A product-mix decision model using green manufacturing technologies under activity-based costing

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ABSTRACT
The purpose of this study is to assess how the integration of activity-based costing (ABC) and the theory of constraints (TOC), as well as the application of a mixed-integer programming (MIP) model, can assist in making decisions about product-mix using green manufacturing technologies (GMTs). This study proposes a mathematical programming model to analyze the profitability of a product-mix decision based on the ABC and TOC, with the adoption of new GMTs. Using a numerical example from a metal component parts manufacturer in the automotive industry, the findings of this study provide insight into the value of mathematical programming approaches for GMTs investment and product-mix decision making based on ABC systems while simultaneously improving the value of green manufacturing technology investments.

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1. Introduction

Since production and consumption activities have been generating negative impact effects on the environment through their production, use and disposal in recent years, environmental issues have increasingly become primary concerns of corporate management. Meanwhile, stakeholders have also begun to put pressure on organizations to be more environmentally responsible for both their products and processes given regulatory requirements, product stewardship, public image and potential competitive advantages. In such circumstances, many organizations are finding that going beyond regulatory compliance can create value for their customers and respond to pressure groups. For example, many facility operators are actively investigating the use of green manufacturing technologies (GMTs), such as aqueous degreasers and powder coatings, in an effort to reduce toxic air emissions and control costs associated with the treatment of contaminated effluent.

However, in practice, most companies continue to struggle to invest their scarce resources to adopt new GMTs because of a lack of proper justification tools that would both identify profitable and non-profitable products and account for resource constraints. Facility operators need accurate information from their management accounting systems in order to make profitable choices regarding environmental spending. The use of activity-based costing (ABC) on profitability analysis is called upon to help break down and analyze the nature of environmental costs in products that generate those costs, in order to lead to the best decisions. Even though ABC provides a systematic approach to analyzing non-value added activities, processes and products, ABC may ignore resource constraints in the production process (Yahya-Zadeh, 1998; Kee and Schmidt, 2000). Under such conditions, the theory of constraints (TOC) may provide a better solution when making decisions about reducing environmentally-damaging products from the mix (Onwubolu and Mutungi, 2001; Lockhart and Taylor, 2007). The purpose of this research paper is to propose a mathematical programming model that analyzes the profitability of a product-mix decision based on the ABC and TOC, with the adoption of new GMTs.

2. Background

Due to the growing awareness of environmental issues, new GMTs have been widely considered for maintaining a competitive advantage (Kong and White, 2010), enhancing production skills (Puurunen and Vasara, 2007) and coping with pressure groups (Tsai et al., 2011a). Nevertheless, facility managers still have difficulty analyzing the impact of GMTs on profits because of a lack of...
appropriate performance measures (Azone and Noci, 1998; Kuldip, 2006), systematic analysis of overhead and fixed operating costs (Kaplan and Cooper, 1998) and considerations of constrained resources in the production process (Wang et al., 2009; Tsai et al., 2011b, 2012b). Recently, researchers have suggested a number of methodologies and evaluation techniques that look promising insofar as the economic justification process for these advanced manufacturing technologies is concerned. The solutions to these shortages should be sought in the ABC and TOC.

2.1. Activity-based costing (ABC)

Currently, ABC has become a popular cost accounting system to overcome the shortcomings of traditional cost accounting systems. There are four key benefits to the ABC system: (1) accurate identification of product cost, especially overhead; (2) more precise information about value-added and non-value-added costs by the identification of cost drivers; (3) direct allotment of costs to products or processes that consume resources; and (4) identification of non-value-adding costs (Kaplan, 1989; Canada et al., 1996; Hilton, 2005). Various surveys have also indicated that the ABC approach has been used to analyze different kinds of management decisions, including: pricing, quoting, product-mix and joint products decisions (Tsai and Lai, 2007; Kee, 2007, 2008), outsourcing decisions (Kee, 1998; Tsai and Lai, 2007; Tsai et al., 2007), quality improvement (Tsai, 1998), design and development (Qian and Ben-Arieh, 2008), financial and physical flows analysis (Comelli et al., 2008) and environmental management, among others (Tsai et al., 2007; Tsai and Hung, 2009a, 2009b; Silva and Amaral, 2009).

