Has the Decline in the Price of Investment Increased Wealth Inequality?*

Simone Civale†

Department of Economics

University of Minnesota

November 23, 2016

Abstract

The price of investment relative to consumption has steadily declined in the United States over the last three decades. Over the same period, inequality at the top of the wealth distribution has increased substantially. This paper develops a framework that allows to associate these two trends. Households are modeled as entrepreneurs who accumulate capital, and are subject to persistent idiosyncratic productivity shocks. In this environment, the decline in the relative price of investment benefits productive entrepreneurs more than unproductive ones, increasing the saving rate differential between them. In turn, this increases wealth inequality. In a version of the model calibrated to match the observed degree of wealth concentration in the United States in 1980, the proposed mechanism can account for half of the increase in wealth inequality observed over the last thirty years.

Link to the latest version of the paper

*I am especially thankful for the guidance and feedback of Fatih Guvenen, Loukas Karabarbounis, and Fabrizio Perri. I also thank Anmol Bhandari, Kyle Herkenhoff, Ellen McGrattan, Luis Díez-Catalán, and Guillaume Sublet for helpful comments and discussions.

†Please direct your questions and comments to cival003@umn.edu

1 Introduction

The price of investment relative to consumption has decreased sharply in the United States over the last three decades. Some important macroeconomic phenomena have been explained in light of this decline as reflecting investment-specific technological change. Greenwood, Hercowitz, and Krusell (1997) consider the impact on growth, Krusell, Ohanian, Ríos-Rull, and Violante (2000) argue that it has caused an increase in wage inequality, whereas Karabarbounis and Neiman (2014) show that it has caused a decline in the labor share of income. This paper contributes to this literature by studying the implications of the decline in the relative price of investment on wealth inequality.

What motivates this line of inquiry is the substantial increase in concentration at the top of the wealth distribution that has occurred in the United States since the 1980s, as documented by Bricker, Henriques, Krimmel, and Sabelhaus (2015) and Saez and Zucman (2016). For instance, these papers estimate that the share of wealth owned by the 1% wealthiest households in United States increased, from 1989 to 2013, by 7 and 15 percentage points, respectively.

To study the effect of the declining price of investment on wealth inequality, this paper considers a framework in which households act as entrepreneurs who accumulate capital, and are subject to persistent idiosyncratic productivity shocks. The adoption of this environment allows the paper to model a meaningful degree of wealth concentration and to investigate its evolution.

The main finding of this paper is that, in a calibrated version of the model that matches the observed wealth distribution of the United States in 1980, the decline in the price of investment can account for half of the increase in wealth inequality observed over the last three decades in the United States.

The increase in wealth inequality is driven in part by a redistributive effect that the decline in the price of investment causes across heterogeneously productive households. In fact, as the price of investment declines, productive households that increase their capital stock can purchase investment goods more cheaply. On the other hand, for unproductive households that disinvest, purchasing a given amount of consumption becomes more costly.

The key mechanism driving the increase in wealth inequality is a change in saving behavior across households with different productivity types. In fact, in response to the drop in price, productive households increase their saving rate more than unproductive households.

This paper relates to the literature that studies the causes and modeling of wealth inequality,[1] and more specifically, to a strand of literature that explores the role of entrepreneurship in generating wealth mobility and inequality, which has its seminal contributions in Quadrini (1999, 2000) and Cagetti and De Nardi (2006). This paper builds on a paradigm of wealth inequality that is based on capital income risk, in the spirit of Angeletos (2007) and Benhabib, Bisin, and Zhu (2011, 2014). These authors show that, in different classes of models, when returns to wealth are heterogeneous, the stationary wealth distribution has a heavy tail that can match the observed wealth distribution in the United States. On the empirical side of this literature, Fagereng, Guiso, Malacrino, and Pistaferri (2015) use administrative Norwegian data to provide evidence of persistent heterogeneity in returns. Moskowitz and Vissing-Jørgensen (2002) show that returns on private equity are also highly dispersed because of a strong lack of diversification.[2]

Finally, this paper contributes to recent literature that investigates the causes of the increase in wealth inequality of the last three decades. Kaymak and Poschke (2016), in the context of an Aiyagari model in the fashion of Castañeda, Díaz-Giménez, and Ríos-Rull (2003), find that the changes in the United States tax system can account for half of the rise in wealth concentration. Hubmer, Krusell, and Smith (2015) build a model with capital income risk, heterogeneity in patience, and a very positively skewed income process. Under this model, the decrease in tax progressivity can explain most of the change in the wealth distribution observed in the United States. Gabaix, Lasry, Lions, and Moll (2015) argue that models with capital income risk are a better candidate to explain wealth inequality because they can generate transition dynamics that are as fast as the changes in wealth concentration observed in the United States data.

The rest of the paper is organized as follows. Section 2 discusses the data on the price of investment, and on wealth inequality, that are relevant to this paper. Section 3

[1] In the context of an Aiyagari (1994) model—where the only source of risk is the labor endowment—several mechanisms have been explored. This literature studies, among others, heterogeneity in patience, bequest motives, human capital transmission across generations, and a high-risk earning process. For a survey, see De Nardi (2015).

[2] Benhabib and Bisin (2016) offer a detailed discussion about the theoretical and empirical background of this strand of literature.

lays out the benchmark model. Section 4 considers a simpler two-period version of the benchmark model to develop the intuition for the effect of the decline in the price of investment on capital accumulation. Section 5 calibrates the model and presents the main quantitative results of the paper. Section 6 tests the robustness of the results to departures from the baseline calibration and assumptions. Section 7 concludes.

2 Data

This section presents the data that are relevant for this paper. The first part shows the data on the decline in the relative price of investment goods in the United States. The second part illustrates the data on wealth inequality.

Relative Price of Investment Goods

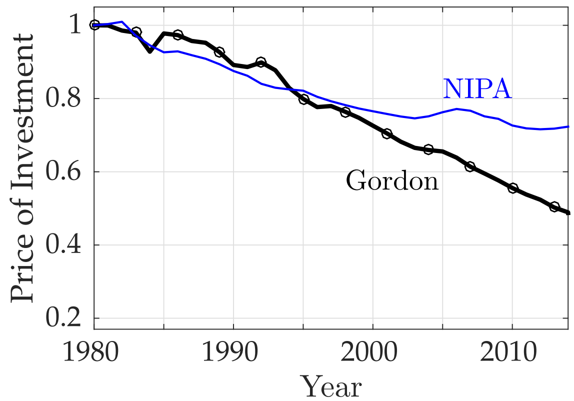
The national income and product accounts (NIPA) report the price deflator for several categories of investment. The price for each of these categories relative to consumption is computed using these deflators.

Since the benchmark model lumps together all types of investment, in the data the relative price of investment goods is computed as the ratio between the deflator of fixed investment and the deflator of personal consumption expenditure. Over the period 1980 to 2010, this relative price exhibits a decline of 27%.

NIPA controls for quality improvement when calculating quantities and prices of its accounts. The work of Gordon (1990), however, shows that NIPA understates quality improvement, and therefore the decline in the relative price of investment is understated. To correct this bias, DiCecio (2009) extrapolates the quality-adjusted price time series of Gordon (1990) to 2010, using the same technique of Cummins and Violante (2002) and Fisher (2006). This paper adopts the extrapolated time series of Gordon (1990) as a benchmark. When controlling for quality improvements, the relative price of investment goods declined by 44% from 1980 to 2010.

Figure 1 concludes this section by showing the time series of the relative price of investment.

Figure 1: Relative Price of Investment



All the time series in this figure are normalized to 1 in 1980. The Gordon time series is computed by DiCecio (2009), extrapolating the quality-adjusted series of Gordon (1990). The figure shows the price of investment relative to the price of consumption. The NIPA series is obtained as the ratio between the deflator of fixed investment and the deflator of personal consumption expenditure.

Wealth Inequality

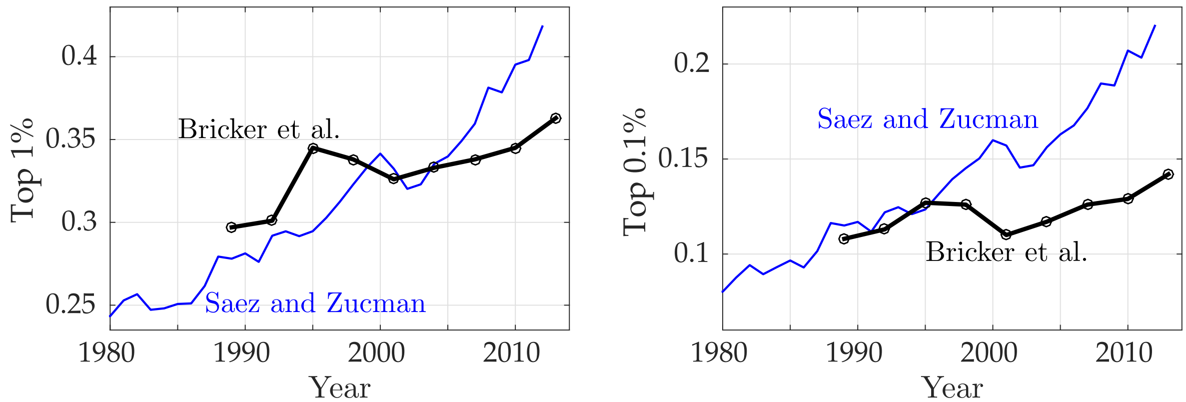
In this paper the word *wealth* refers to household net worth in the United States. To measure inequality at the top of the wealth distribution, this paper considers the share of wealth owned by the $x\%$ wealthiest households in the distribution, which is referred to as Top $x\%$.

Since taxes are not levied on wealth in the United States,[3] there is no administrative data set on wealth holdings. For this reason, the primary data source on wealth is a cross-sectional survey data set: the Survey of Consumer Finances (SCF). The SCF oversamples income-rich households to obtain a good representation of wealthy households. However, the nonresponse rate increases in the high-income strata, a problem that can be corrected for only in part with an appropriate system of weights (Kennickell and Woodburn, 1999).

