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Risk Adjustment in Private Equity Returns

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Abstract

This article reviews empirical methods to assess risk and return in private equity. I discuss data and econometric issues for fund-level, deal-level, and publicly traded partnerships data. Risk-adjusted return estimates vary substantially by method, time period, and data source. The weight of evidence suggests that, relative to a similarly risky investment in the stock market, the average venture capital (VC) fund earned positive risk-adjusted returns before the turn of the millennium, but net-of-fee returns have been zero or even negative since. Average leveraged buyout (BO) investments have generally earned positive risk-adjusted returns both before and after fees, compared with a levered stock portfolio. Based on an expanded set of risk factors from the literature, VC resembles a small-growth investment, while BO loads mostly on value. I also discuss the empirical evidence on liquidity and idiosyncratic volatility risks.

1. INTRODUCTION

A sizable academic literature analyzes the average returns to private equity (PE) investments (see Kaplan & Sensoy 2015 for a thorough review).[1] Historically, institutional investors' allocations to PE were very low, and investors were effectively risk neutral to such small bets. In recent years, PE investments have grown dramatically, reaching double-digit portfolio weights for many US pension funds and endowments.[2] Consequently, assessing the risks borne by PE investors has become a first-order concern.

The main goal of this review is to organize, compare, and contrast the many empirical methodologies in the growing literature on risk adjustment of PE returns. I separate methods developed for portfolio company returns [e.g., start-up companies in the case of venture capital (VC)] from techniques for fund-level data, because these data sources involve distinct econometric issues. The methods are broadly applicable to all asset classes under the PE umbrella, but research to date has focused mainly on VC and leveraged buyout (BO). I discuss results for these two types of assets in detail, and briefly touch upon applications to other asset classes where relevant.

Performance evaluation in PE is inherently complicated because returns cannot be observed on a regular basis and payoff distributions are highly skewed, resembling option payoffs. Fund data typically consist of fund cash flows that occur at irregular times, with reported net asset values (NAVs) that are often stale and in some cases biased. Deal-level data are valuations that are observed at the time of investment or when a portfolio company is successfully sold or goes public, but failures are often missing. For these reasons, it is not possible to apply standard techniques developed for public equities, mutual funds, and hedge funds. Instead, the PE literature employs a wide array of approaches, including methods to construct return indices, such as repeat-sales models and corrections for stale valuations, and methods to value fund cash flows, including stochastic discount factor (SDF) techniques. At the deal level, selection corrections are important to control for success bias, especially in VC. Another complication is that data sources are incomplete and noisy, although more and better data sets have recently become available.

Table 1 gives an overview of risk and return estimates from the literature. The table is not intended to be comprehensive, and it ignores important subsample results in both the time series and cross section, as well as differences in risk estimates with respect to alternative market indices (S&P 500 results are shown if available; the Nasdaq and Russell 2000 are popular alternative choices for VC), and factor models other than the capital asset pricing model (CAPM) and the Fama–French three-factor model (Fama & French 1993). This review discusses many of these additional results.

A glance at **Table 1** reveals a substantial degree of heterogeneity in risk-adjusted return estimates, depending on the time period, empirical method, and data source used. There is currently no consensus as to what the main empirical approach should be toward estimating risk and return in PE, or how relevant benchmark returns should be constructed. This lack of agreement has likely contributed to the lack of formal quantitative risk adjustment in practice. For example, in a survey of 79 BO investors, Gompers, Kaplan & Mukharlyamov (2016) conclude that managers rely primarily on internal rates of return (IRRs) and cash multiples to evaluate investments, and that fund investors focus more on absolute performance rather than risk-adjusted returns.

[1] For the purpose of this review, the term private equity (PE) encompasses all types of private equity, including but not limited to leveraged buyout (BO), venture capital (VC), real estate, distressed debt, infrastructure, and natural resources.

[2] For example, in 2016, the 200 largest US defined benefit public (corporate) pension funds invested an average of 9.0% (5.8%) of their assets in VC and BO, and another 8.3% (5%) in real estate. University endowments allocated 17% of total assets to VC and BO, and another 6% to noncampus PE real estate (NACUBO 2016, Pensions & Investments 2017).

Table 1 Overview of references[a]

Reference	Sample period[b]	Alpha (%)[c]	β_M[d]	β_{SMB}[d]	β_{HML}[d]	Data source	Method and notes
i Fund-level returns (net of fees)							
VC							
Gompers & Lerner (1997)	1972–1997	2.0[q]	1.4			WP	Index method
	1972–1997	1.7[q]	1.3	0.8	0.1	WP	Index method
Anson (2007)	1985–2005	0.2[q]	1.4			VE	Dimson betas (three lags)
Woodward (2009)	1996–2008	0.5[q]	2.2			CA	Dimson betas (five lags)
Driessen, Lin & Phalippou (2012)	1980–1993	−1.1[m]	2.7			VE	NPV method
	1980–1993	−0.7[m]	2.4	0.9	−0.2	VE	NPV method
Ewens, Jones & Rhodes-Kropf (2013)	1980–2007	−0.2[q]	1.2			VE+P+LPS	Dimson betas (four lags)
	1980–2007	0.5[q]	1.1	−0.1	−0.9	VE+P+LPS	Dimson betas (four lags)
Buchner & Stucke (2014)	1980–2001	13.2[y]	2.7			B	Matching fund distributions
Jegadeesh, Kräussl & Pollet (2015)	1997–2008	−0.3[m]	1.0			MKT	Funds-of-funds
	1997–2008	0.0[m]	1.2			MKT	Partnerships
Ang et al. (2018)	1994–2008	0[y]	1.8			P	Bayesian filtering of realized PE returns
	1994–2008	1[y]	1.7	0.8	−0.6	P	Bayesian filtering of realized PE returns
Boyer et al. (2018)	1906–2017	−6[y]	1.0			S	Secondary transactions–based index
Peters (2018)	NR	0.6[q]	1.4			CA	Index returns, Dimson betas (five lags)
McCourt (2018)	1995–2015	0.1[m]	1.4	1.1	−1.4	MKT	Partnerships, including momentum factor
Buyout							
Anson (2007)	1985–2005	0.8[q]	0.7			VE	Dimson betas (three lags)
Woodward (2009)	1996–2008	1.4[q]	1.0			CA	Dimson betas (five lags)
Driessen, Lin & Phalippou (2012)	1980–1993	−0.4[m]	1.3			VE	NPV method
	1980–1993	−1.0[m]	1.7	−0.9	1.4	VE	NPV method
Ewens, Jones & Rhodes-Kropf (2013)	1980–2007	1.2[q]	0.7			VE+P+LPS	Dimson betas (four lags)
	1980–2007	0.9[q]	0.8	0.1	0.2	VE+P+LPS	Dimson betas (four lags)
Buchner & Stucke (2014)	1980–2001	4.5[y]	2.7			B	Matching fund distributions
Jegadeesh, Kräussl & Pollet (2015)	1997–2008	0.2[m]	0.7			MKT	Fund-of-funds
	1997–2008	−0.0[m]	0.9			MKT	Partnerships
Ang et al. (2018)	1994–2008	4[y]	1.3			P	Bayesian filtering of realized PE returns
	1994–2008	1[y]	1.2	0.5	0.6	P	Bayesian filtering of realized PE returns
Boyer et al. (2018)	2006–2017	−2[y]	2.4			S	Secondary transactions–based index
McCourt (2018)	1995–2015	0.8[m]	1.1	0.6	0.3	MKT	Partnerships, including momentum factor
Mixed VC and buyout							
Ljungqvist & Richardson (2003)	1981–1993	7.2[y]	1 (assumed)			LP1	IRR minus matched market investment

(*Continued*)

Table 1 (*Continued*)

Reference	Sample period[b]	Alpha (%)[c]	Data source	Method and notes
ii Fund-level returns (net of fees) using SDFs				
VC				
Kaplan & Schoar (2005)	1980–1994	0.96	VE	PME
McKenzie & Janeway (2008)	1980–2002	2.0	LP2	PME
Harris, Jenkinson & Kaplan (2014)	1984–2008	1.36	B	PME
Fang, Ivashina & Lerner (2015)	1991–2010	1.01	LP7	Direct investments PME
Robinson & Sensoy (2016)	1984–2009	1.01	LP1	2× levered PME
Korteweg & Nagel (2016)	1979–2008	−0.10	P	GPME
Gredil, Sorensen & Waller (2018)	1979–2008	0.08	B	Excess NPV, long-run risk model
	1979–2008	0.11	B	Excess NPV, habit model
Gupta & van Nieuwerburgh (2018)	1990–2010	−0.01	P	GPME extension, SDF: five priced factors
Harris et al. (2018)	1987–2007	1.16	B	Fund-of-funds PME
Lerner et al. (2018)	1987–2017	0.06	SS	Discretionary funds, excess PME
	1991–2017	−0.06	SS	GP-directed funds, excess PME
Buyout				
Kaplan & Schoar (2005)	1980–1994	0.97	VE	PME
Higson & Stucke (2012)	1980–2008	1.12	CA+CalPERS	PME
Axelson et al. (2013)	1987–2009	1.36	P	PME
Phalippou (2014)	1993–2010	0.97	P	1.3× levered PME
Harris, Jenkinson & Kaplan (2014)	1984–2008	1.22	B	PME
Fang, Ivashina & Lerner (2015)	1991–2010	1.34	LP7	Direct investments PME
Robinson & Sensoy (2016)	1984–2009	1.15	LP1	1.3× levered PME
Gredil, Sorensen & Waller (2018)	1985–2008	0.02	B	Excess NPV, long-run risk model
	1985–2008	0.05	B	Excess NPV, habit model
Harris et al. (2018)	1987–2007	1.14	B	Fund-of-funds PME
Lerner et al. (2018)	1987–2017	0.51	SS	Discretionary funds, excess PME
	1991–2017	0.91	SS	GP-directed funds, excess PME
Gupta & van Nieuwerburgh (2018)	1990–2010	0.01	P	GPME extension, SDF: five priced factors
Mixed VC and buyout				
Phalippou & Gottschalg (2009)	1980–1993	0.92	VE	Value-weighted PME