The ABC uses a two-stage procedure to assign resource costs to cost objects (Turney, 1991; Tsai, 2010). In the first stage, resource costs are assigned by resource drivers to activity cost pools that can be classified by activity levels such as unit, batch, product and facility (Cooper, 1990). Each activity level can have several activity cost pools. In the second stage, activity cost pools are assigned to cost objects by activity drivers, which are activities that incur costs (Hilton, 2005). Because of the problem of assessing the benefits and cost drivers of the new GMTs are less well studied and understood, in this study, we follow Kim’s (2009) approach and attempt to identify the resource and activity drivers of production processes using new GMTs. First, we estimate overall costs and possible resources of various production processes using new GMTs. Second, we construct resource cost pools and develop resource cost drivers. Third, we define the main activities that consume resources of production and calculate a total cost of each activity. Finally, we identify the second stage activity drivers and allocate activity costs to various products using the cost driver.

2.2. The theory of constraints (TOC)

In early ABC research, numerous studies demonstrated the differences between production costs using ABC and traditional costing systems. Nevertheless, a method to reach the optimal product-mix under ABC was seldom mentioned. Furthermore, ABC has been criticized for its failure to incorporate constraints into production-mix decisions (Kee and Schmidt, 2000). On account of this, Kee (1995) began to integrate ABC with TOC in the product-mix decision analysis. Several researchers proposed various mathematical programming models in succession to conduct the product-mix decision analysis under ABC (Kee, 1995; Malik and Sullivan, 1995; Yahya-Zadeh, 1998; Kee and Schmidt, 2000; Tsai et al., 2012c).

According to TOC, the performance of any production system is determined by its slowest process (Goldratt et al., 1986; Yahya-Zadeh, 1998). Thus, managers should focus their attention on capacity constraint resources (CCRs) and removing bottleneck processes (Onwubolu and Mutingi, 2001). Since TOC treats overhead and operating expenses as given, TOC can also be viewed as a short-run optimization procedure for managing resources and opening bottlenecks, with the goal of maximizing throughput (Holmen, 1995). The ABC and TOC approaches appear to complement one another in addressing short-run operational and long-run cost management problems (Kee, 1995; Lockhart and Taylor, 2007). Several researchers have suggested that TOC should be used for short-term production-mix decisions where costs are predominantly fixed (Noreen et al., 1995) and that ABC should be used to determine any increases or decreases in capacity and products because all costs tend toward being variable in the long term.

The following steps, called the TOC procedure, provide on-going improvement to the throughput of a system (Goldratt, 1990):

1. Identify the system’s constraints.
2. Decide how to exploit the system’s constraints.
3. Subordinate everything else to the above decision.
4. Elevate the system’s constraints.
5. If in the previous steps, a constraint has been broken, go back to step 1.

The above five steps of the TOC can be applied to the product-mix decision problem. Through the cycle of these five steps, TOC successively relieves bottlenecks and their associated constraints by expanding the obtainable resources (capacities) or by improving the company’s operations (Tsai and Lin, 1990; Tsai et al., 2011a). In addition, the product-mix decision problem that is solved using TOC can also be formulated as a linear programming (LP) model (Tsai and Lin, 1990; Luebbe and Finch, 1992; Plenert, 1993; Tsai et al., 2011b, 2012a).