To address concerns about non response and to obtain as detailed a picture of the top of the wealth distribution as possible, several scholars have proposed alternative ways to study wealth concentration at the top. Kopczuk and Saez (2004) use administrative data on estate taxation and Vermeulen (2014) corrects SCF estimates of inequality by including the net worth of the members of the Forbes 400 list, who are by construction excluded from sampling in the SCF. Finally, in an influential paper, Saez and Zucman (2016) use administrative data on capital income taxation to reconstruct

[3] A notable exeption is estate taxation.

Figure 2: Top 1% and Top 0.1% Share of Wealth



The time series in these figures are computed by Bricker, Henriques, Krimmel, and Sabelhaus (2015) and Saez and Zucman (2016). The left panel shows the Top 1%, the share of household net worth owned by the wealthiest 1% of households in the United States. The right panel shows the Top 0.1%, the share of household net worth owned by the wealthiest 0.1% of households in the United States.

wealth holdings by capitalizing the flow of capital income.

These different data sources sometimes produce very different estimates of the level and trend of wealth inequality in the United States. Bricker, Henriques, Krimmel, and Sabelhaus (2015) argue that the estimates produced using the SCF are of high quality, and that a large part of the difference between these estimates and the figures of Saez and Zucman (2016) can be explained as conceptual differences between the object measured using these two data sources.

Bricker et al. (2015), over the period 1989 to 2013, find a 7 percentage points increase in the Top 1%, of which 3 percentage points accrued to the Top 0.1%. Over the same period of time, Saez and Zucman (2016) find a sharper change, with a 15 percentage points increase in the Top 1%, of which 13 percentage points accrued to the Top 0.1%. Using the SCF, Wolff (2014) finds that the Gini coefficient of wealth increased by 7 percentage points in the period 1983 to 2013.

In light of this discussion, the strategy of this paper is to use the estimates produced using the SCF by Bricker et al. (2015) and Wolff (2014) as a benchmark. The statistics of Saez and Zucman (2016) are used as a supplement to the SCF estimates and as an upper bound to gauge the increase in wealth inequality.

Figure 2 concludes this section and shows the time series describing the trend in wealth inequality at the top of the distribution.

3 Model

This section develops a model for studying the impact of the declining relative price of investment on wealth inequality. Throughout the paper, the model introduced in this section is referred to as the *benchmark model*. After presenting the benchmark model, this section describes the notion of equilibrium for the model. The last part of the section presents a simplified two-period version of the benchmark model that is used to discuss the forces at work in the benchmark model as the relative price of investment declines.

To have a meaningful discussion of wealth inequality, this section draws on a model environment that can reproduce the degree of wealth concentration observed in the United States, where each household is an entrepreneur with stochastic idiosyncratic productivity.

This class of models is appealing because the decline in the price of investment interacts in a significant way with the saving decision, therefore affecting wealth inequality.

Time is discrete and the horizon is infinite, $t = 1, 2, \ldots$. There is a continuum of measure one of households indexed by i. There are three layers—optimization problems—to this economy: (i) A household supplies labor and owns a monopolistically competitive firm producing one good variety. (ii) The final good producer competitively aggregates good varieties and labor to produce a final good. (iii) Investment and consumption goods are produced competitively using the final good as an input.

In what follows, the description of the model begins with the problem of the final good producer and of the consumption and investment goods producers. This section first characterizes the competitive price functions from these optimization problems, and then presents the household problem.

Final Good Producer

An individual household is denoted using index i. Let $y_{i,t}$ be the quantity of goods produced by household i at time t.

The final good producer uses a Cobb-Douglas technology to produce the final good,

$$Y_t = Q_t^{\alpha} L_t^{1-\alpha}.$$

Aggregate capital Q_t is an aggregation of household varieties using a constant elasticity of substitution (CES) production technology,

$$Q_t = \left(\int_i y_{i,t}^{\mu} \right)^{\frac{1}{\mu}}.$$

Let us denote by μ the CES curvature parameter, which can also be expressed in terms of the elasticity of substitution (ES) between varieties as $\mu = (\mathrm{ES} - 1)/\mathrm{ES}$. The capital share of income is α.

The final good producer solves the following problem:

$$\max_{\{y_{i,t}\}_i, L_t} Y_t - w_t L_t - \int_i y_{i,t} p_{i,t},$$

where w denotes the wage rate and p denotes the price of a good variety. This implies price functions

$$\begin{aligned} w_t &= (1-\alpha) Q_t^{\alpha} L_t^{-\alpha}, \\ p_{i,t} &= L_t^{1-\alpha} \alpha Q_t^{-\mu+\alpha} y_{i,t}^{\mu-1}. \end{aligned}$$

Consumption and Investment Good Producers

The consumption good producer uses a linear technology that turns one unit of the final good into one unit of the consumption good $C_t = Y_t^c$.

Let p^x denote the price of one unit of the investment good. The investment good producer uses a linear technology that turns one unit of the final good into A_t^x units of the investment good X_t. Therefore, it solves

$$\max_{Y_t^x} X_t p_t^x - Y_t^x$$

subject to

$$X_t = Y_t^x A_t^x.$$

Hence $p_t^x = 1/A_t^x$. This modeling strategy implies that the relative price of investment is driven entirely by investment-specific production efficiency. In the study of transitional dynamics, A_t^x changes exogenously.

Households

Let k denote capital and θ the entrepreneurial type. The entrepreneurial type of the household evolves according to an AR(1) process. Each household operates a firm with its capital stock and its entrepreneurial type, earning profit $\pi(k, \theta)$. Capital depreciates at rate δ, and investment is denoted by x. Each household provides one unit of labor inelastically.

The Bellman equation of household i is

$$V_t\left(k_{i,t}, \theta_{i,t}; p_t^x, Q_t, w_t\right) = \max_{c_{i,t}, k_{i,t+1}, x_{i,t}} \left\{u\left(c_{i,t}\right) + \beta \mathbb{E} V_{t+1}\left(k_{i,t+1}, \theta_{i,t+1}; p_{t+1}^x, Q_{t+1}, w_{t+1}\right)\right\}$$

subject to

$$\begin{aligned} c_{i,t} + x_{i,t} p_t^x &= w_t + \pi\left(k_{i,t}, \theta_{i,t}\right), \\ x_{i,t} &= k_{i,t+1} - k_{i,t}\left(1-\delta\right), \\ \log\theta_{i,t+1} &= \rho \log\theta_{i,t} + \varepsilon_{i,t}, \end{aligned}$$

with $k_{i,t+1} \geq 0$, $c_{i,t} > 0$, and $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma)$. Each household operates a firm that employs a linear technology

$$y_{i,t} = \theta_{i,t} k_{i,t}$$

and earns profit π given by

$$\begin{aligned} \pi\left(k_{i,t}, \theta_{i,t}\right) &= y_{i,t} p_{i,t}, \\ &= \alpha Q_t^{-\mu+\alpha}\left(\theta_{i,t} k_{i,t}\right)^{\mu}. \end{aligned} \tag{1}$$

Notice that these equations incorporate the market-clearing condition $L_t = 1$.

Since the household runs a monopolistically competitive firm, if $\mu < 1$, it faces decreasing marginal profits. The curvate parameter μ governs the rate at which marginal profits decline. Declining marginal profits imply that the distribution of capital is stationary on a bounded support.

Equation 1 also shows that aggregate capital Q_t feeds into the firm profit. This has interesting implications because, as the price of investment declines, aggregate capital increases. Sections 4 and 5 consider in more detail the consequences of an increase in aggregate capital on household capital income.

Market Clearing

Two markets need to clear in this model: the market for consumption and investment goods,

$$\begin{aligned} Y_t &= Y_t^c + Y_t^x, \\ &= \int_i c_{i,t} + \frac{x_{i,t}}{A_t^x}, \end{aligned}$$

and the labor market,

$$\int_i l_{i,t} = L_t = 1.$$

Equilibrium

In this economy, idiosyncratic uncertainty washes out at the aggregate. Consequently there is no aggregate uncertainty. For this reason, aggregate quantities and prices follow a deterministic path. Households choose optimal contingent plans that at the equilibrium are consistent with the aggregate path. More formally, a **competitive equilibrium** for this economy is as follows:

(i) exogenous sequence $\{A_t^x\}$,

(ii) sequence of prices $\{p_t^x, w_t\}$ and quantities $\{Q_t\}$,

(iii) collection of contingent household plans $\{c_{i,t}, x_{i,t}, k_{i,t+1}\}$,

that satisfy the following conditions:

(1) given prices and quantities $\{c_{i,t}, x_{i,t}, k_{i,t+1}\}$ solve the household problem,

(2) prices $\{p_t^x, w_t, p_{i,t}\}$ are competitive,

(3) markets clear,

(4) aggregation holds.

A **stationary competitive equilibrium** is a competitive equilibrium such that $\{A_t^x, p_t^x, Q_t, w_t\}$ are constant and the measure of households over (k_t, θ_t) is constant.

4 Simple Model

In the benchmark model, investment-specific technological progress decreases the price of investment; that is, as A_t^x increases, $p_t^x = 1/A_t^x$ decreases.

This section considers the effect of a drop in the price of investment on wealth inequality. The reason why, in this model, a drop in price can affect wealth inequality is that agents with heterogeneous productivity types respond differently to a drop in p_t^x.

To see why this type of asymmetric effect is important, consider what happens if the decline in the price provides incentives to increase saving that are stronger for more productive agents. Because in this model more productive agents are wealthier and have higher saving rates on average, a further increase in their saving rates increases wealth inequality.

Since in the benchmark model, household wealth is in the form of capital, capital accumulation uniquely determines the evolution of wealth inequality. For this reason, this section considers a two-period model of capital accumulation that is based on a simplification of the benchmark model, which is referred to as the *simple model*.