(*Continued*)

Table 1 (*Continued*)

Reference	Sample period[b]	Alpha (%)[c]	β_M[d]	β_{SMB}[d]	β_{HML}[d]	Data source	Method and notes
iii Portfolio company returns (gross of fees)							
VC							
Peng (2001)	1987–1999	-0.2^{m}	1.3			VO	Index returns; annual β_M is 2.4
Cochrane (2005a)	1987–2000	32^{y}	1.9			VO	Selection model
Hwang, Quigley & Woodward (2005)	1987–2003	0.9^{q}	0.6			SHE	Index returns, Heckman selection model
Korteweg & Sorensen (2010)	1987–2005	3.3^{m}	2.8			SHE	Selection model
	1987–2005	3.5^{m}	2.3	1.1	−1.6	SHE	Selection model
Buchner & Stucke (2014)	1980–2001	15.1^{y}	2.5			C	Matching fund distributions
Peters (2018)	NR	0.8^{m}	1.6	0.0	−0.7	SHE	Index returns
Buyout							
Kaplan (1989)	1980–1985	41.9^{x}	1 (assumed)			G	Market-adjusted return to all buyout capital over the deal (average: 2.7 years)
Groh & Gottschalg (2011)	1984–2004	NR	1.4			PPM	Mimicking portfolios
Franzoni, Nowak & Phalippou (2012)	1975–2006	9.3^{y}	0.9			C	Log-MIRR regressions
	1975–2006	3.1^{y}	1.4	−0.1	0.7	C	Log-MIRR regressions
Axelson, Sorensen & Stromberg (2014)	1994–2007	8.6^{y}	2.4			LP1	Log-CAPM with heteroscedasticity
Buchner & Stucke (2014)	1980–2001	4.8^{y}	3.2			C	Matching fund distributions
Stafford (2017)	1986–2016	0.7^{m}	1.8			R	Mimicking portfolios
Reference	**Sample period[b]**	**Alpha (%)[c]**				**Data source**	**Method and notes**
iv Portfolio company returns (gross of fees) using SDFs							
VC							
Korteweg & Nagel (2016)	1987–2005	0.52				SHE	GPME
Buyout							
Acharya et al. (2013)	1991–2008	1.88				MK+LP1	PME using sector return as discount rate
Braun, Jenkinson & Stoff (2017)	1974–2013	1.3				LP3	Median PME

[a]This table shows the main results for references discussed in this review. It is not intended to be comprehensive. References are shown in chronological order by asset class (VC or buyout). (*i*,*ii*) Results for fund-level data. (*iii*,*iv*) Results for deal-level data. For references that estimate both the CAPM and Fama–French three-factor models, both are shown on consecutive lines.

[b]The column labeled "Sample Period" contains the period over which fund vintages (*i*,*ii*) or deal returns (*iii*,*iv*) are observed.

[c]The column labeled "Alpha" shows the arithmetic, equal-weighted, risk-adjusted return for US data in percentages (e.g., 5.0 means 5%), to the extent that these numbers are reported. Each superscript indicates the data frequency (m, monthly; q, quarterly; x, some other frequency, as described in the "Method and Notes" column; y, yearly). For SDF-based methods in panels *ii* and *iv*, risk loadings are usually not separately estimated, and the "Alpha" column shows the risk-adjusted return metric described in the "Method and Notes" column.

[d]The column labeled "β_M" reports the market beta estimates, using the S&P 500 as market index if available. If the Fama–French three-factor model was estimated, the columns labeled "β_{SMB}" and "β_{HML}" show the loadings on the size (SMB) and value (HML) factors, respectively.

Abbreviations: B, Burgiss; C, CEPRES; CA, Cambridge Associates; CalPERS, California Public Employees' Retirement System; CAPM, capital asset pricing model; G, stock delistings and Grimm's Mergerstat Review; GP, general partner; GPME, generalized public market equivalent; HML, high-minus-low book-to-market value factor; IRR, internal rate of return; LP*n*, sourced from *n* limited partners (LPs); LPS, LP source; MIRR, modified internal rate of return; MK, McKinsey; MKT, publicly listed market data; NPV, net present value; NR, not reported; P, Preqin; PE, private equity; PME, public market equivalent; PPM, private placement memoranda; R, Thomson Reuters M&A; S, secondary transactions intermediary; SDF, stochastic discount factor; SHE, Sand Hill Econometrics; SMB, small-minus-big size factor; SS, State Street; VC, venture capital; VE, Venture Economics (now VentureXpert); VO, VentureOne (now VentureSource); WP, Warburg, Pincus & Co.

Gompers et al. (2019) find that of 546 institutional VCs, 64% say that they adjust their investments' target metrics for risk, though it is not quite clear how those benchmarks are determined. Recently, the literature has begun to shift toward SDF approaches, in particular the public market equivalent (PME) metric and its generalizations. This metric has also started to catch on among investors in PE funds.

Apart from evaluating manager skill, proper risk measurement is also important in the study of managerial style, persistence, the role of PE in portfolio allocations, and agency issues such as pay-for-performance and the risk-taking incentives of contractual features. These are not the main questions of interest for this review, but I touch upon some of them where appropriate. Many questions are yet to be addressed by the literature, and I conclude with thoughts for future research.

The remainder of this review is organized as follows. Section 2 describes methods for fund-level data, Section 3 considers deal-level data, and Section 4 discusses publicly traded PE. Section 5 concludes with thoughts for future research.

2. FUND-LEVEL DATA

PE funds are portfolios of individual deals, typically organized as 10-year limited partnerships. The fund manager [called general partner (GP)] makes investment and exit decisions on behalf of investors such as pension funds or endowments [called limited partners (LPs)]. The main difference between VC and BO is that the investments in VC are minority stakes in start-up companies, while BO typically purchases all the equity of established firms, using leverage to help finance the purchase. Fund data consist of a series of cash inflows and outflows to LPs, and quarterly reported NAVs. Investors are not required to put up their committed capital at the start of the fund. Rather, the GP calls capital as investment opportunities arise. As such, cash contributions by LPs are high early in the fund's life, followed by large distributions in later years when portfolio companies are sold or go public.

Risk-adjusted return estimates that are based on fund cash flows should be interpreted from the LPs' perspective, because the cash flows are reported net of fees to the GP. Fees consist of a management fee (usually between 1% and 2%, but with considerable variation in the percentage and the basis to which it is applied) and a profit-sharing fee called carried interest (typically 20%).[3] Net-of-fee returns are important for answering questions such as LPs' optimal portfolio allocation to PE, but less can be said about managerial skill because GPs may absorb rents through fees, in the spirit of Berk & Green (2004).[4] For studies that use both cash flows and NAVs, the interpretation is unclear because NAVs do not adjust for GP carried interest, so that return measures are somewhere between net and gross of fees, depending on how heavily NAVs influence the results.

In one of the earliest academic papers on risk and return in PE, Gompers & Lerner (1997) use data from one PE firm (Warburg, Pincus & Co.) between 1972 and 1997.[5] It would appear straightforward to compute a quarterly return series for this GP from its net cash disbursements to LPs (distributions minus capital calls) and the change in reported quarterly NAVs of portfolio

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[3] Buyout funds often also charge fees to portfolio companies, some of which may be shared with LPs (see Phalippou, Rauch & Umber 2018 for a detailed analysis of buyout fees).

[4] There is evidence suggesting that in practice, LPs may share in those rents (see, e.g., Hochberg, Ljungqvist & Vissing-Jorgensen 2014, Harris et al. 2014, Korteweg & Sorensen 2017, and the sizeable literature on return persistence in PE fund returns more generally). GP skill is a necessary condition for any persistence in LP performance.

[5] Although Gompers & Lerner (1997) use PE firm-level rather than fund-level data, the issues are the same as in other fund data papers.

companies. However, reported NAVs have a number of drawbacks. Fund managers typically do not (aggressively) update portfolio companies' valuations, often leaving them at cost or, for VC, at the most recent valuation from a financing round.[6] This results in a stale index. Standard factor model regressions yield downward-biased risk estimates, since risk is determined by the covariance of the portfolio's value with the risk factors (see Stafford 2017 for a clear illustration). Moreover, some GPs appear to strategically manipulate their reported NAVs, especially underperforming managers with poor reputations who are trying to raise the next fund (Jenkinson, Sousa & Stucke 2013, Barber & Yasuda 2017, Brown, Gredil & Kaplan 2019). Finally, specifically for VC, to the extent that NAVs are updated using new financing round valuations, the common equity contract assumption in the post money valuation calculation means that these valuations may not reflect their true market values, as discussed in Section 3 below.

