorporates information about both the lottery and the decision-maker's utility function. From Equation (8), we see that it can be used to choose between lotteries if we select the lottery that has the highest probability of meeting its aspiration equivalent. Thus, if the organization delegates the aspiration equivalents as targets to the manager, and if the manager chooses the lottery that has the highest probability of meeting its own target, then he will also choose the lottery that has the highest expected utility for the organization. Furthermore, if the manager discovers a new lottery that is better for the organization, it is in his best interest to bring it forward to the executive, as it will result in a new target that the manager has a higher probability of exceeding. The manager will therefore choose the new lottery in the best interest of the organization and will have an incentive to identify new lotteries with a higher expected utility. This choice creates a win–win situation for both the executive and the manager. The aspiration equivalent thus satisfies the two criteria (target setting and choice among lotteries).

The aspiration equivalents are a set of lottery-dependent deterministic targets that are revealed before the manager chooses among projects. Deterministic targets are a very common way that executives actually reward their managers. Because of their appeal, most corporations spend considerable effort setting up budgetary targets before making decisions about projects. However, as we have shown, simple deterministic targets that do not vary with the lotteries do not induce the desired utility-maximizing behavior. By selecting the correct targets that vary with the selected alternative, aspiration-equivalent targets achieve the *optimality* of the normative expected utility method using the widespread practice and *simplicity* of deterministic target setting. This makes it an attractive candidate to be adopted in practice.

CONCLUSIONS

We have developed a new approach to normative target-based decision making that yields decisions identical to that of Von Neumann–Morgenstern utility maximization. This normative method requires a separate target for each alternative lottery under consideration, as calculated by the new concept of the aspiration equivalent. Picking the alternative with the highest probability of meeting its own target is the maximum expected utility, normative choice.

The methodology we have developed in this paper has several implications for decision making in organizations. For example, when an executive puts forth a fixed target as a goal, and then asks his organization to search for alternatives that maximize the probability of meeting that fixed target, he is not acting normatively—rather he is likely to make a sub-optimal choice. The goal may be a motivator for seeking creative alternatives, but in choosing the final alternative to pursue, our normative conclusion is that he should set a different target for each alternative, based on the individual properties of its associated lottery and the organization's risk attitude. The organization should then choose the alternative that has the highest probability of meeting its own target. Similarly, pursuing a fixed goal may by operationally motivational when things are going smoothly, but when major impacts, such as setbacks or new opportunities, create a need to re-evaluate alternatives, the normative approach demands determining new targets for each new alternative. Simply maximizing the probability of reaching the old target is no longer optimal.

The normative target-based method we have discussed paves the way for several directions of future research. One direction would be in behavioral research, looking at real situations or experimental data to see where this method might be used as a descriptive or predictive model of behavior and where it might be used to improve behavior. Another direction would be to carry out further normative research, such as extending this method to dynamic situations by exploring whether and how normative target-based methods can be used in making sequential decisions.

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