The simplifying assumptions are the following: (i) There is no labor income. (ii) Capital income takes the form $k\theta\phi$, where θ is the idiosyncratic productivity type of the agent and ϕ is the component of return to capital that is common to all agents, net of depreciation.

Under these assumptions, and suppressing all time and individual subscripts, the budget equations of the benchmark model simplify to

$$\begin{aligned} c + xp &= k\theta\phi, \\ k' &= k + x, \end{aligned}$$

so that

$$x = \frac{\theta\phi k - c}{p} = \frac{\theta\phi k}{p}\left(1 - \frac{c}{\theta\phi k}\right) = k\frac{\theta\phi}{p}s,$$

where $s = 1 - c/\theta\phi k$ is the saving rate of the agent. Notice that $s < 1$ because the agent can save at most all of her income. Furthermore, since the agent can dissave at most all of her capital, it follows that $s > -p/(\theta\phi)$. Finally, the law of motion for capital is

$$k' = k\left(1 + \frac{\theta\phi s}{p}\right). \tag{2}$$

The law of motion for capital in the simple model is used to measure the effect of a drop in the price of investment on k' and whether this effect differs across productivity types.

Let us consider two types of effect: (i) The *direct* effect is the change in k' caused by the drop in p_t^x, while holding fixed the saving rate. This effect is direct because it occurs without affecting the saving rates. (ii) The *incentive* effect is the effect that occurs on k' when the agent changes her saving rate in response to a drop in p_t^x. This effect quantifies the change in the agents' incentives to save in response to the drop in price.

The Effect of the Price of Investment

Previewing the results, this section finds that the incentive effect is positive and increasing in the household type. Therefore, when the price of investment drops more productive agents have stronger incentives to increase their saving rates.

Let us describe the direct effect of a decline in the price of investment on capital accumulation using the following elasticity:

$$\mathcal{E}^p = \frac{\partial \log k'}{-\partial \log p} = \frac{\theta\phi s}{p + \theta\phi s}, \tag{3}$$

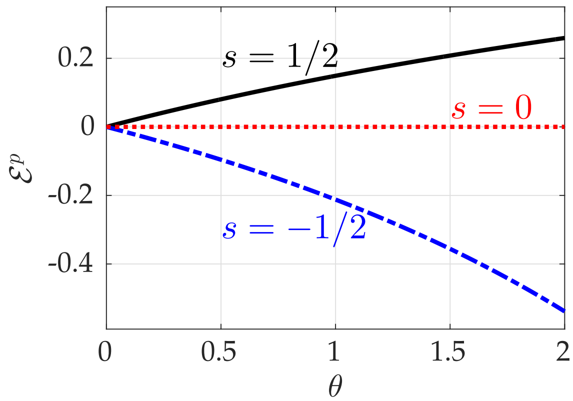
which measures by what percentage accumulated capital changes tomorrow following a 1% drop in price today. Notice that measuring the relative effect—as opposed to the effect in levels—is the correct strategy for gauging the effect on wealth inequality because wealth inequality is measured as the degree of relative dispersion in wealth.

Let us consider the sign of $\mathcal{E}^p$. Since $s > -p/(\theta\phi)$, the denominator is always positive. Since θ and ϕ are positive, $\mathcal{E}^p$ is positive if $s > 0$. Furthermore, $\mathcal{E}^p$ is increasing in θ if $s > 0$ and decreasing otherwise. Figure 3 illustrates this by plotting $\mathcal{E}^p$ as a function of θ for three different values of s.

As the figure shows, there are two scenarios based on whether the saving rate is positive or negative. For an agent who is increasing her capital holding, the decline in price entails a positive direct effect because as p declines, the same amount of capital income $\theta\phi s$ purchases more capital. In this scenario, a more productive agent stands to benefit more from the drop in price because she has more capital income to purchase capital. When $s < 0$, the direct effect is negative because to purchase a given amount of consumption, more capital needs to be dissaved.

The direct effect is therefore ambiguous with respect to θ. In fact, whether $\mathcal{E}^p$ is

Figure 3: The Direct Effect of the Drop in Price of Investment



This figure illustrates the direct effect of the drop in price, $\mathcal{E}^p$. This represents by what percentage accumulated capital varies in response to a 1% drop in price. The direct effect is positive and increasing in θ if $s > 0$, whereas it is negative and decreasing in θ if $s < 0$.

increasing or decreasing depends on s, which in the benchmark model is endogenous and itself a function of θ.[4]

The next step is to consider the incentive effect, that is, the effect of a drop in p on the incentives to save. Let us describe the incentive effect using the following derivative:

$$\mathcal{E}^p_s = \frac{\partial \mathcal{E}^p}{\partial s} = \frac{p\phi\theta}{(p + \theta\phi s)^2}, \tag{4}$$

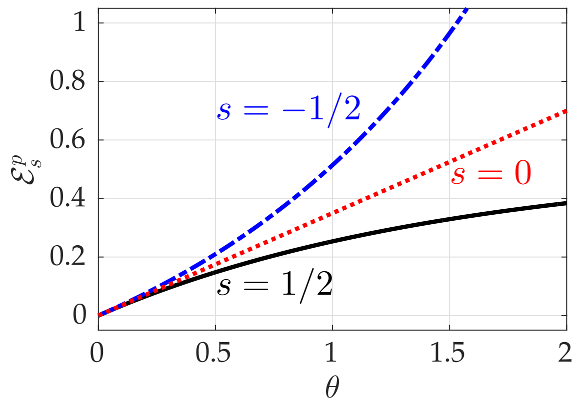
which measures—in percentage terms—the effect on capital accumulation of an increase in the saving rate in response to a 1% drop in price. Figure 4 illustrates this by plotting $\mathcal{E}^p_s$ as a function of θ for three different values of s.

Equation 4 shows that $\mathcal{E}^p_s$ is positive, meaning that increasing saving magnifies the effect of the drop in price on capital accumulation. Therefore, the drop in price provides an incentive to increase saving across the board. Furthermore, $\mathcal{E}^p_s$ is an increasing function of θ. This implies that the incentives to increase saving are stronger for more productive agents; that is, the incentive effect is stronger for more productive agents. In turn, this increases wealth inequality because in the benchmark model productive households on average are wealthier and have higher saving rates.

In accordance with the intuition that Equation 4 helps develop, Section 5.3 shows

[4] In the benchmark model, the saving rate is monotonically increasing in the productivity type. Section 5.3 shows that the direct effect of the drop in the price causes wealth inequality to increase.

Figure 4: The Incentive Effect of the Drop in the Price of Investment



This figure illustrates the incentive effect of the drop in price, $\mathcal{E}_s^p$. This represents—in percentage terms—the effect on capital accumulation of an increase in the saving rate, in response to a 1% drop in price. The effect is positive and increasing in θ.

that, in the benchmark model, the change in wealth inequality is mostly caused by the change in the saving behavior of the agents.

The Effect of Aggregate Capital

In the benchmark model, profit has the following functional form:

$$\pi\left(k_{i,t}, \theta_{i,t}\right) = \alpha Q_t^{\alpha-\mu} \theta_{i,t}^{\mu} k_{i,t}^{\mu}, \tag{5}$$

and therefore decreases in Q_t if $\alpha < \mu$, where α represents the capital income share in the economy, and the parameter μ governs the returns to scale of the household's firm. When $\mu = 1$ the firm has constant marginal profits, whereas $\mu < 1$ implies decreasing marginal profits. The calibration exercise of Section 5.1 shows that the empirically relevant case is $\alpha < \mu$. Therefore capital income is decreasing in aggregate capital.

As investment becomes cheaper, households accumulate more capital, causing Q_t to increase. In light of the discussion about Equation 5, as aggregate capital increases, marginal profit decreases across the board for all households.

In the context of the simple model, where capital income is written as $k\theta\phi$, a decline in ϕ captures an increase in Q_t.

Following the same steps as in the previous section, let us study the effect of the decline in ϕ on capital accumulation. The direct effect is measured by what percentage accumulated capital varies following a 1% drop in ϕ, using the following elasticity:

$$\mathcal{E}^{\phi} = \frac{\partial \log k'}{-\partial \log \phi} = -\frac{\theta \phi s}{p + \theta \phi s}. \tag{6}$$

Figure 5 illustrates $\mathcal{E}^{\phi}$ as a function of θ for three different values of s. If $s > 0$, then $\mathcal{E}^{\phi}$ is negative and decreasing in θ, whereas if $s < 0$, $\mathcal{E}^{\phi}$ is positive and increasing in θ. The direct effect is therefore ambiguous with respect to θ.

The following derivative describes the incentive effect:

$$\mathcal{E}_s^{\phi} = \frac{\partial \mathcal{E}^{\phi}}{\partial s} = -\frac{p \phi \theta}{(p + \theta \phi s)^2}, \tag{7}$$

which measures—in percentage terms—the effect on capital accumulation of an increase in the saving rate in response to a 1% drop in ϕ. Figure 6 illustrates $\mathcal{E}_s^{\phi}$ as a function of θ for different values of s. Equation 7 shows that $\mathcal{E}_s^{\phi}$ is negative. This provides an incentive to decrease saving across the board. Also in this case, the effect is heterogeneous across productivity types. In particular, $\mathcal{E}_s^{\phi}$ is a decreasing function of θ, so that the incentive to dissave is greater for agents who are more productive.