Instead of relying on reported NAVs, Gompers & Lerner (1997) value each portfolio company in each quarter by taking a firm's most recently observed value and updating it to the present period using the return to an equal-weighted index of publicly traded firms in the same three-digit Standard Industrial Classification (SIC) industry. They find a CAPM quarterly alpha of 2% with a beta of 1.4. The alpha for the Fama–French three-factor model is similar.[7]

The Gompers–Lerner method is difficult to apply in general, in part because portfolio company data has historically not been available in larger data sets. Emery (2003) instead advocates the use of longer-horizon returns to mitigate the stale NAV problem, and shows that the correlation between PE index returns (constructed from fund cash flows and NAVs) and proxies for the market portfolio increases markedly when using annual rather than quarterly returns. He suggests using lagged market returns in regressions but does not report the regression coefficients. This is essentially the Dimson (1979) correction for infrequent trading, omitting the leading stock market returns, which should be negligible since the public stock market is very frequently traded. Anson (2002, 2007) also suggests the use of lagged market returns. Using index returns from Venture Economics and including three lagged quarters of S&P 500 returns, Anson (2007) reports a VC (BO) alpha of 0.2% (0.8%) per quarter, and a beta of 1.4 (0.7). Woodward (2009) includes five lagged quarters and finds a market beta of 2.2 for VC and 1.0 for BO (based on the Wilshire 5000 stock market index), with quarterly alphas of 0.5% and 1.4% in Cambridge Associates index data.

Boyer et al. (2018) take a different approach and use secondary market transactions instead of NAVs to construct PE indices. In theory, secondary market transactions should reflect arm's-length valuations and therefore avoid the problems with NAV, but the market is small and frictions are substantial. They find a high BO beta of 2.4 and a negative CAPM alpha of −2% per year, which is surprising given that the literature consistently finds positive buyout alphas. For VC, they estimate a beta of 1.0, which is considerably lower than most estimates in the literature, and an alpha of −6%.

[6]Leaving portfolio companies at cost or at the most recent round valuation used to be common practice. In 2007, Financial Accounting Standard 157, now known as Accounting Standards Code Topic 820, came into effect; it requires the fair valuation of portfolio companies. However, there is no clear market to which to mark illiquid assets such as BO or VC portfolio companies (so-called level-three assets), so GPs and their consultants and auditors must rely on the pricing of recent deals and valuations of comparable firms. This is a subjective exercise, and many GPs maintain a conservative policy of marking assets up slowly while being quicker to mark them down if they believe their value has dropped (Anson 2002, 2007).

[7]For convenience, I refer to the intercept of factor regressions as alpha, though it is not clear in all cases that that this is a true risk-adjusted return. The intercept is not a true alpha if one or more risk factor(s) is not a traded asset. It also contains the average return of any omitted risk factors.

A different way around the myriad problems with NAVs is to simply avoid their use altogether. For fully liquidated funds, a fund's IRR is computed from cash flows only.[8] Fund IRRs are commonly reported but suffer from a number of drawbacks. Most importantly, there is no adjustment for risk.[9]

Many papers use IRR as a return metric, and typically compare IRR with the return on the market portfolio over the fund's life (see the review by Kaplan & Sensoy 2015). This comparison is not as straightforward as it may appear, since IRR is a money-weighted rate of return. With cash going in and out of a fund at various times during the life of the fund, relating a fund IRR to a simple (time-weighted) market return over the fund's life is not an apples-to-apples comparison, even if PE were as risky as the market. Recognizing this issue, Ljungqvist & Richardson (2003) compare IRRs with a matched investment strategy that invests in the market portfolio using the drawdown schedule of the fund (or, alternatively, of an average fund). They hold the portfolio until year 10, despite the fact that many exits (and hence cash distributions) occur before the final year, and it is not uncommon for exits to occur later. Using data on 19 VC and 54 BO funds from a large LP, Ljungqvist & Richardson (2003) find that PE funds (lumping VC and BO funds together) outperform the S&P 500 by 5% to 8% per year across their various measures.

Kaplan & Schoar (2005) estimate market betas by regressing fund-level IRRs on the realized market return over the first 5 years of the fund, and report a coefficient of 1.2 for VC and 0.4 for BO. The relation between factor models and IRRs, and the econometric properties of such regressions, is largely unknown. Axelson, Sorensen & Stromberg (2014) show that in simulated data, beta estimates from this type of regression tend to be downward biased. It may be more accurate to regress fund IRRs on the IRRs of matched factor-invested portfolios, but the econometrics of such a procedure are also unknown.[10] This is a topic for future research to explore.

2.1. Stochastic Discount Factors

Kaplan & Schoar (2005) introduce the PME performance metric, building on Long & Nickels (1996). The PME takes a fund's cash distributions to LPs (and any residual NAV if the fund has not yet been liquidated), discounts them back to fund inception at the realized public stock market's rate of return, and divides this number by the similarly discounted value of all cash contributions made by LPs to the fund (i.e., capital calls).[11] If the PME is greater than one, then this is interpreted as the fund having outperformed the public stock market.

Sorensen & Jagannathan (2015) and Korteweg & Nagel (2016) point out that the PME is in fact an application of an SDF valuation, which states that the time t price of a cash flow that realizes

[8]For funds that have not yet been liquidated, it is common practice to include the latest observed NAV as final pseudodistribution. Doing so leaves some of the NAV issues and muddies the interpretation (because NAVs are not adjusted for fees), although the problem is not as prominent as in metrics that use all quarterly NAVs.
[9]Other problems with IRR are that it may be manipulated by GPs through their choice of the size and timing of investments (possibly through the use of credit lines), it may not exist, it may not be unique, the IRR of a portfolio of investments is not a weighted average of the component IRRs, and there is an implicit reinvestment assumption (modified IRR addresses the last issue, but there is no guidance on the appropriate reinvestment rate). See Kocis et al. (2009) for a detailed discussion of the IRR and its shortcomings.
[10]In unreported research I find that IRR regressions using matched factor-invested portfolios still underestimate betas, and that this problem is more severe if expected returns are time varying or if cash flows are highly correlated with factor returns. Results are available on request.
[11]Ljungqvist & Richardson (2003) also consider an NPV-type measure, discounting capital calls at the risk-free rate and the distributions at the market return (or, alternatively, a cost of capital derived from public firms from the same industries as the fund's portfolio firms). The NPV-type measure is closely related to the PME measure.

at time $t + h$ equals the expectation of the cash flow multiplied by an SDF, $M_{t,t+h}$:

$$P_t = E_t(M_{t,t+h}C_{t+h}). \tag{1}$$

The SDF can roughly be thought of as a state price, the time t value of a \$1 payoff at time $t + h$. Given that the state of nature (and, hence, the value of having a dollar) at time $t + h$ is unknown at time t, the SDF is a random variable. The expected PME is thus, approximately, the present value of fund distributions divided by the present value of the capital calls, using the reciprocal of the market return as the SDF. This is exactly the SDF of an investor with log-utility preferences. Note that the SDF approach takes the expectation of realized cash flows discounted at realized discount factors, as opposed to discounting expected cash flows at expected rates of return, but with additional assumptions there is a well-known equivalence between SDF pricing and expected returns from factor models [see, for example, Cochrane's (2005b) textbook]. Notwithstanding this equivalence, the PME makes no distributional assumptions on the cash flows, unlike most factor models. This is important because PE payoffs are highly skewed, resembling option payoffs, and standard factor models such as the CAPM do not work well for options (unlike log-utility models; see Rubinstein 1976) because betas are highly time varying. Note also that the endogeneity of cash flows is not important, as their riskiness is properly accounted for by the SDF.

PME estimates vary across periods and data sources. In a sample of 577 VC funds and 169 BO funds from Venture Economics, Kaplan & Schoar (2005) find average PMEs close to 1 for both asset classes. Size-weighted PMEs are 1.21 and 0.93 for VC and BO funds, respectively, indicating that larger VC funds tend to perform better. Phalippou & Gottschalg (2009) argue that the final NAVs of funds for which no cash flows are observed for many quarters should be set to zero. Using a slightly updated version of the Venture Economics data set, they find a lower average PME of 0.92. Stucke (2011) documents that Venture Economics fund data have an updating problem with reported NAVs and cash flows, and researchers have mainly utilized different fund data sets since. Using data on 387 VC funds from two large LPs, McKenzie & Janeway (2008) estimate an average PME of 2. Higson & Stucke (2012) have a large merged data set of 1,169 BO funds and find a PME of 1.12. Axelson et al. (2013) find a PME of 1.36 on the basis of 706 BO funds in Preqin. Using data from Burgiss, arguably the most comprehensive high-quality fund cash flow data set currently available, Harris, Jenkinson & Kaplan (2014) report average PMEs of 1.36 and 1.22 for VC and BO, respectively. BO performance has been consistently high over time (Braun, Jenkinson & Stoff 2017 also find this result in deal-level PMEs), but VC performance has been markedly worse after the turn of the millennium, with a PME of 0.91 for post-2000 vintages. Andonov, Kräussl & Rauh (2019) find that the average PMEs for infrastructure and real estate funds are 0.93 and 0.94, respectively.