As compared to Kee (1995), the model proposed in this paper has the following additional characteristics: (1) considering Environmental Regulatory Cost (fixed cost), Volatile Organic Compounds (VOCs) emission cost (piecewise linear cost) and the associated constraints to restrain the VOCs emission quantity, (2) considering the facility-level activity cost increasing with a stepwise fixed cost function, and (3) considering the capacity expansions for direct labor hours and machine hours. In terms of point (3) mentioned above, regarding capacity expansion, we consider capacity expansion for direct labor hours by using overtime work with higher wage rates, which can be formulated as the piecewise linear cost function. Herein, we also consider capacity expansion for machine hours by renting machines or by buying machines, which can be formulated as a stepwise linear cost function. Currently, this method is applied in various research topics (Tsai and Kuo, 2004; Tsai and Lai, 2007; Tsai et al., 2007, 2008, 2010).

2.3. Priming and top coating processes and activities in the automotive industry

Many products and parts manufactured in the metal industries are required to receive both priming and top coating processes. Typical primer-top coat technologies are adopted for miscellaneous metal work pieces, plastic part, and the automotive industry. The typical sequence of operations performed by such a primer-topcoat system in the automotive industry includes multiple stages, as shown in Fig. 1.

In the beginning of a primer-topcoat process, the incoming metal component parts or metalwork pieces for the car body are often cleaned (e.g., degreased or steam-cleaned) and are passed through a zinc phosphate stage before being immersed in a large
ranging from 300 to 400 °F. In the sealers and sound deadening phase, the metal work pieces receive a polyvinyl chloride (PVC) coating that provides sound-deadening attributes. After undergoing light sanding, a second coat of primer is applied and then the metal work pieces enter the top coating spray booth and a basecoat, a solid color top coat or a wet-on-wet clear coat may be applied. Eventually, the metal work pieces enter the final baking oven in which the top coats are cured.

Given the importance of the primer-top coat process in the automotive industry, new GMTs that prevent pollution are important issues for operations. Facilities typically use new GMTs such as thermal oxidizers, catalytic incinerators, and carbon absorbers, or a combination of these technologies, to control VOCs. Below, we will describe the elements of product cost that will be used in developing the product-mix decision model under ABC systems.

2.4. Product-mix decision model with capacity expansions

In this section, we develop a product-mix decision model using the primer-top coat system as an example. In addition to the assumption about the primer-top coat process, the product-mix decision model presented includes the following assumptions:

1. The activities in this green, multi-product primer-topcoat system have been classified as unit-level, batch-level, product-level, and facility-level activities. The related resource drivers and activity drivers have also been chosen by the company’s ABC team through an ABC study.
2. The unit selling prices are constant within the relevant range.
3. The specific process is regarded as a stepwise fixed cost that varies with machine hours.
4. Machine hour resources can be expanded by renting or purchasing additional machines.
5. Direct labor resources can be expanded by using overtime work or additional night shifts with higher wage rates.
6. VOC emissions are taxed at different rates, which are dependent on emission quantities, and the cost of VOC emissions is regarded as a piecewise variable cost.

3. A mathematical programming model

The following is a discussion on capacity expansions, capacity constraints and VOC emission costs, which are incorporated into the mathematical programming model for determining the optimal decision. The cost elements related to the product-mix decision model are described below:

1. Total revenue:

   The terms in Eq. (1) represent the total revenue of bid prices for products.

   \[ \sum_{i=1}^{n} p_i X_i \]  

2. Total direct labor cost:

   We assume that direct labor resources can be expanded using overtime work or additional night shifts, or by hiring workers at a higher wage rate. The total direct labor cost function will be a piecewise linear function, as shown in Fig. 2. The available normal direct labor hours are \( LH_1 \) and the direct labor hours can be expanded to \( LH_2 \) with the total direct labor cost being \( LC_1 \) and \( LC_2 \) at \( LH_1 \) and \( LH_2 \), respectively.