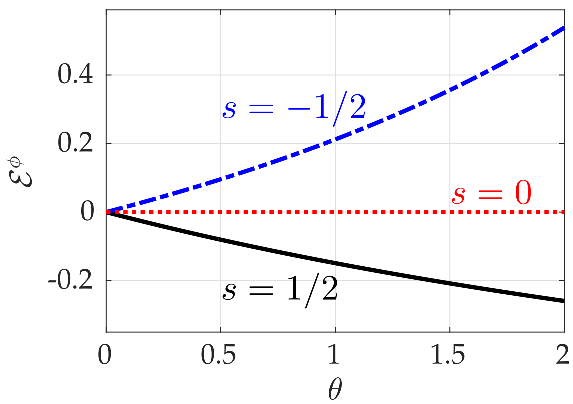
This last point is important to understand how an increase in aggregate capital affects wealth inequality. In fact, more productive households have incentives to decrease their saving rates that are stronger than for less productive households. This decreases wealth inequality because it compresses the saving rate differential between productive and unproductive households. In fact, Section 5.3 shows that, in the benchmark model, the increase in aggregate capital decreases wealth inequality.

Wealth Inequality

The discussion of the simple model shows that the decline in the price of investment and the increase in aggregate capital have asymmetric effects on agents with different productivity types.

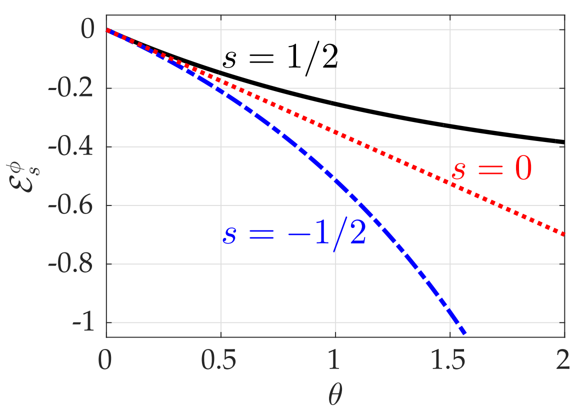
In particular, the decline in price provides stronger incentives to increase saving for more productive agents. This effect can increase wealth inequality in a model of entrepreneurship because more productive agents operate bigger firms, are wealthier, and have higher saving rates. On the other hand, an increase in aggregate capital can

Figure 5: The Direct Effect of the Increase in Aggregate Capital



This figure illustrates the direct effect of an increase in aggregate capital, represented as a drop in ϕ, that is, $\mathcal{E}^{\phi}$. The direct effect is positive and increasing in θ if $s < 0$, whereas it is negative and decreasing in θ if $s > 0$.

Figure 6: The Incentive Effect of the Increase in Aggregate Capital



This figure illustrates the incentive effect of the increase in aggregate capital represented as a drop in ϕ, that is, $\mathcal{E}^{\phi}_{s}$. The incentive effect is negative and decreasing in θ.

decrease wealth inequality because it provides incentives to decrease saving that are stronger for more productive agents.

This intuition is based on the two-period time horizon and on the set of simplifying assumptions of the simple model. Section 5 considers the quantitative solution of the benchmark model and uses the intuition gained from the simple model to analyze the results. Drawing a parallel with the simple model, it isolates the direct effect of the decline in price and finds that it increases wealth inequality. However, the incentive effect of the decline in price is the main driver of the quantitative result of this paper. In fact, wealth inequality increases because of a change in saving behavior as the price of investment declines.

Both the direct effect and the incentive effect of the increase in aggregate capital work in the direction of decreasing wealth inequality. Also in this case the incentive effect is the more important.

Interestingly, the increase in Q_t occurs more slowly than the decline in p_t^x, so that in the benchmark model, wealth inequality increases for the first three decades and then decreases again, reverting to levels that are comparable with those of the initial stationary equilibrium.

5 Quantitative Results

This section calibrates the model and presents the main quantitative results of this paper. The purpose of this quantitative exercise is to study the consequences of the decline in the price of investment on wealth inequality and to compare the model predictions with the path of wealth inequality observed in the United States from 1980 to 2010.

To accomplish this, the model is calibrated so that in the stationary equilibrium, it matches several characteristics of the United States economy in 1980. Then the declining time series of the relative price of investment—shown in Figure 1—is fed into the model. In this exercise, once the agents learn about the new declining path of the price of investment, they have perfect foresight. Section 6.1 considers the robustness of the model's implications for the perfect foresight assumption.

As the economy responds to the decline in the price of investment, wealth inequality increases over the transition to the new stationary equilibrium. The model can account for approximately half of the increase in wealth inequality observed in the United States from 1989 to 2013, as measured by the SCF.

5.1 Calibration

This section calibrates the model. Throughout the paper, this calibration of the model is referred to as the *benchmark calibration*. Each period in the model corresponds to a year. Table 1 reports the parameters of the model that are calibrated independently, whereas Table 2 reports the parameters that are jointly calibrated to match the data moments reported in Table 3.

Preferences

Let us assume a constant relative risk aversion (CRRA) utility function, with relative risk aversion (RRA) of 2. The discount rate β is calibrated to generate a wealth-over-output ratio of 3 in the stationary equilibrium. The calibrated value of β is 0.93.

Entrepreneurial Skills

Entrepreneurial skills of the household evolve according to an AR(1) process. The parameters that govern this process are the autocorrelation, ρ, and the standard deviation of the innovation, σ. These two parameters are calibrated to match the moments of the distribution of wealth in the stationary equilibrium. The calibrated value of ρ is 0.98, whereas the calibrated value of σ is 0.08.

Production and Capital Depreciation

Let us assume a capital share of income α of 0.35 and a capital depreciation rate δ of 0.04. The curvature parameter μ determines how steeply decreasing marginal profits are, and consequently the shape and moments of the wealth distribution. For this reason, μ is jointly calibrated to match the moments of the wealth distribution in the stationary equilibrium. The calibrated value of μ is 0.86.

Moments of the Wealth Distribution

In the benchmark model the household's only asset is capital. Therefore, household wealth equals the current value of the household's stock of capital. To ensure that the wealth distribution in the stationary equilibrium is close to the 1980 wealth distribution in the United States, the calibration targets the following moments: a wealth share of 1% for the bottom four deciles of the wealth distribution, as reported by Wolff (2014) for the 1983 SCF; a Gini coefficient of wealth of 80%, as reported by Wolff (2014) in 1983. For the Top 1%, the figures reported in the years close to 1980 differ substantially across different sources. Wolff (2014) reports 34% in the year 1983, Saez and Zucman (2016) find a value of 24% in the year 1980, and Bricker, Henriques, Krimmel, and Sabelhaus (2015) report a value of 30% in 1989. Therefore, the calibration targets a value of 30%, in the middle of this range. Finally, for the Top 0.1% and Top 0.01%, let us target the values of Saez and Zucman (2016), 8% and 3%, respectively. Table 3 summarizes the information on the moments and the performance of the model.

5.2 Transition

This section shows the main quantitative results of the benchmark model. In 1980 the modeled economy is in the stationary equilibrium. Then, agents learn about the future declining path of the price of investment, and with perfect foresight, they acquire knowledge of the future path of all the aggregate prices and quantities of the economy. In the benchmark exercise, the price of investment declines as shown in Figure 1 and remains constant after 2013 for the rest of the transition. Figure 7 shows the evolution of aggregate prices and quantities. As investment becomes cheaper, aggregate capital Q_t increases, therefore increasing w_t.

Table 4 shows different measures of wealth inequality from 1989 to 2013, comparing the output of the model with the SCF data. The model can account for half of the increase in wealth inequality at the top of the wealth distribution, as shown by the Top 0.1% and the Top 1% shares. The model also captures half of the increase in more global measures of wealth inequality such as the Gini coefficient and the Top 10%. Figure 8 shows in more detail the time series of these measures of wealth inequality comparing the SCF with the model transition.

5.3 Decomposition

Section 4 developed the intuition for the asymmetric effect of the decline in the price of investment on heterogeneously productive households. This section uses the intuition

Table 1: Benchmark Parameters Calibrated Independently

Parameter		Value
Capital depreciation	δ	0.04
Capital share in production	α	0.35
Relative risk aversion	RRA	2

This table lists the parameters of the benchmark model that are calibrated independently.

Table 2: Benchmark Parameters Calibrated Jointly in Equilibrium

Parameter		Value
Discount factor	β	0.93
Curvature parameter of the aggregator	μ	0.86
Std. dev. of entrepreneurial ability	σ	0.08
Persistence of entrepreneurial ability	ρ	0.98

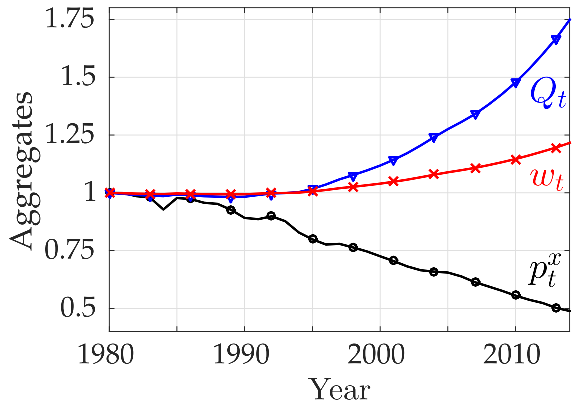
This table lists the parameters of the benchmark model that are calibrated jointly to match in the initial stationary equilibrium the moments reported in Table 3.

Table 3: Data and Benchmark Model Moments

Targets	Data	Model
Bottom 40% share of wealth	0.01	0.02
Top 1% share of wealth	0.30	0.33
Top 0.1% share of wealth	0.08	0.10
Top 0.01% share of wealth	0.03	0.03
Gini of Wealth	0.80	0.83
Wealth/Output	3.00	3.00

This table summarizes the joint calibration exercise. The data column reports the targets of the calibration, whereas the model column reports the moments of the calibrated model. The calibrated parameters that generate the model moments are reported in Table 2.