Venturing outside the realm of traditional direct PE fund investments, Harris et al. (2018) consider the performance of PE funds-of-funds. They find average PMEs of 1.16 for VC and 1.14 for BO, on par with VC performance of direct fund investments but below the PME of direct BO funds. Lerner et al. (2018) consider alternative PE investment vehicles that are offered to select LPs. The excess PMEs of discretionary and GP-directed buyout funds relative to the main fund vehicles of the same GP are 0.51 and 0.91, respectively. The performance of alternative VC vehicles is close to the PME of the GP's main funds. There is heterogeneity, however, and performance depends on GP quality and the relative bargaining position of LPs. Fang, Ivashina & Lerner (2015) estimate PMEs for LPs that are invited to coinvest with a GP in a specific deal, as well as for solo deals by LPs. Although the PMEs are above one, only solo transactions appear to outperform on average when compared with the regular PE fund investments, and outperformance is concentrated in BO.

Gredil, Griffiths & Stucke (2014) propose a measure that effectively annualizes PME, called direct alpha. Annualization matters because there is material variation in the effective duration of funds. Direct alpha forges a closer connection between SDF-type performance measures and the intercept in factor models.

A few papers (e.g., Ljungqvist & Richardson 2003, Phalippou & Gottschalg 2009, Harris, Jenkinson & Kaplan 2014, Phalippou 2014) recognize that the market return may not accurately reflect the riskiness of PE. They employ alternate discount factors such as the post–initial public offering (IPO) cost of capital for similar firms, or growth (for VC) or value portfolio returns (for BO). Harris, Jenkinson & Kaplan (2014), Phalippou (2014), and Robinson & Sensoy (2016) use a levered market return to compute PMEs. The resulting levered PMEs tend to be lower than standard unlevered PMEs. Instead of assuming a leverage number, Driessen, Lin & Phalippou (2012) estimate the loading on the market return. They also add an alpha term to the discount rate and estimate the parameters so as to most closely force all fund net present values (NPVs) to be equal to zero.[12] They estimate large negative CAPM alphas of −1.1% for VC and −0.4% for BO, with similar results for the Fama–French three-factor model. These alphas are more negative compared with those from other papers that span the same period. This could be due to Driessen et al.'s use of Venture Economics, whose fund data are now known to be downward biased (Stucke 2011). Like most papers that consider the Fama–French three-factor model, they find that VC investments behave like small-growth firms, with a positive small-minus-big (SMB) size loading and a negative high-minus-low book-to-market (HML) value beta. BO behaves like large-value investments. The positive BO value loading is consistent across papers, but the negative loading on SMB is not. Most other research reports either insignificant or positive SMB loadings for BO funds and deals.

Korteweg & Nagel (2016) generalize the PME by allowing for more flexibility in the SDF. Specifically, they compute a generalized PME (GPME) using an exponential-affine SDF, $M_{t,t+h} = \exp(a - b \cdot \log(R_{\mathrm{M}}))$, where R_{M} is the market return from time t to $t + h$. This is the SDF for an agent with constant relative risk aversion equal to b.[13] With the additional assumption of jointly log-normal payoffs, this model is equivalent to the CAPM in log-returns, although this assumption is not necessary. The PME is the special case when $a = 0$ and $b = 1$. Korteweg & Nagel (2016) define the (G)PME over a fund's net cash flows (scaled by the total fund size), which has better statistical properties than the ratio of discounted distributions to contributions of prior research. An additional benefit is that the (G)PME of a portfolio of funds is a weighted average of the constituent (G)PMEs, if the same scaling is used. With this change, the benchmark (G)PME number of no under- or overperformance is now zero.

Contrary to the popular claim that PME assumes a market beta of one, the PME in fact makes no assumption regarding betas, since the riskiness of cash flows is accounted for by the SDF. Rather, the constraints are on the risk-free rate and the equity market premium implied by the SDF. For example, the risk-free rate for a log-utility investor equals $1/E(1/R_{\mathrm{M}})$. This is important, as **Table 2** shows. The table reports summary statistics of the annualized risk-free rate and market return by decade. For example, in the 1990s—a heavily populated decade in most current PE fund data sets—the natural logarithm of the risk-free rate and average market return were 4.8% and 17.5%, respectively, with a market volatility of 13.9% per year. Based on these numbers, the PME-implied risk-free rate over the decade was 15.6% and the market premium was 1.9% (both in logs).

[12]Another way to view the Driessen, Lin & Phalippou (2012) approach is that they introduce a factor model into the fund IRR calculation. Buchner & Stucke (2014) propose a similar approach that aims to match the observed distributions, under an additional assumption regarding the dividend process.

[13]In this utility interpretation, the leverage factor in the levered PME is actually the degree of risk aversion of the agent.

Table 2 Market and SDF-implied returns

	1960–1969	1970–1979	1980–1989	1990–1999	2000–2009	2010–2017
i Market return statistics						
Risk-free rate	3.8	6.1	8.5	4.8	2.7	0.2
Market return	8.7	7.3	16.9	17.5	1.0	14.0
Market volatility	12.6	16.9	17.0	13.9	16.9	12.3
ii PME-implied market returns						
Risk-free rate	7.1	4.5	14.1	15.6	−1.9	12.5
Market premium	1.6	2.9	2.9	1.9	2.9	1.5
iii Benchmark returns						
β = 1						
PME	8.7	7.3	16.9	17.5	1.0	14.0
Log-CAPM	8.7	7.3	16.9	17.5	1.0	14.0
β = 2						
PME	10.3	10.2	19.9	19.4	3.8	15.5
Log-CAPM	13.6	8.5	25.4	30.1	−0.8	27.8
β = 3						
PME	11.9	13.0	22.8	21.3	6.7	17.0
Log-CAPM	18.5	9.7	33.8	42.8	−2.6	41.6

(*i*) The natural logarithm of the average annual risk-free rate and the average market return, as well as the standard deviation of log stock market returns by decade, computed from monthly data downloaded from Kenneth French's data library (**https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html**). (*ii*) The log risk-free rate and market premium implied by the SDF underlying the PME metric. (*iii*) The benchmark rates of return (in logs) for various market risk levels (β). All returns are annualized and in percentages. See Korteweg & Nagel (2016) for more details.
Abbreviations: CAPM, capital asset pricing model; PME, public market equivalent; SDF, stochastic discount factor.

It is true that with a beta of one, the PME recovers the correct benchmark return (17.5% for the 1990s). However, with higher betas, the PME benchmark can be above or below the log-CAPM benchmark depending on the relative size of the market's average return and its volatility (e.g., compare the 1960s and the 1970s in **Table 2**). This example illustrates that the interpretation of PME as assuming a beta of one may lead to incorrect inference.

There are two ways to interpret the (G)PME. The first is the utility interpretation described above. A drawback is that this interpretation requires knowledge of LPs' preferences, which may vary by investor and may depend on variables that are difficult to measure. The second interpretation, favored by Korteweg & Nagel (2016), is as a pure benchmarking exercise. They fit the SDF to price public stocks and bonds, and ask whether that same SDF can also price VC funds. A GPME above zero indicates that there is a component of VC that cannot be replicated by investing in stocks and bonds. They reject the PME restrictions for their sample of VC funds, and find an overall negative GPME of −0.103. This means that, relative to investing in public assets, a $1 commitment to VC led to a risk-adjusted loss of $0.103 over a fund's lifetime in present value terms. GPME was positive but insignificant for pre-1998 vintages.

The GPME method can accommodate additional benchmark factors. Korteweg & Nagel (2016) find little role for a small-growth stock portfolio in addition to the market factor. Gredil, Sorensen & Waller (2018) use SDFs implied by habit formation and long-run risk models, and find a small positive outperformance for VC (though insignificant in some specifications) and insignificant results for BO. Gupta & van Nieuwerburgh (2018) include a total of five priced risk factors in their SDF. Together, they pin down the level and slope of the yield curve and the unconditional risk premium on publicly traded stocks, real estate, and infrastructure. Their model allows for richer risk price dynamics by pricing the benchmark assets period by period. Average risk-adjusted returns to VC, BO, real estate, and infrastructure funds are close to zero (higher in the pre-2000 vintages and lower afterward).

Korteweg & Nagel (2018) argue that while GPME works well when averaged across many funds, the performance metric is noisy for individual funds. They propose a benchmark return that is consistent with aggregate GPME but reduces the noise in individual fund returns. Giacoletti & Westrupp (2018) apply this method to real estate investments.

Ang et al. (2018) also consider the benchmarking question. Using Bayesian estimation, they filter a quarterly time series of realized PE returns. Their main goal is to explore whether PE is spanned by publicly traded assets, rather than estimating expected returns, but they do show risk loadings from regressing their realized PE returns on factor returns. These risk estimates are in line with those from other studies. They find evidence of a PE-specific factor, such that PE returns cannot be perfectly replicated by simple passive strategies of traded assets.

Stafford (2017) takes a somewhat different approach to constructing a benchmark portfolio that matches the riskiness of PE investments. He shows that a portfolio of publicly traded small firms with low EBITDA (earnings before interest, tax, depreciation, and amortization) multiples can produce returns that are consistent with prefee BO index returns. A replicating portfolio of public equities earns a monthly alpha of 0.7% at a CAPM beta of 1.8, which is in the range of prefee BO estimates in the literature.