   Thus, the total direct labor cost and associated constraints are as follows (Tsai and Lin, 1990; Tsai et al., 2011b):

   \[ \text{Total direct labor cost} = LC_1 \mu_1 + LC_2 \mu_2 \]  

   Constraints:

   \[ TL = LH_1 \mu_1 + LH_2 \mu_2 \]  

   \[ \mu_0 - \eta_1 \leq 0 \]  

   \[ \mu_1 - \eta_1 - \eta_2 \leq 0 \]  

   \[ \mu_2 - \eta_2 \leq 0 \]  

   \[ \mu_0 + \mu_1 + \mu_2 = 1 \]  

   \[ \eta_1 + \eta_2 = 1 \]  

\((\eta_1, \eta_2)\) is an SOS1 set of 0–1 variables, within which exactly one variable must be non-zero; \((\mu_0, \mu_1, \mu_2)\) is an SOS2 set of non-negative variables, within which at most two adjacent variables, in the order given to the set, can be non-zero (Beale and Tomlin, 1970; Williams, 1985); \( TL \) is the total direct labor hours we need, and its function depends on the case under study.

3. Stepwise machine cost:

   The total machine cost is regarded as a common fixed cost, and its cost function is assumed to be a stepwise function (as shown in Fig. 2).
that varies with machine hours, as observed from prior cost behavior analysis. The total machine cost is defined as $FC_k$ under the current capacity of $MH_0$ machine hours. If the capacity is successively expanded to $MH_1$, $MH_2$, ..., $MH_t$ machine hours, then the total machine cost increases to $FC_1$, $FC_2$, ..., $FC_t$, respectively. Let $\lambda_{ih}$ be the requirement of machine hours for one unit of product $i$. Thus, the total machine cost and associated machine hour constraints are as follows (Tsai and Lin, 1990):

$$\text{Total machine cost } = \sum_{k=0}^{t} FC_k \theta_k$$  \hfill (9)\)

Constraints:

$$\sum_{i=1}^{n} \lambda_{ih} X_i \leq \sum_{k=0}^{t} MH_k \theta_k$$  \hfill (10)\)

$$\sum_{k=0}^{t} \theta_k = 1$$  \hfill (11)\)

($\theta_0, \theta_1, ..., \theta_t$) is an SOS1 set of 0–1 variables, within which exactly one variable must be non-zero (Beale and Tomlin, 1970; Williams, 1985). When $\theta_m = 1 \ (m \neq 0)$, the capacity needs to be expanded to the $m$th level, i.e. $MH_m$ machine hour.

4. Environmental and social cost – VOC emission costs:

Even though new GMTs can reduce emissions, there are still small amounts of VOC emissions from the topcoat spray processes. The emission costs are measured by the life cycle assessment (LCA) method (Tsai et al., 2011a). LCA is a method of comparison of environmental impacts of products, technologies or services with a view of their whole life cycle. The VOC emissions to all areas of the environment during product production, use and disposal are considered. Following Ward and Chapman’s (1995) approaches, Eq. (12) is used to quantize the VOC emissions. VOC emissions are also assumed to be taxed at different rates depending on the quantity of these. Hence, the total VOCs emission cost function will be a piecewise linear function (see Fig. 4). With increasing VOC emissions, taxation will be increased. The quantity of VOC emissions can be increased from $VOQ_1$ to $VOQ_2$ and $VOQ_3$. Therefore, the total VOC emission cost is $VOP_1$, $VOP_2$, and $VOP_3$ at $VOQ_1$, $VOQ_2$ and $VOQ_3$, respectively (Tsai et al., 2011b).

The total VOC emission costs and associated constraints are as follows:

$$\text{Total VOC emission costs } = VOP_1 \alpha_1 + VOP_2 \alpha_2 + VOP_3 \alpha_3$$  \hfill (12)\)

VOC emission constraints:

$$TVOC = VOQ_1 \alpha_1 + VOQ_2 \alpha_2 + VOQ_3 \alpha_3$$  \hfill (13)\)

$$\alpha_0 - \alpha_1 \leq 0$$  \hfill (14)\)

$$\alpha_1 - \alpha_2 - \alpha_2 \leq 0$$  \hfill (15)\)

$$\alpha_2 - \alpha_3 \leq 0$$  \hfill (16)\)