Figure 7: Aggregates



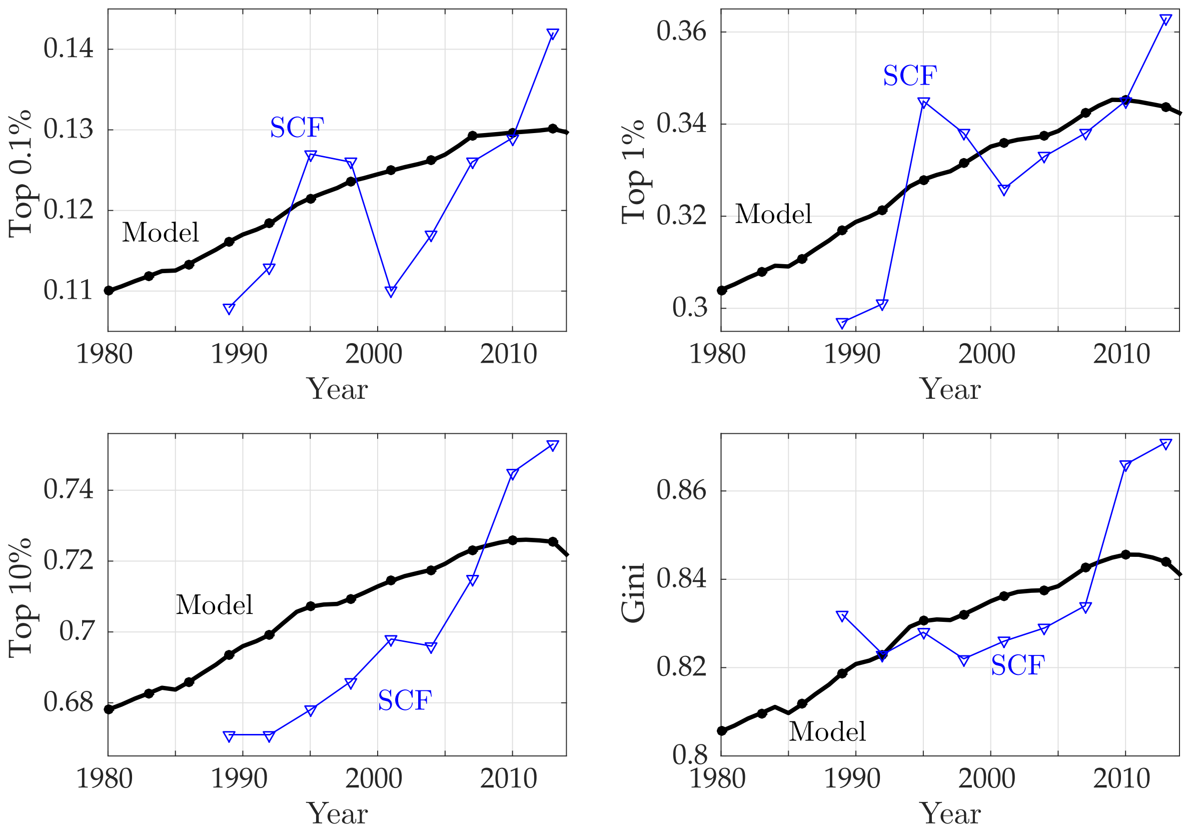
All the time series in this figure are normalized to 1 in 1980. This is a plot of the aggregate prices and quantities over the transition. The time series for p_t^x is fed into the model, whereas Q_t and w_t are the result of general equilibrium.

Table 4: Measures of Wealth Inequality: Data and Model

	Change from 1989 to 2013	
	SCF	Model
Top 0.1%	3 pp	2 pp
Top 1%	7 pp	3 pp
Top 10%	8 pp	4 pp
Gini%	4 pp	3 pp

This table compares the output from the benchmark model with the data from the SCF. The results are displayed in percentage points (pp) differences. The model accounts on average for half of the rise in wealth inequality observed in the SCF. The 1980 calibration of the model that generates this output is reported in Tables 1, 2 and 3. Figure 8 plots in detail the time series of these statistics comparing the SCF with the model transition.

Figure 8: Measures of Wealth Inequality: Data and Model



This figure compares the output from the benchmark model with the data from the SCF. Each panel of the figure shows a different statistic of wealth inequality, comparing the SCF time series with the time series generated by the model over the transition. The model accounts on average for half of the rise in wealth inequality observed in the SCF. The 1980 calibration that generates this output is reported in Tables 1, 2 and 3. Table 4 shows a direct comparison of the change in the different statistics of wealth inequality from 1989 to 2013.

gained from the simple model to analyze the effect on wealth inequality presented in the transition of the benchmark model.

Drawing a parallel with the simple model, this section decomposes the effect in two ways. (i) In the first part of the experiment, the saving rates are fixed over the state space at the 1980 levels. Then, the economy is simulated again, allowing the aggregates to vary as in the transition of the benchmark model. This isolates the *direct* effect on wealth inequality caused by the changing aggregates, while the saving behavior remains fixed. (ii) In the second part of the experiment, the aggregates are fixed at the levels of 1980. Then, the economy is simulated again, allowing the saving rates to vary as in the transition of the benchmark model. This isolates the *incentive* effect on wealth inequality caused by the changing saving behavior, while the aggregates remain fixed. Notice that these decomposition exercises, though informative, are not additive.

The decomposition exercise takes the following steps. The model is solved over the transition, and the aggregates $\{p_t^x, w_t, Q_t\}_{t=1980}^T$ and policy functions $\{g_k(k,\theta,t)\}_{t=1980}^T$ are stored. T denotes the last period of the transition. Using the policy functions, the optimal saving rates are computed as

$$g_s(k,\theta,t) = \frac{\{g_k(k,\theta,t) - k(1-\delta)\}\, p_t^x}{w_t + \alpha Q_t^{-\mu+\alpha}(\theta k)^\mu},$$

where the numerator is investment and the denominator is income in the benchmark model. Notice that the sequence of policy functions is indexed by time to highlight that, over the transition, the policy functions depend on the path of the aggregates.

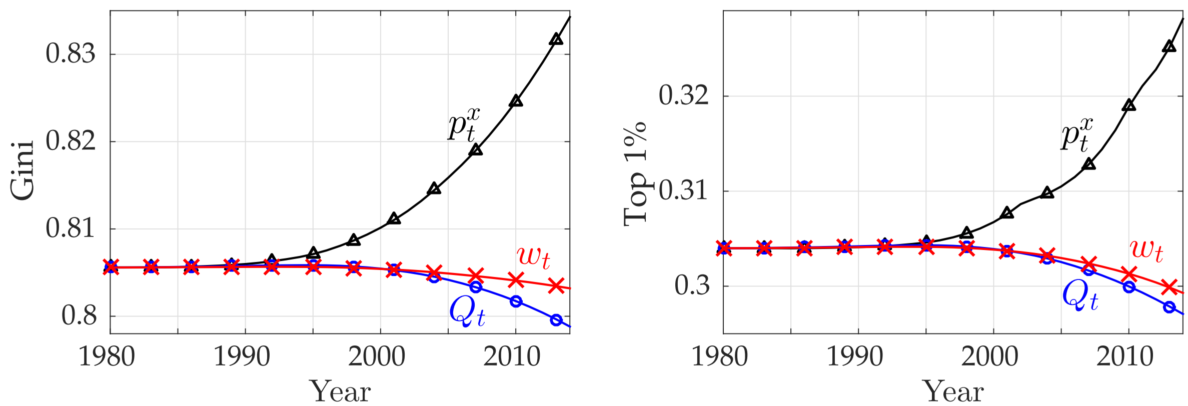
Direct Effect

The first part of the experiment studies the direct effect on wealth inequality. Saving rates are fixed at their 1980 levels, that is, $g_s(k,\theta,t) = g_s(k,\theta,1980)$. New policy functions are computed, based on the fixed saving rates, as the aggregates change:

$$\hat{g}_k(k,\theta,t) = g_s(k,\theta,1980)\left\{w_t + \alpha Q_t^{-\mu+\alpha}(\theta k)^\mu\right\}/p_t^x + k(1-\delta).$$

The new policy functions are used to simulate a new transition path for the economy. Figure 9 shows the evolution of the Gini and Top 1% share of wealth, while only one aggregate at a time changes. The direct effect of the decline in the price of investment

FIGURE 9: DIRECT EFFECT, ONE AGGREGATE AT A TIME



This figure shows the result of a decomposition exercise. The saving rates of the agents are fixed at the levels of 1980. Then the transition is recomputed, allowing only one aggregate at a time to change. For any such experiment, this figure illustrates the evolution of the Gini and Top 1% share of wealth. Next to each time series is marked which of the three aggregates changes when the transition is recomputed. For example, the blue line in the left panel represents the evolution of the Gini of wealth when only Q_t increases.

is to increase wealth inequality, whereas the increase in wage and aggregate capital decreases wealth concentration.

Figure 10 considers the combined direct effect of all the aggregates, comparing it with the output of the benchmark model. The figure shows that the direct effects of the aggregates tend to cancel each other out. In conclusion, the direct effect of the aggregates is to increase wealth inequality, although the size of the effect is small.