Given the disagreement in the literature, especially on BO, it is clear that more research is needed to determine whether PE returns can be replicated with publicly traded assets. At present, the weight of evidence is that there appears to be something special about BO, though its magnitude has shrunk over time, while for VC it may have disappeared altogether (at least for the average fund). There are still many questions to explore, most importantly regarding the set of risk factors in PE. I discuss some further results along these lines in the next section.

2.2. Other Risk Factors

The bulk of the literature estimates the CAPM or Fama–French three-factor model.[14] Recent research has started to push beyond these standard models to explore other potential sources of risk. The two main risk factors that have been considered are liquidity and idiosyncratic volatility.

2.2.1. Liquidity. With LPs committing to PE funds that last 10 years or longer, it would seem obvious that there should be a large liquidity risk premium in PE returns. However, since not all capital is called immediately and many exits occur before the 10 years are up, effective duration is well under 10 years. Moreover, many investors select into PE because they are less prone to liquidity shocks, and PE allocations used to be quite small. All of these forces may lower the required illiquidity premium.

Franzoni, Nowak & Phalippou (2012) add the PS liquidity factor (named after Pastor & Stambaugh 2003) to the Fama–French three-factor model and find a positive and statistically significant loading for their BO data. They compute a liquidity premium of approximately 3% per year. Ang et al. (2018) also include the PS factor and find a quantitatively similar positive and significant loading for BO, but the loading for VC is insignificant. Buchner (2016) also finds a negligible effect of including the PS factor in VC.

Notwithstanding some supportive results, the PS factor was developed for public equities and is geared toward liquidity risk stemming from order flow fluctuations. This is relevant for public equity markets, but PE investors are more concerned with illiquidity due to search and information

[14] Ang et al. (2018) also use the five-factor Fama–French model.

frictions from locating a counterparty in an over-the-counter market. These liquidity costs are substantial: Nadauld et al. (2019) find that the LPs who sell their PE fund stakes in the secondary market accept an average discount to NAV of 13.8%, though this fluctuates with fund age and market conditions. For the most common transactions (which occur outside of the financial crisis), the discount is 9%. The buyers realize a higher return than the sellers, approximately 5% per year in both IRR and PME terms.

Sorensen, Wang & Yang (2014) specify a contingent-claims model of PE valuations. If PE is spanned by traded assets, there is no illiquidity cost because investors can hedge their PE exposure. Their model quantifies the cost of illiquidity due to unspanned risk. Calibrating the model to a typical PE fund, they find that the cost of illiquidity is high, on the same order of magnitude as GP fees (management fee plus carried interest). Bollen & Sensoy (2016) build a valuation model that explicitly allows for a secondary market (as well as stochastic capital flows). Calibrations suggest that a moderately risk-averse LP should be indifferent between a 20% portfolio allocation to PE (with the remainder invested 60% in public stocks and 20% in Treasury bills) and an 80–20 portfolio of public stocks and Treasury bills.

An interesting as-yet-unanswered question is whether the liquidity premium has changed over time. On one hand, the financial crisis may have heightened awareness of liquidity concerns, given that some LPs defaulted on their commitments or sold out at large discounts. On the other hand, the growth of a (currently still small) secondary market has increased overall liquidity. Having a more model-free liquidity factor that is pertinent to PE could help answer these and other important questions.

2.2.2. Idiosyncratic volatility. Ang et al. (2006) uncover evidence that idiosyncratic risk is negatively priced in the public market (i.e., stocks with high exposure to idiosyncratic risk shocks have low average returns). In PE, idiosyncratic volatility may instead be positively priced due to the underdiversification of GPs, who demand compensation by pricing their exposure into contracts when negotiating with entrepreneurs. Ewens, Jones & Rhodes-Kropf (2013) find supporting evidence for this hypothesis in VC and BO returns. Peters (2018) documents a positive loading on aggregate idiosyncratic risk shocks in VC, and attributes this to the option-like nature of VC payoffs. Gompers et al. (2019), in their survey of VCs, find that 42% of VCs treat systematic and idiosyncratic risk the same, 5% discount systematic risk more, and 14% discount idiosyncratic risk more.

Entrepreneurs are also underdiversified, even more so than GPs. They do not appear to be compensated for idiosyncratic risk, as average returns to entrepreneurs are quite poor (Moskowitz & Vissing-Jorgensen 2002, Hall & Woodward 2010). Whether this is due to a preference for skewness, overoptimism, overconfidence, or some other channel remains an open question.

2.2.3. Summary. PE research has traditionally confined itself to the CAPM and Fama–French three-factor models, but a recent movement has started to consider other risk factors that may play a role in PE. Some of these risks are also present to some degree in public markets (e.g., liquidity, idiosyncratic volatility), but the nature of many of these risks in PE tends to be different from that of publicly traded assets. Term structure features must be important for PE returns measured over long horizons, and though there has been some exploration along these lines (Gupta & van Nieuwerburgh 2018), more work remains to be done. Finally, there is some (mixed) evidence that suggests the existence of a PE-specific return component that is not spanned by public assets (e.g., Korteweg & Sorensen 2010, Ang et al. 2018), the nature of which has not yet been explored in the literature.

3. DEAL-LEVEL DATA

The main advantage of analyzing individual deal data is that returns are computed before GP fees are taken out, enabling direct assessment of GP skill.[15] In addition, one can fairly cleanly estimate risk and return by industry, stage of investment (e.g., seed, early, or late rounds in VC), strategy (e.g., health care versus Internet technology), or geography, which are often mixed inside PE funds. It is even possible in principle to consider individual managing partners' performance, though this has not yet been explored using a formal risk and return model (but see Ewens & Rhodes-Kropf 2015).

Panels *iii* and *iv* of **Table 1** give an overview of the risk-adjusted returns literature that uses deal-level data. Historically, most papers have considered VC only, as start-up company returns have traditionally been more readily available. VC returns are also easier to construct because start-ups rarely pay dividends and VCs do not charge fees to their portfolio companies, unlike buyout firms (Phalippou, Rauch & Umber 2018), and data on dividends and fees are difficult to collect. However, researchers have recently gained access to large data sets of buyout deal returns.

The literature on deal-level data uses two main approaches. The earlier approach is to build an index from the individual deals, and then regress the index returns on a set of risk factors. The later approach estimates factor models from the individual company data directly. I discuss each approach in turn below.

3.1. Index Method

Peng (2001) constructs an index based on valuations of 5,643 start-up companies that were funded by venture capitalists between 1987 and 1999. A key challenge that permeates the literature on deal-level data is that market values are observed only when companies raise money from investors or when there is an exit event, such as an IPO or an acquisition by another firm. In the intermediate periods between financing events, which can last several months to several years, no arm's-length market prices are observed.[16]

Peng (2001) bases his approach on the repeat-sales regression (RSR) approach, originally developed in the real estate literature (Bailey, Muth & Nourse 1963; Case & Shiller 1987, 1989), to fill in the missing values and compute the index. The RSR method is used, for example, to compute the well-known S&P/Case–Shiller home price index. The main assumption is that all firms have the same expected return (and thus the same risk factor loadings) in the same period.

Success bias poses an additional key challenge in PE deal data. While it is relatively easy to backfill data for start-ups that successfully went public (from IPO filings), and large acquisitions are more likely to have details disclosed, failures and minor acquisitions (which are often disguised failures) are not well publicized, and it is difficult to find their dates and values. This means that returns for good outcomes are more likely to be known.[17] It also leaves a substantial number of so-called zombie firms by the end of the sample period that appear to be alive, in the sense that they have not been confirmed bankrupt or sold, but are probably dead, since they have not raised

[15]While the management fee is essentially predetermined, gross-of-fee beta estimates may differ from net-of-fee estimates because the GP's performance fee (carried interest) is essentially a call option contract between the GP and LPs on the fund's cash flow realizations, and therefore has a systematic component. LPs are short this call option. Therefore, net-of-fee betas are lower than gross-of-fee betas, all else equal.

[16]The indexing problem in deal-level data is more complicated than in fund-level data, where NAVs are regularly observed. But, as discussed in Section 2, NAVs have their own distinct set of issues.

[17]Note that fund-level data do not generally suffer from success bias, since all investments are accounted for when funds have liquidated. For not-yet-liquidated funds there may be a bias, if NAVs are not properly marked to market, as discussed in Section 2.

a financing round in many years.[18] To deal with the resulting selection bias, Peng (2001) first constructs separate repeat-sales indices for known successful and unsuccessful investments (making assumptions about the returns to the unsuccessful deals). For the zombie companies with unknown outcomes at the end of the sample period, he assigns each company a probability of success in each period using a nonparametric model, and uses that probability to distribute their value into the successful and unsuccessful indices. Finally, he combines the two indices with weights proportional to the NAVs of each index.[19]

With the final index in hand, it is straightforward to compute returns at regular intervals (typically monthly or quarterly) and regress these returns on a set of risk factors, following standard practice in the asset pricing literature. Using the S&P 500 as the market index, Peng (2001) finds a beta of 1.3. The alpha is −0.2% per month and is statistically insignificant. Both alpha and beta magnify with the frequency of returns: Using annual returns, Peng finds a beta of 2.4 and an alpha of −0.9% per year. Using the Nasdaq as an alternative market index that is more weighted toward smaller technology companies, he finds a monthly (annual) beta of 0.8 (4.7) and an alpha of 0.3% per month (−3.8% per year). Unfortunately, there is no clear explanation for why the annual alpha is not a properly scaled counterpart to the monthly alpha, or why the beta changes with measurement frequency.