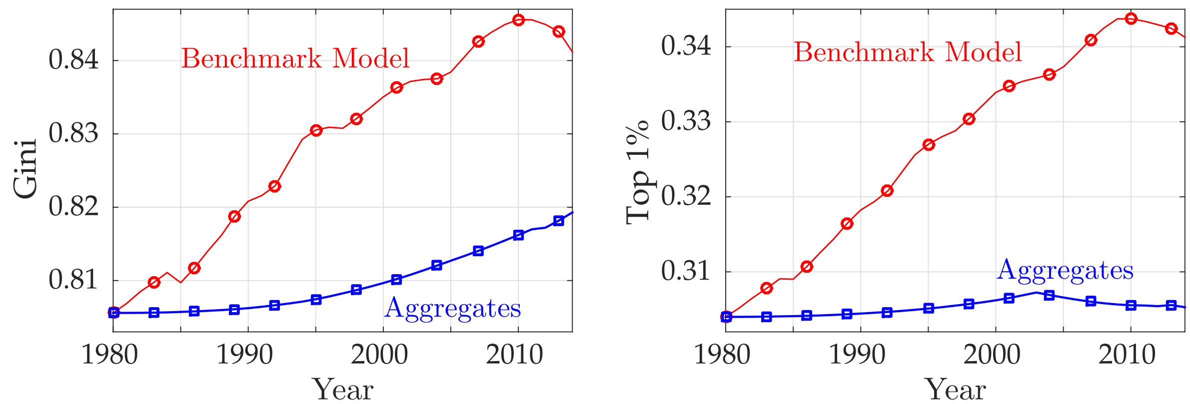
Incentive Effect

The second part of the experiment considers the incentive effect, which works through the change in the saving behavior. To do this, the aggregates are fixed, that is, $Q_t = Q_{1980}$ $p_t^x = p_{1980}^x$ and $w_t = w_{1980}$. New policy functions are obtained based on the fixed aggregates, while the saving rates are allowed to change:

$$\hat{g}_k(k,\theta,t) = g_s(k,\theta,t)\left\{w_{1980} + \alpha Q_{1980}^{-\mu+\alpha}(\theta k)^{\mu}\right\}/p_{1980}^x + k(1-\delta).$$

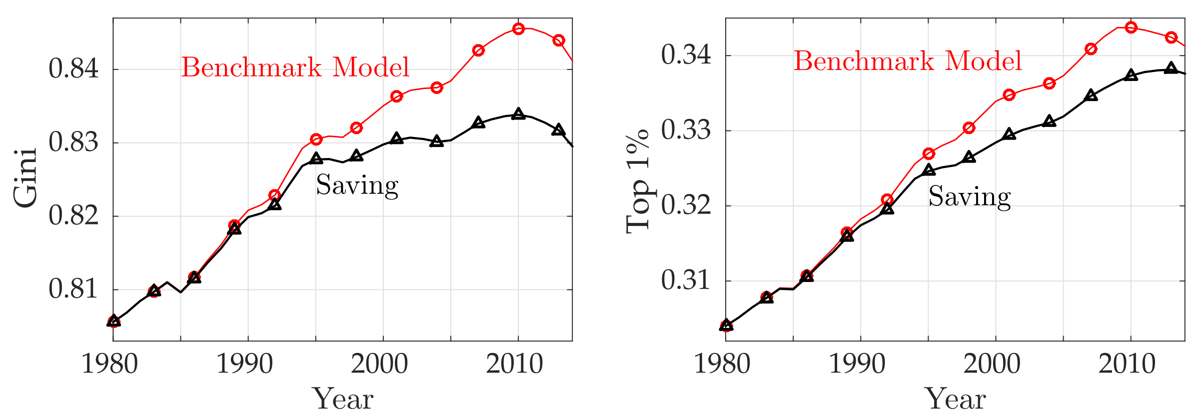
A new transition is computed using the new policy functions. Figure 11 shows the evolution of the Gini and the Top 1% share of wealth as the saving behavior of the agents changes. A comparison of this figure with Figure 10 shows that most of the increase in wealth inequality is driven by the incentive effect.

Figure 10: Direct Effect, All Aggregates



This figure shows the result of a decomposition exercise. The saving rates of the agents are fixed at the levels of 1980. Then the transition is recomputed, allowing all the aggregates to change contemporaneously, as shown in Figure 7. This figure plots the Gini and the Top 1% share of wealth from this experiment (in blue) and compares it with the output generated by the benchmark model (in red).

Figure 11: Incentive Effect



This figure shows the result of a decomposition exercise. All the aggregates are fixed at the levels of 1980. The transition is recomputed, allowing the saving rates of the agents to change, as in the transition of the benchmark model. The Gini and the Top 1% share of wealth from this experiment are represented in black and compared with the output of the benchmark model in red.

5.4 Saving Behavior

The decomposition in Section 5.3 illustrates that the change in the household's saving behavior causes most of the increase in wealth inequality. This section analyzes the change in saving behavior in more detail.

In what follows, the saving rate of a household is defined as the share of income that it spends to purchase investment goods:

$$s\left(\theta_{i,t}, k_{i,t}\right) = \frac{x_{i,t} p_t^x}{w_t + \alpha Q_t^{-\mu+\alpha} \left(\theta_{i,t} k_{i,t}\right)^{\mu}}.$$

The sign of $s\left(\theta_{i,t}, k_{i,t}\right)$ is negative when the household decumulates capital in order to purchase consumption.

The simple model of Section 4 shows that in response to the decline in the price of investment, productive households have incentives to save more. To analyze this in the benchmark model, let us compare the average saving rates across households with different productivity types.

When solving the benchmark model numerically, the support for θ is discretized using 10 grid points. Let us denote the highest and lowest grid points as θ_H and θ_L, respectively. The fifth point is denoted as θ_M, where M stands for medium. Then for $j \in \{L, M, H\}$, the average saving rate is defined as

$$s_t\left(\theta_j\right) = \int_{i|\theta_{i,t}=\theta_j} s\left(\theta_{i,t}, k_{i,t}\right).$$

To show the divergence of the saving rates across household types, Figure 12 represents $s\left(\theta_H\right) - s\left(\theta_M\right)$ and $s\left(\theta_H\right) - s\left(\theta_L\right)$. The figure shows that these time series are fanning out, therefore increasing wealth inequality across types.

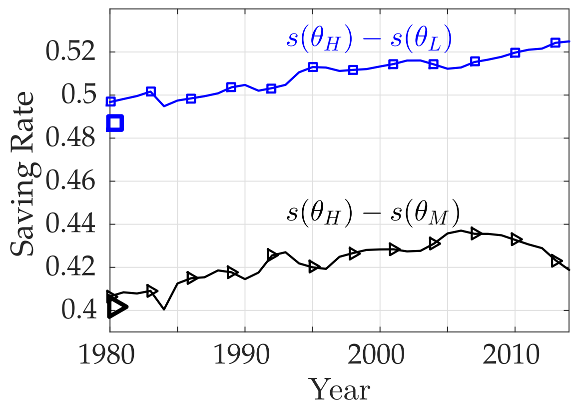
In the figure, the differences in saving rates at the initial stationary equilibrium of 1980 are marked on the left vertical axis. Then, as the households learn about the new path of aggregates they face, they reoptimize and change their saving behavior.

To understand the effect on wealth accumulation of different types of households, let us consider average capital holding by household type, which is denoted as

$$K_t\left(\theta_j\right) = \int_{i|\theta_{i,t}=\theta_j} k_{i,t},$$

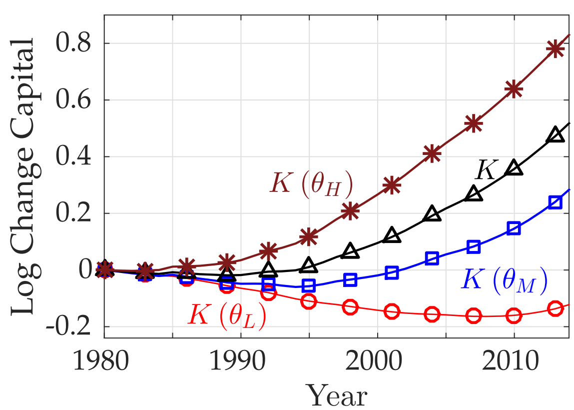
so that average capital holding across all households is denoted K_t. Figure 13 plots the log changes of $K_t(\theta_j)$ using 1980 as the reference year, that is, $\log K_t(\theta_j) - \log K_{1980}(\theta_j)$. The figure shows that the capital holding of different types of households fans out over the transition.

Figure 12: Fanning Out of Saving Rates



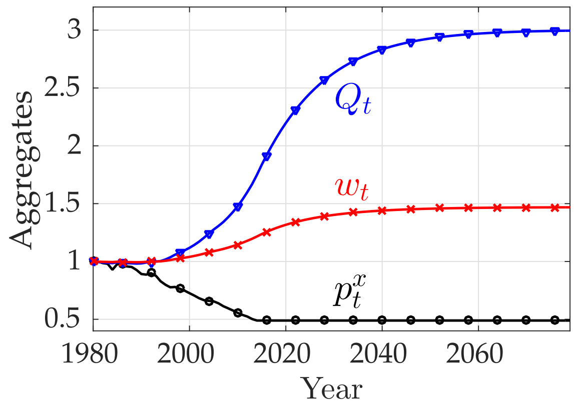
This figure shows the difference between the average saving rate of θ_H, θ_M, and θ_L households, that is, $s(\theta_H) - s(\theta_M)$ and $s(\theta_H) - s(\theta_L)$. The markers on the vertical axis indicate the value of $s(\theta_H) - s(\theta_M)$ and $s(\theta_H) - s(\theta_L)$ in the 1980 stationary equilibrium.

Figure 13: Average Capital Holding



This figure displays the log changes in average capital holding by household type. The types are θ_H, θ_M, and θ_L. All the time series are normalized in 1980, so that for type j the time series is computed as $\log K_t(\theta_j) - \log K_{1980}(\theta_j)$. The figure also shows the log changes in average capital holding across all types, that is, $\log K_t - \log K_{1980}$.

Figure 14: Aggregates, Long Run



All the time series in this figure are normalized to 1 in 1980. This is a plot of the aggregate prices and quantities over the transition in the long run. The time series for p_t^x is fed into the model from 1980 to 2013 and then remains constant throughout the rest of the transition. Q_t and w_t are the result of the general equilibrium.

5.5 Wealth Inequality in the Long Run

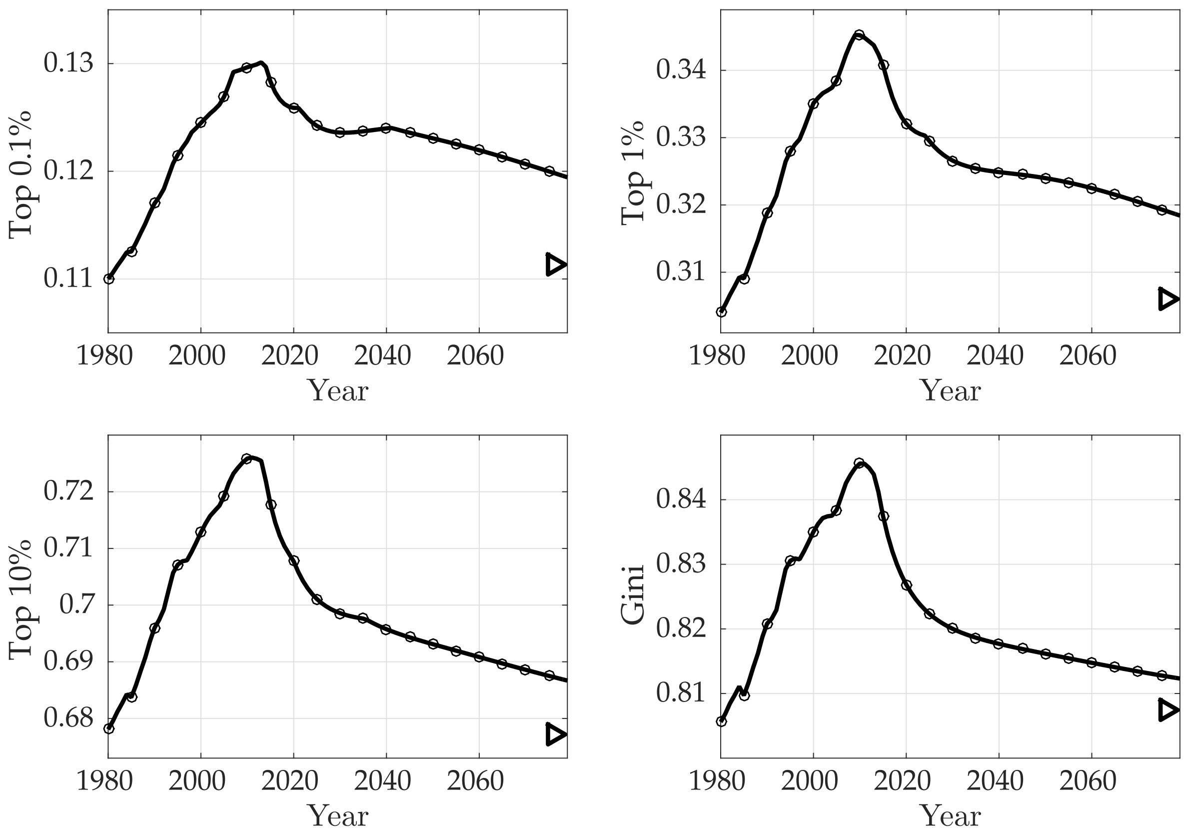
This section considers the model long-run predictions for wealth inequality. The increase in aggregate capital Q_t plays an important role for these predictions. For this reason, the discussion begins by considering the long-run time series of the aggregates in the benchmark model, displayed in Figure 14.

The figure shows that aggregate capital increases the fastest precisely when the price of investment stops decreasing. Q_t then increases at a declining rate until it stabilizes at a value of around three times Q_{1980}.

Section 4 explores with the simple model the asymmetric effect of an increase in Q_t on heterogeneously productive households. It shows that an increase in Q_t provides incentives to decrease saving that are stronger for more productive agents. Furthermore, Section 5.3 shows that the direct effect of an increase in Q_t is to decrease wealth inequality. Based on these considerations, the increase in Q_t has the potential to reduce wealth inequality in the model.

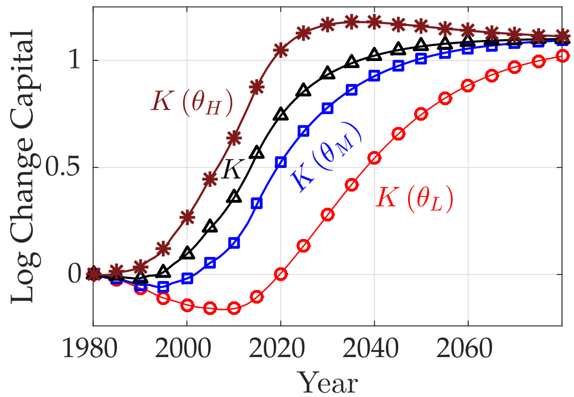
In fact, Figure 15 shows that in the benchmark model, wealth inequality decreases in the long run. The figure illustrates the same statistics of wealth inequality as in Figure 8 over a time horizon of 100 periods. A small triangle marks the value of each statistic at the final stationary equilibrium. This shows that, eventually, the degree of wealth concentration returns to levels that are comparable with those of 1980.

Figure 15: Measures of Wealth Inequality: Model, Long Run



This figure presents the output from the benchmark model in the long run. Each panel of the figure shows a different statistic of wealth inequality. The marker on the right vertical axis of each panel indicates the value of the statistic at the final stationary equilibrium.

Figure 16: Average Capital Holding, Long Run



This figure displays the log changes in average capital holding by household type in the long run. The types are θ_H, θ_M, and θ_L. All the time series are normalized in 1980, so that for type j the time series is computed as $\log K_t(\theta_j) - \log K_{1980}(\theta_j)$. The figure also shows the log changes in average capital holding across all types, that is, $\log K_t - \log K_{1980}$.

Figure 16 illustrates average capital holding by household type in the long run. This figure shows an increase in wealth dispersion across household types in the first three decades. However, this increase in dispersion eventually vanishes.

6 Robustness

This section examines the robustness of the results of the benchmark model to departures from the baseline calibration and assumptions.

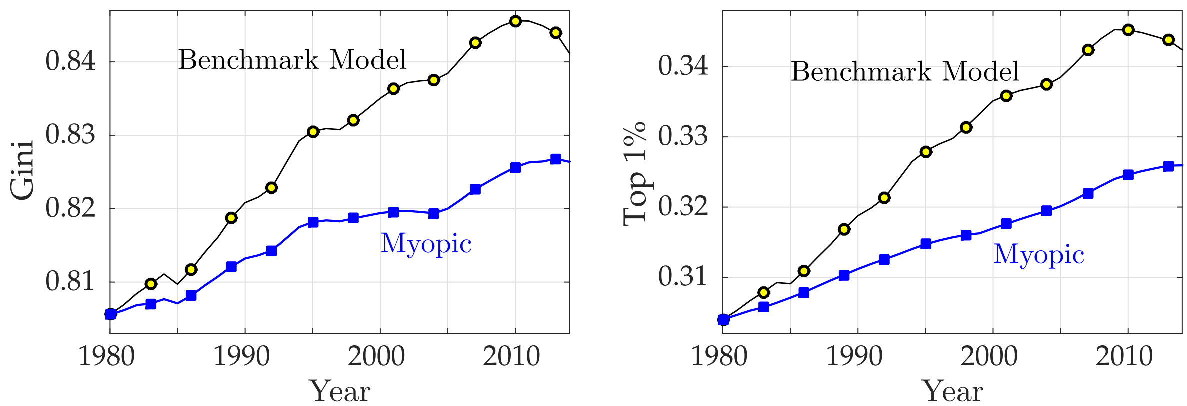
6.1 Myopic Agents

In the benchmark model, in 1980 the agents acquire knowledge of the future path of all the aggregates.

This section considers the evolution of wealth inequality in the model, when agents do not have perfect foresight. To bound the effect between the two extreme cases, let us consider the opposite of perfect foresight, that is, perfectly myopic agents.

In this version of the model, agents learn about the contemporaneous change in the price of investment each period from 1980 to 2013, but do not anticipate the future declining price path. In response to each surprising change the agents optimally choose new contingent consumption and saving plans.

Figure 17: Myopic Agents



This figure illustrates the result of a robustness exercise. Each panel of the figure shows a different statistic of wealth inequality, comparing the output from the benchmark model with the time series generated by a model where agents are perfectly myopic.

Figure 17 shows the evolution of the Gini and the Top 1% share of wealth over the transition, under the assumption that agents are perfectly myopic. To facilitate the comparison with the benchmark model, the statistics of the benchmark model are also represented in the graph. The figure shows that the effect on wealth inequality is dampened and that it takes place with a slightly slower timing. In fact, wealth inequality peaks in 2013 instead of 2010. In conclusion, assuming perfectly myopic agents does not affect the qualitative findings of the benchmark model, though it affects the size of the effect.

7 Conclusions

This paper discusses the consequences of the decline in the price of investment on wealth inequality in the United States.

The paper shows that the decrease in the price of investment affects the saving behavior of households with different productivity types asymmetrically. In fact, productive households respond by purchasing more capital than unproductive households, therefore increasing wealth inequality.

This theory of the increase in wealth inequality reproduces half of the rise in wealth concentration observed in the United States since the 1980s. Under the assumption that investment specific technological progress stops in the 2010s, the model further predicts that the increase in wealth inequality will vanish in the long run.

References

Rao S. Aiyagari. Uninsured Idiosyncratic Risk and Aggregate Saving. *Quarterly Journal of Economics*, 109(3):659–684, August 1994.

George-Marios Angeletos. Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic Dynamics*, 10(1):1–30, 2007.

Jess Benhabib and Alberto Bisin. Skewed wealth distributions: Theory and empirics. NBER Working Papers 21924, National Bureau of Economic Research, Inc, 2016.

Jess Benhabib, Alberto Bisin, and Shenghao Zhu. The distribution of wealth and fiscal policy in economies with finitely lived agents. *Econometrica*, 79(1):123–157, 2011.

Jess Benhabib, Alberto Bisin, and Shenghao Zhu. The wealth distribution in Bewley models with investment risk. NBER Working Papers 20157, National Bureau of Economic Research, Inc, 2014.

Jesse Bricker, Alice M. Henriques, Jacob Krimmel, and John Sabelhaus. Measuring income and wealth at the top using administrative and survey data. (2015-30), 2015.