Hwang, Quigley & Woodward (2005) use an updated version of the data set used by Peng (2001) to construct a VC index between 1987 and 2003. Following Heckman (1979), they treat selection bias as an omitted-variables problem and use a two-step estimation to correct the repeat-sales index regression. The first stage is an ordered probit that models the probability of observing various outcomes for a given start-up in a given period, most importantly whether or not a valuation is observed. The second stage adds the inverse Mills ratios from the first stage to the RSR to account for the truncated error distribution that stems from the fact that high-value firms are more likely to be observed. Regressing index returns on S&P 500 (Nasdaq) returns yields a beta of 0.6 (0.4) with a quarterly alpha of 0.9% (1.0%), which is not statistically different from zero. Returns are higher in the pre-2000 period, which includes the Internet boom of the late 1990s but not the subsequent bust of the early 2000s, with statistically significant alphas of around 3.5% per quarter. However, similar to other authors who use the index method, Hwang, Quigley & Woodward (2005) do not adjust standard errors for the fact that the index itself is estimated, thereby overstating statistical significance.

3.2. Individual Deal Returns

More recent VC papers bypass the index construction step and directly use the returns to the individual start-up companies, computed between financing rounds or from financing round until exit, to estimate risk and return. These returns are irregularly spaced, which is not a problem in itself. If valuations were observed randomly, then one could simply regress these returns on risk

[18]Other than zombie firms that were not observed to have gone bankrupt, companies that are still private by the end of the sample period include firms that have only recently started operations and have not had enough time to exit (a right-censoring problem), and so-called lifestyle companies that are still in operation even though they have not raised a round in years. The lifestyle firms provide a living for the entrepreneur but are not attracting interest from potential buyers and are too small for a public offering, and their value to the VC is likely low.

[19]The sample selection issue in repeat-sales estimators was first explored in the real estate literature. Haurin & Hendershott (1991) hypothesize its existence, and Wallace & Meese (1997) show empirical evidence. Real estate studies that apply standard selection models to this problem include those by Jud & Seaks (1994), Gatzlaff & Haurin (1997, 1998), Munneke & Slade (2000, 2001), Hwang & Quigley (2004), and Goetzmann & Peng (2006).

factors measured over the corresponding periods, and compute standard errors to account for any cross-correlation in residuals due to the overlap in return horizons. But as discussed above, successful companies are more likely to be observed than failed ones. To address this issue, Cochrane (2005a) specifies a selection correction that models the probability that a valuation is observed in a given period for a given firm as a function of the firm's value, and estimates the model using maximum likelihood. He assumes that the underlying process for firm values follows a CAPM in logs (i.e., the natural logarithm of returns follows the CAPM) with normally distributed residuals. The log-CAPM is convenient because it compounds very nicely over time, as long-horizon returns are simple summations of short-horizon returns. By contrast, compounding the regular (arithmetic return) CAPM to compute multiperiod discount rates becomes problematic, especially when factor returns are serially correlated, or with time-varying interest rates or factor loadings (Constantinides 1980, Brennan 1997, Ang & Liu 2004). Moreover, arithmetic returns computed from Cochrane's model follow a log-normal distribution, whose right skew appears to fit the data better, since VC deal returns look somewhat like long call option returns.[20] A drawback of the log-CAPM is that standard portfolio theory does not apply to log returns, but Cochrane shows how to adjust the parameters from the log-CAPM to recover arithmetic return alphas and betas. From 16,638 round-to-exit returns (accounting for dilution due to intermediate financing rounds) with the S&P 500 (Nasdaq) as the market factor, he estimates a beta of 1.9 (1.4) and an annual alpha of 32% (39%). He also finds that both the alpha and beta decline for later-stage financing rounds. In the round-to-round returns data, Cochrane estimates a beta of 0.6 and an alpha of 45% per year.

Korteweg & Sorensen (2010) extend Cochrane's approach. They point out that the selection problem does not conform to the standard Heckman model assumption that unobserved outcomes (here, values) are uncorrelated with observed outcomes. Since values accumulate over time, a start-up's unobserved valuations are strongly related to its observed valuations, resulting in a more general dynamic selection problem. They formulate the problem as a state space model, where the underlying state is the value of the start-up. Returns are assumed to follow a factor model in logs, and values are observed according to a generic selection equation. The time since the last financing round serves as an instrument, as it changes the probability of observing a new round (or an exit), while it should not predict future unexpected returns. Using Bayesian Markov chain Monte Carlo methods to estimate their model, Korteweg & Sorensen (2010) find a CAPM beta of 2.8 and a monthly alpha of 3.3%. The Fama–French three-factor model alpha is nearly identical. Though the beta estimate is higher, the alpha is roughly in the ballpark of Cochrane's estimate. Splitting the sample by time period, they find an alpha of 1.6% before 1993, 5.8% during the Internet boom of 1994–2000, and −2.6% from 2000 to 2005. They also find suggestive evidence of a VC-specific factor that is related to the percentage change in aggregate VC investments. The Korteweg–Sorensen model has been applied to other asset classes: Korteweg & Sorensen (2016) use it to estimate real estate indices and distributions of loan-to-value ratios, and Korteweg, Kräussl & Verwijmeren (2016) estimate a more general version of the model in the art market.

Korteweg & Nagel (2016) estimate risk-adjusted returns from individual start-up data using the SDF model discussed in Section 2.1, above, which allows for arbitrary return distributions. They report that a \$1 investment in VC generates an NPV of approximately \$0.50–0.60. With the average time between rounds of roughly 1 year, this corresponds to approximately a 50–60% annual return, which is in the vicinity of Cochrane's round-to-round estimates.

[20]Note that the RSRs in the index method are usually specified on log returns as well, which affects the index calculation (see Goetzmann 1992).

A puzzling result across studies is that the alphas estimated directly from the individual deal data are substantially higher than the alphas from the index methods. It is unlikely that data are the driver of this difference, as Korteweg & Sorensen (2010) use the same data source (though updated and extended) as Peng (2001) and Hwang, Quigley & Woodward (2005).

Finally, the VC deal-return literature typically assumes that VCs own common equity claims, as it has been difficult to collect large samples of contract data. In reality, VCs usually own convertible preferred equity with bells and whistles such as liquidation preferences, participation rights, and board representation, among others. The standard argument for ignoring non–common equity features is that results are driven mainly by the large successes rather than the liquidation values of failures, and for reasonably large IPOs, VCs are automatically converted to common equity. However, IPOs have become less common since the turn of the millennium, so an increasingly large share of the payoffs has come from mergers and acquisitions. Moreover, recent studies find that reported (post money) valuations are biased when non–common equity features are ignored (e.g., Ewens, Gorbenko & Korteweg 2019, Gornall & Strebulaev 2019), which could affect the calculation of round-to-round returns. How contracts affect deal-level returns is still an open question. With more detailed contract data becoming available, this may be a fruitful avenue of future research. By contrast, note that in fund-level data the contract between entrepreneur and GP is not an issue for return computations, because the fund cash flows reflect the contractual division of payoffs.

Turning to buyout data, survivorship is somewhat less of a concern compared with VC, though selection bias remains an issue because data are often sourced from one or a few LPs. Kaplan (1989) finds an average market-adjusted return of 42% (median 28%) to 25 public-to-private BO deals from the early 1980s. This is the return to all the buyout capital, debt plus equity, from the initial deal until final exit, which takes 2.7 years on average. The equity investors' return (i.e., the GPs' return) is considerably higher. BO returns have come down since the 1980s as the industry has grown and become more competitive. Groh & Gottschalg (2011) collect data from private placement memoranda, which GPs provide to potential investors when fundraising. They contain the full history of deals of a given GP, but there may be some bias due to the identity of LPs who are willing to share data, and because more successful GPs are more likely to raise a follow-on fund in the first place. The average (median) IRR to the GP across 133 US buyouts is 50.1% (35.7%) per year. In comparison, the average return on a mimicking investment in the levered market portfolio is 9% to 12.6%, depending on assumptions. Franzoni, Nowak & Phalippou (2012) collect a large data set of 4,403 individual BO investments. They regress the log modified IRR to the GP of monthly portfolios of BO deals on log factor returns over the same period. The log-CAPM beta is 0.9, and the authors find a positive loading on value and an insignificant loading on size in the Fama–French three-factor model (in logs). The annualized alpha is 9.3% in the log-CAPM and 3.1% in the Fama–French model.[21] Axelson, Sorensen & Stromberg (2014) have data on all 2,075 deals of a single BO fund-of-funds, and find an annual alpha of 8.6% and a log-CAPM beta of 2.4. They also estimate a CAPM with jumps, motivated by the illiquid nature of buyouts. This does not alter the beta estimate significantly, but raises the alpha to 16.3% per year. Their beta estimate is considerably higher than in most other BO papers. They rationalize this number from a Modigliani–Miller calculation (similar to that of Groh & Gottschalg 2011), assuming the underlying company is representative of traded equities, has an (unlevered) asset beta of 0.66, and has a typical leverage ratio for buyouts. Buchner & Stucke (2014) also find a high BO beta in deal-level data. Fees are unlikely to explain the difference, and this remains a puzzle for future research to resolve.