Marco Cagetti and Mariacristina De Nardi. Entrepreneurship, frictions, and wealth. *Journal of Political Economy*, 114(5):835–870, 2006.

Christopher Carroll. The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320, 2006.

Ana Castañeda, Javier Díaz-Giménez, and José-Víctor Ríos-Rull. Accounting for the u.s. earnings and wealth inequality. *Journal of Political Economy*, 111(4):818–857, 2003.

Jason Cummins and Giovanni Violante. Investment-specific technical change in the us (1947-2000): Measurement and macroeconomic consequences. *Review of Economic Dynamics*, 5(2):243–284, 2002.

Mariacristina De Nardi. Quantitative models of wealth inequality: A survey. NBER Working Papers 21106, National Bureau of Economic Research, Inc, 2015.

Riccardo DiCecio. Sticky wages and sectoral labor comovement. *Journal of Economic Dynamics and Control*, 33(3):538–553, 2009.

Andreas Fagereng, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri. Wealth returns dynamics and heterogeneity. Mimeo, Stanford University, 2015.

Jonas Fisher. The dynamic effects of neutral and investment-specific technology shocks. *Journal of Political Economy*, 114(3):413–451, 2006.

Xavier Gabaix, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. The dynamics of inequality. NBER Working Papers 21363, National Bureau of Economic Research, Inc, 2015.

Robert Gordon. *The Measurement of Durable Goods Prices.* National Bureau of Economic Research, Inc, 1990.

Jeremy Greenwood, Zvi Hercowitz, and Per Krusell. Long-run implications of investment-specific technological change. *American Economic Review*, 87(3):342–62, 1997.

Joachim Hubmer, Per Krusell, and Anthony Smith. The historical evolution of the wealth distribution: A quantitative-theoretic investigation. 2015 Meeting Papers 1406, Society for Economic Dynamics, 2015.

Loukas Karabarbounis and Brent Neiman. The global decline of the labor share. *The Quarterly Journal of Economics*, 129(1):61–103, 2014.

Barış Kaymak and Markus Poschke. The evolution of wealth inequality over half a century: The role of taxes, transfers and technology. *Journal of Monetary Economics*, 77:1 – 25, 2016. ISSN 0304-3932.

Arthur B. Kennickell and R. Louise Woodburn. Consistent weight design for the 1989, 1992 and 1995 SCFs, and the distribution of wealth. *Review of Income and Wealth*, 45(2):193–215, 1999. ISSN 1475-4991.

Wojciech Kopczuk and Emmanuel Saez. Top wealth shares in the united states, 1916-2000: Evidence from estate tax returns. *National Tax Journal*, 57(2):445–87, 2004.

Per Krusell, Lee Ohanian, José-Víctor Ríos-Rull, and Giovanni Violante. Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica*, 68(5): 1029–1054, 2000.

Tobias J. Moskowitz and Annette Vissing-Jørgensen. The returns to entrepreneurial investment: A private equity premium puzzle? *American Economic Review*, 92(4): 745–778, September 2002.

Vincenzo Quadrini. The importance of entrepreneurship for wealth concentration and mobility. *Review of Income and Wealth*, 45(1):1–19, 1999.

Vincenzo Quadrini. Entrepreneurship, saving, and social mobility. *Review of Economic Dynamics*, 3(1):1 – 40, 2000. ISSN 1094-2025.

Geert K. Rouwenhorst. *Asset Pricing Implications of Equilibrium Business Cycle Models*. Princeton University Press, 1995.

Emmanuel Saez and Gabriel Zucman. Editor's choice wealth inequality in the united states since 1913: Evidence from capitalized income tax data. *The Quarterly Journal of Economics*, 131(2):519–578, 2016.

Philip Vermeulen. How fat is the top tail of the wealth distribution? Working Paper Series 1692, European Central Bank, 2014.

Edward N. Wolff. Household wealth trends in the united states, 1962-2013: What happened over the great recession? Working Paper 20733, National Bureau of Economic Research, December 2014.

APPENDIX

A1 Computational Appendix

This appendix describes the computational strategy used to solve the the benchmark model.

Household Problem

The households problem is solved using the endogenous grid method (EGM) of Carroll (2006). The states of the problem are cash-on-hand and the the productivity type of the households.

The productivity type is discretized using Rouwenhorst (1995) using a discrete grid with 10 gridpoints.

The support for capital, though bounded, is large, with an upper bound of 10^{20}. For this reason, the grid for capital is exponential, that is, the spacing between the grid points grows exponentially. The model is solved with 200 gridpoint.

The interpolation necessary for the EGM is performed using cubic spline interpolation.

Stationary Equilibrium

This paper discusses statistics of the wealth distribution that can be reliably estimated only if the distribution is computed very precisely. For this reason, the solution does not rely on a simulation but solves directly for the stationary distribution. This section considers the procedure to do this.

Let $g(k, \theta)$ denote the policy function that solves the households problem, and let θ_i be the i-th element in the discrete grid for the type. The ergodic set for capital is bounded by nodes n_1 and n_{N+1} such that $g(n_1, \theta) > n_1$ for all θ and $g(n_{N+1}, \theta) < n_{N+1}$ for all θ. Then, a given assignment of nodes $n_1, n_2, \ldots, n_{N+1}$ defines bins b_i, $i = 1, \ldots, N$, where $x \in b_i \iff n_i < x \leq n_{i+1}$. Let Π denote the transition matrix associated with the productivity type, with typical element $\Pi_{j,k}$

Then the transition probability between any two points in the discrete state space is given by

$$
\begin{aligned}
&\Pr\left\{ k' \in b_j \wedge \theta' = \theta_k \middle| k \in b_i \wedge \theta = \theta_h \right\}, \\
&= \Pr\left\{ \theta' = \theta_k \middle| k \in b_i \wedge \theta = \theta_h \right\} \times \Pr\left\{ k' \in b_j \middle| k \in b_i \wedge \theta = \theta_h \right\}, \\
&= \Pr\left\{ \theta' = \theta_k \middle| \theta = \theta_h \right\} \times \Pr\left\{ k' \in b_j \middle| k \in b_i \wedge \theta = \theta_h \right\}, \\
&= \Pi_{h,k} \times \Pr\left\{ n_j < g\left(k, \theta_h\right) \leq n_{j+1} \middle| k \in b_i \right\}, \\
&= \Pi_{h,k} \times \frac{\min\left\{n_{i+1}, g^{-1}\left(n_{j+1}, \theta_h\right)\right\} - \max\left\{n_i, g^{-1}\left(n_j, \theta_h\right)\right\}}{n_{j+1} - n_j},
\end{aligned}
$$

where the last line comes from assuming that within each bin the distribution is uniform. Computing the transition probability between any two point in the discrete type-bin state space completes the description of a first order Markov process, whose stationary distribution can be obtained directly as the eigenvector associated with the eigenvalue 1. The transition matrix of this Markov process has a band matrix structure that can be exploited to make the computation of the stationary distribution faster.

Model Transition

Solving the transition requires first solving the initial and final stationary equilibrium. The number of periods for the transition is set to T=400. Let $G = G_k \times G_\theta$ denote the grid over capital and productivity type of the household, and let Φ denote a probability distribution over G. The tolerance for the convergence of this problem is set to 10^{-6}. The following describes the procedure to solve for the transition.

(i) For a given path of aggregates, the value functions are solved backward from the final stationary equilibrium to the first period, obtaining $V_2, \ldots, V_T$.

(ii) Using this sequence of value functions, the initial stationary distribution Φ_1 is rolled forward.

The following describes the generic forward-rolling step at time t, when Φ_t and V_{t+1} are available, and for a given guess of prices, denoted $\left\{\hat{w}_t, \hat{Q}_t\right\}$.

(1) Given $\left\{\hat{w}_t, \hat{Q}_t\right\}$ compute cash on hand available on G, that is for each $z \in G_\theta$

$$
\hat{y}_t\left(z\right) = G_k\left(1-\delta\right) p_t^x + \hat{w}_t + \pi_t\left(G_k, \hat{Q}_t, z\right).
$$

(2) Given V_{t+1} and $\left\{\hat{w}_t, \hat{Q}_t\right\}$ obtain policy function over G, $\hat{g}_t\left(V_{t+1}, \hat{w}_t, \hat{Q}_t\right)$, that solves for each $z \in G_\theta$

$$p_t^x u'\left(\hat{y}_t\left(z\right) - \hat{g}_t\left(z\right) p_t^x\right) = \beta \mathbb{E}\left(V_{t+1}\left(z\right)\right)'.$$

(3) Given $\hat{g}_t$ compute demand for labor and investment, and denote them $\hat{l}_t$ and $\hat{x}_t$, respectively.

(4) Given $\left\{\hat{l}_t, \hat{x}_t\right\}$ and Φ_t compute excess demand for labor and intermediate good

$$ED\left(\hat{w}_t, \hat{Q}_t, \Phi_t\right).$$

(5) Find $\left\{\hat{w}_t, \hat{Q}_t\right\}$ such that $ED = 0$ iterating over (1)-(5), and denote it $\left\{\bar{w}_t, \bar{Q}_t\right\}$, so that the market clearing value function is denoted by $\bar{g}_t\left(V_{t+1}, \bar{w}_t, \bar{Q}_t\right)$.

(6) Given $\bar{g}_t$ compute the discrete transition matrix—as in the previous section of the appendix—and call it Tr.

(7) Compute

$$\bar{\Phi}_{t+1} = \text{Tr} \times \Phi_t$$

(iii) Step (ii) generates a new path of aggregates as substep (5) explains. Compare this new path of aggregates with the initial one, used as input in step (i). If the maximum difference is smaller than the tolerance then the code has converged. Otherwise, iterate over steps (i)-(iii).

The solution of the transition with perfectly myopic agents is more involved because it requires solving the problem showed above every time the price of investment changes and surprises the agents.