[21]See Section 2 for further discussion of regressions that use IRR as the return metric.

Using the SDF perspective on risk and return, Acharya et al. (2013) find a sector-adjusted PME of 1.9 for 395 buyout deals sourced from McKinsey (a consulting firm that serves large PE firms) and a large LP, and Braun, Jenkinson & Stoff (2017) report a median PME of 1.3 for 12,541 buyout deals from three large fund-of-fund managers. Braun et al. find that buyout returns have been consistently high over time, although the persistence of individual GPs has declined as the asset class has matured and competition has increased.

4. PUBLICLY TRADED PARTNERSHIPS

In order to avoid the problems with fund- and deal-level data, researchers have turned to publicly traded PE partnership data. Jegadeesh, Kräussl & Pollet (2015) collect data on 24 traded funds-of-funds and 129 traded PE partnerships. They find statistically insignificant VC and BO alphas for both the CAPM and a four-factor (Fama–French three-factor plus momentum) model.[22] These results are quite different from those of Harris et al. (2018), who find that funds-of-funds on average outperform the market, and the weight of evidence that the average BO fund outperforms. McCourt (2018) finds results that are more in line with the unlisted fund data: On the basis of 134 listed PE funds, he finds positive excess returns for BO funds and insignificant performance for VC, using the same four-factor model as Jegadeesh, Kräussl & Pollet (2015). Using traditional tools from the mutual fund literature to separate skill and luck, he also finds evidence of skill in BO, but not in VC (by contrast, Hochberg, Ljungqvist & Vissing-Jorgensen 2014 and Korteweg & Sorensen 2017 find skill in both). One reason for the difference in results between the Jegadeesh, Kräussl & Pollet (2015) and McCourt (2018) papers may be that the latter includes the period following the financial crisis.

The publicly traded partnerships are a useful alternate lens through which to view PE risk-adjusted returns. However, it is not clear how the results should be compared with the literature on unlisted funds. First, because it is difficult to get a good sense of true NAVs (due to the issues discussed above), and payout policies tend to be highly smoothed, the market returns of listed funds may not capture all the underlying variation in PE returns in a timely manner, which could explain why estimated betas are at the lower end of the spectrum. Perhaps this is not a concern in the long run when funds liquidate (reducing the reliance on NAVs), but the time series of PE data is short: Traded funds data go back only 20 years, and the time series of unlisted funds data is not much longer. The second issue is that the sample of traded funds is small and possibly selected, although both Jegadeesh, Kräussl & Pollet (2015) and McCourt (2018) argue that they are representative. Third, the fact that the funds are publicly traded may change GPs' incentives compared with the same fund if it were unlisted, complicating direct comparisons. Finally, to the extent that results are based on publicly traded firms such as Blackstone or KKR, these more closely reflect the GPs' share of returns rather than the LPs' share, and they tend to reflect a mix of investments that is not pure-play VC or BO, or even limited to only PE.

5. CONCLUSION

Many methods have been proposed to evaluate risk and return in PE. The recent literature appears to be converging toward the use of SDFs and benchmarking approaches, but much work remains to be done. A key open question involves the set of risk factors in PE. Are PE returns spanned by publicly traded assets, or is there a component of returns that cannot be captured otherwise? If

[22]Jegadeesh, Kräussl & Pollet (2015) also exploit the divergence between market prices and NAVs to obtain an alternate estimate of manager skill.

the latter, is that component a risk premium unique to PE, or is it pure alpha? How much cross-sectional and time-series variation (both in calendar time and over the life of a fund or portfolio company) is there in factor loadings and, ultimately, in risk-adjusted returns? Can we reconcile prefee and postfee returns, how does the fee structure affect risk taking, and what does that imply for GP compensation? With more insight into the risk and return question, many applications remain to be explored in more depth. For example, what is the degree of persistence in GP and LP risk-adjusted returns (and why do LP returns persist in the first place)? Do managers have styles? How do agency problems or measurement issues from contractual arrangements (between GPs and LPs, and between GPs and portfolio companies) affect returns? And how does PE fit into a broader portfolio of assets? Finally, the methods developed for PE can be applied to other assets that are infrequently traded (e.g., real estate, distressed debt), as they are plagued by similar empirical problems.

With the increasing availability of PE data sets of higher quantity and quality, and with recent developments in methodology, risk adjustment in PE returns looks to be a promising area for future research.

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LITERATURE CITED

Acharya VV, Gottschalg OF, Hahn M, Kehoe C. 2013. Corporate governance and value creation: evidence from private equity. *Rev. Financ. Stud.* 26:368–402

Andonov A, Kräussl R, Rauh JD. 2019. *The subsidy to infrastructure as an asset class*. Work. Pap., Univ. Amsterdam/Univ. Luxembourg/Stanford Univ.

Ang A, Chen B, Goetzmann WN, Phalippou L. 2018. Estimating private equity returns from limited partner cash flows. *J. Finance* 73:1751–83

Ang A, Hodrick RJ, Xing Y, Zhang X. 2006. The cross-section of volatility and expected returns. *J. Finance* 61:259–99

Ang A, Liu J. 2004. How to discount cashflows with time-varying expected returns. *J. Finance* 59:2745–83

Anson MJP. 2002. Managed pricing and the rule of conservatism in private equity portfolios. *J. Priv. Equity* 5:18–30

Anson MJP. 2007. Performance measurement in private equity: another look. *J. Priv. Equity* 10:7–21

Axelson U, Jenkinson T, Stromberg P, Weisbach MS. 2013. Borrow cheap, buy high: the determinants of leverage and pricing in buyouts. *J. Finance* 68:2223–67

Axelson U, Sorensen M, Stromberg P. 2014. *Alpha and beta of buyout deals: a jump CAPM for long-term illiquid investments*. Work. Pap., London Sch. Econ./Columbia Bus. Sch./Stockholm Sch. Econ.

Bailey MJ, Muth RF, Nourse HO. 1963. A regression method for real estate price index construction. *J. Am. Stat. Assoc.* 58:933–42

Barber BM, Yasuda A. 2017. Interim fund performance and fundraising in private equity. *J. Financ. Econ.* 124:172–94

Berk JB, Green RC. 2004. Mutual fund flows and performance in rational markets. *J. Political Econ.* 112:1269–95

Bollen NPB, Sensoy BA. 2016. *How much for a haircut? Illiquidity, secondary markets, and the value of private equity*. Work. Pap., Vanderbilt Univ., Nashville, TN
Boyer B, Nadauld TD, Vorkink KP, Weisbach MS. 2018. *Private equity indices based on secondary market transactions*. NBER Work. Pap. 25207
Braun R, Jenkinson T, Stoff I. 2017. How persistent is private equity performance? Evidence from deal-level data. *J. Financ. Econ.* 123:273–91
Brennan MJ. 1997. The term structure of discount rates. *Financ. Manag.* 26:81–90
Brown GW, Gredil O, Kaplan SN. 2019. Do private equity funds manipulate reported returns? *J. Financ. Econ.* 132:267–97
Buchner A. 2016. Risk-adjusting the returns of private equity using the CAPM and multi-factor extensions. *Finance Res. Lett.* 16:154–61
Buchner A, Stucke R. 2014. *The systematic risk of private equity*. Work. Pap., Univ. Passau/Univ. Oxford
Case KE, Shiller RJ. 1987. Prices of single-family homes since 1970: new indexes for four cities. *N. Engl. Econ. Rev.* 1987:46–56
Case KE, Shiller RJ. 1989. The efficiency of the market for single-family homes. *Am. Econ. Rev.* 79:125–37
Cochrane JH. 2005a. The risk and return of venture capital. *J. Financ. Econ.* 75:3–52
Cochrane JH. 2005b. *Asset Pricing*. Princeton, NJ: Princeton Univ. Press. Revis. ed.
Constantinides GM. 1980. Admissible uncertainty in the intertemporal asset pricing model. *J. Financ. Econ.* 8:71–86
Dimson E. 1979. Risk measurement when shares are subject to infrequent trading. *J. Financ. Econ.* 7:197–226
Driessen J, Lin T-C, Phalippou L. 2012. A new method to estimate risk and return of nontraded assets from cash flows: the case of private equity funds. *J. Financ. Quant. Anal.* 47:511–35
Emery K. 2003. Private equity risk and reward: assessing the stale pricing problem. *J. Priv. Equity* 6:43–50
Ewens M, Gorbenko A, Korteweg A. 2019. *Venture capital contracts*. Work. Pap., Calif. Inst. Technol./Univ. South. Calif.
Ewens M, Jones CM, Rhodes-Kropf M. 2013. The price of diversifiable risk in venture capital and private equity. *Rev. Financ. Stud.* 26:1854–89
Ewens M, Rhodes-Kropf M. 2015. Is a VC partnership greater than the sum of its partners? *J. Finance* 70:1081–113
Fama EF, French KR. 1993. Common risk factors in the returns on stocks and bonds. *J. Financ. Econ.* 33:3–56
Fang L, Ivashina V, Lerner J. 2015. The disintermediation of financial markets: direct investing in private equity. *J. Financ. Econ.* 116:160–78
Franzoni F, Nowak E, Phalippou L. 2012. Private equity performance and liquidity risk. *J. Finance* 67:2341–73
Gatzlaff DH, Haurin DR. 1997. Sample selection bias and repeat-sales index estimates. *J. Real Estate Finance Econ.* 14:33–50
Gatzlaff DH, Haurin DR. 1998. Sample selection and biases in local house value indices. *J. Urban Econ.* 43:199–222
Giacoletti M, Westrupp V. 2018. *The risk-adjusted performance of asset flippers*. Work. Pap., Univ. South. Calif./AQR
Goetzmann WN. 1992. The accuracy of real estate indices: repeat sale estimators. *J. Real Estate Finance Econ.* 5:5–53
Goetzmann WN, Peng L. 2006. Estimating house price indexes in the presence of seller reservation prices. *Rev. Econ. Stat.* 88:100–12
Gompers PA, Gornall W, Kaplan SN, Strebulaev IA. 2019. How do venture capitalists make decisions? *J. Financ. Econ.* In press
Gompers PA, Kaplan SN, Mukharlyamov V. 2016. What do private equity firms say they do? *J. Financ. Econ.* 121:449–76
Gompers PA, Lerner J. 1997. Risk and reward in private equity investments: the challenge of performance assessment. *J. Priv. Equity* 1:5–12
Gornall W, Strebulaev IA. 2019. Squaring venture capital valuations with reality. *J. Financ. Econ.* In press
Gredil O, Griffiths BE, Stucke R. 2014. *Benchmarking private equity: the direct alpha method*. Work. Pap., Tulane Univ./Landmark Partners/Univ. Oxford

Gredil O, Sorensen M, Waller W. 2018. *Evaluating private equity performance using stochastic discount factors*. Work. Pap., Tulane Univ./Copenhagen Bus. Sch.

Groh AP, Gottschalg O. 2011. The effect of leverage on the cost of capital of US buyouts. *J. Bank. Finance* 35:2099–110

Gupta A, van Nieuwerburgh S. 2018. *Valuing private equity investments strip by strip*. Work. Pap., NYU, New York

Hall RE, Woodward SE. 2010. The burden of the nondiversifiable risk of entrepreneurship. *Am. Econ. Rev.* 100:1163–94

Harris RS, Jenkinson T, Kaplan SN. 2014. Private equity performance: What do we know? *J. Finance* 69:1851–82

Harris RS, Jenkinson T, Kaplan SN, Stucke R. 2014. *Has persistence persisted in private equity? Evidence from buyout and venture capital funds*. Work. Pap., Univ. Va./Univ. Chicago/Univ. Oxford

Harris RS, Jenkinson T, Kaplan SN, Stucke R. 2018. Financial intermediation in private equity: How well do funds of funds perform? *J. Financ. Econ.* 129:287–305

Haurin DR, Hendershott PH. 1991. House price indexes: issues and results. *Real Estate Econ*. 19:259–69

Heckman JJ. 1979. Sample selection bias as a specification error. *Econometrica* 47:153–61

Higson C, Stucke R. 2012. *The performance of private equity*. Work. Pap., London Bus. Sch./Univ. Oxford

Hochberg YV, Ljungqvist A, Vissing-Jorgensen A. 2014. Informational holdup and performance persistence in venture capital. *Rev. Financ. Stud.* 27:102–52

Hwang M, Quigley JM. 2004. Selectivity, quality adjustment and mean reversion in the measurement of house values. *J. Real Estate Finance Econ.* 28:161–78

Hwang M, Quigley JM, Woodward SE. 2005. An index for venture capital, 1987–2003. *Contrib. Econ. Anal. Policy* 4:1–43

Jegadeesh N, Kräussl R, Pollet JM. 2015. Risk and expected returns of private equity investments: evidence based on market prices. *Rev. Financ. Stud.* 28:3269–302

Jenkinson T, Sousa M, Stucke R. 2013. *How fair are the valuations of private equity funds?* Work. Pap., Univ. Oxford, Oxford, UK

Jud GD, Seaks TG. 1994. Sample selection bias in estimating housing sales prices. *J. Real Estate Res.* 9:289–98

Kaplan SN. 1989. The effects of management buyouts on operating performance and value. *J. Financ. Econ.* 24:217–54

Kaplan SN, Schoar A. 2005. Private equity performance: returns, persistence, and capital flows. *J. Finance* 60:1791–823

Kaplan SN, Sensoy BA. 2015. Private equity performance: a survey. *Annu. Rev. Financ. Econ.* 7:597–614

Kocis JM, Bachman JC, Long AM, Nickels CJ. 2009. *Inside Private Equity*. Hoboken, NJ: Wiley

Korteweg A, Kräussl R, Verwijmeren P. 2016. Does it pay to invest in art? A selection-corrected returns perspective. *Rev. Financ. Stud.* 29:1007–38

Korteweg A, Nagel S. 2016. Risk-adjusting the returns to venture capital. *J. Finance* 71:1437–70

Korteweg A, Nagel S. 2018. *Risk-adjusted returns of private equity funds: a new approach*. Work. Pap., Univ. South. Calif./Univ. Chicago

Korteweg A, Sorensen M. 2010. Risk and return characteristics of venture capital–backed entrepreneurial companies. *Rev. Financ. Stud.* 23:3738–72

Korteweg A, Sorensen M. 2016. Estimating loan-to-value distributions. *Real Estate Econ*. 44:41–86

Korteweg A, Sorensen M. 2017. Skill and luck in private equity performance. *J. Financ. Econ.* 124:535–62

Lerner J, Schoar A, Mao J, Zhang NR. 2018. *Investing outside the box: evidence from alternative vehicles in private capital*. NBER Work. Pap. 24941

Ljungqvist A, Richardson M. 2003. *The cash flow, return and risk characteristics of private equity*. NBER Work. Pap. 9454

Long AM, Nickels CJ. 1996. *A private investment benchmark*. Work. Pap., Univ. Tex. Invest. Manag. Co., Austin

McCourt M. 2018. *Estimating skill in private equity performance using market data*. Work. Pap., Univ. Melbourne, Melbourne, Aust.

McKenzie M, Janeway W. 2008. *Venture capital fund performance and the IPO market*. Work. Pap., Univ. Cambridge, Cambridge, UK

Moskowitz TJ, Vissing-Jorgensen A. 2002. The returns to entrepreneurial investment: a private equity premium puzzle? *Am. Econ. Rev.* 92:745–78

Munneke HJ, Slade BA. 2000. An empirical study of sample-selection bias in indices of commercial real estate. *J. Real Estate Finance Econ.* 21:45–64

Munneke HJ, Slade BA. 2001. A metropolitan transaction-based commercial price index: a time-varying parameter approach. *Real Estate Econ*. 29:55–84

NACUBO (Natl. Assoc. Coll. Univ. Bus. Off.). 2016. *2016 NACUBO–Commonfund Study of Endowments*. Final Rep., NACUBO, Washington, DC

Nadauld TD, Sensoy BA, Vorkink K, Weisbach MS. 2019. The liquidity cost of private equity investments: evidence from secondary market transactions. *J. Financ. Econ.* 132:158–81

Pastor L, Stambaugh RF. 2003. Liquidity risk and expected stock returns. *J. Political Econ.* 111:642–85

Peng L. 2001. *Building a venture capital index*. Work. Pap., Univ. Colo., Boulder

Pensions & Investments. 2017. Assets of top funds up 6.2% to $9.4 trillion. *Defined Benefit Blog*, Febr. 6. **https://www.pionline.com/article/20170206/PRINT/302069976/assets-of-top-funds-up-6-2-to-9-4-trillion**

Peters RH. 2018. *Volatility and venture capital*. Work. Pap., Tulane Univ., New Orleans

Phalippou L. 2014. Performance of buyout funds revisited? *Rev. Finance* 18:189–218

Phalippou L, Gottschalg O. 2009. The performance of private equity funds. *Rev. Financ. Stud.* 22:1747–76

Phalippou L, Rauch C, Umber M. 2018. Private equity portfolio company fees. *J. Financ. Econ.* 129:559–85

Robinson DT, Sensoy BA. 2016. Cyclicality, performance measurement, and cash flow liquidity in private equity. *J. Financ. Econ.* 122:521–43

Rubinstein M. 1976. The strong case for the generalized logarithmic utility model as the premier model of financial markets. *J. Finance* 31:551–71

Sorensen M, Jagannathan R. 2015. The public market equivalent and private equity performance. *Financ. Anal. J.* 71:43–50

Sorensen M, Wang N, Yang J. 2014. Valuing private equity. *Rev. Financ. Stud.* 27:1977–2021

Stafford E. 2017. *Replicating private equity with value investing, homemade leverage, and hold-to-maturity accounting*. Work. Pap., Harvard Bus. Sch., Boston

Stucke R. 2011. *Updating history*. Work. Pap., Univ. Oxford, Oxford, UK

Wallace NE, Meese RA. 1997. The construction of residential housing price indices: a comparison of repeat-sales, hedonic-regression, and hybrid approaches. *J. Real Estate Finance Econ.* 14:51–73

Woodward SE. 2009. *Measuring risk for venture capital and private equity portfolios*. Work. Pap., Sand Hill Econom., Palo Alto, CA

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