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 1$$  \hfill (17)\)

$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$  \hfill (18)\)

Similarly, ($\alpha_0$, $\alpha_1$, $\alpha_2$, $\alpha_3$) is also an SOS1 set of 0–1 variables, within which exactly one variable must be non-zero; ($\alpha_0$, $\alpha_1$, $\alpha_2$, $\alpha_3$) is an SOS2 set of non-negative variables, within which at most two adjacent variables, in the order given to the set, can be non-zero. $TVOC$ in Eq. (13) is the total quantity of VOC emissions; the function of $TVOC$ will depend on the case problem. [Note that the function of $TVOC$ in Section 4. “Numerical example” is $(2X_1 + X_2 + X_3)$, which means that the quantity of VOC emissions for one unit of Products 1, 2 and 3 are 2, 1, and 1, respectively.]

3.1. Integrated cost models

The model for product-mix decision making with capacity expansions under activity-based costing is as follows: Maximize:

$$\pi = (\text{Total Revenue}) - (\text{Total Direct Material Cost})$$

- (Total Direct Labor Cost)
- (Total Unit – Level Activity Cost)
- (Total Batch – Level Activity Cost)
- (Total Product – Level Activity Cost)
- (Total Facility – Level Activity Cost)
- (Total VOCs Emission Cost)

$$= \sum_{i=1}^{n} p_i X_i - \sum_{i=1}^{n} \sum_{m=1}^{s} c_m a_{im} X_i - (LC_1 \mu_1 + LC_2 \mu_2) - \sum_{i=1}^{n}$$

$$\times \sum_{j \in U} d_{ij} \lambda_{ij} X_i - \sum_{i=1}^{n} \sum_{j \in B} d_{ij} \beta_{ij} B_j - \sum_{i=1}^{n} \sum_{j \in P} d_{ij} \rho_{ij} R_i - \sum_{k=0}^{t} FC_k \theta_k$$

- $(VOP_1 \alpha_1 + VOP_2 \alpha_2 + VOP_3 \alpha_3)$

$$= \sum_{i=1}^{n} p_i X_i - \sum_{i=1}^{n} \sum_{m=1}^{s} c_m a_{im} X_i - (LC_1 \mu_1 + LC_2 \mu_2) - \sum_{i=1}^{n}$$

$$\times \sum_{j \in U} d_{ij} \lambda_{ij} X_i - \sum_{i=1}^{n} \sum_{j \in B} d_{ij} \beta_{ij} B_j - \sum_{i=1}^{n} \sum_{j \in P} d_{ij} \rho_{ij} R_i - \sum_{k=0}^{t} FC_k \theta_k$$

- $(VOP_1 \alpha_1 + VOP_2 \alpha_2 + VOP_3 \alpha_3)$

$$= \sum_{i=1}^{n} p_i X_i - \sum_{i=1}^{n} \sum_{m=1}^{s} c_m a_{im} X_i - (LC_1 \mu_1 + LC_2 \mu_2) - \sum_{i=1}^{n}$$

$$\times \sum_{j \in U} d_{ij} \lambda_{ij} X_i - \sum_{i=1}^{n} \sum_{j \in B} d_{ij} \beta_{ij} B_j - \sum_{i=1}^{n} \sum_{j \in P} d_{ij} \rho_{ij} R_i - \sum_{k=0}^{t} FC_k \theta_k$$

- $(VOP_1 \alpha_1 + VOP_2 \alpha_2 + VOP_3 \alpha_3)$.  

\[\text{Cost} = FC_0 + FC_1 + FC_2 + FC_3 + FC_4 + FC_5 + FC_6 + FC_7 + FC_8 + FC_9 + FC_{10}\]  \hfill (20)
Subject to:

Direct material constraints:

\[ \sum_{i=1}^{n} a_{im} x_i \leq Q_m, \quad m = 1, 2, \ldots, s \]  

(21)

Direct labor constraints:

\[ TL = LH_1 \mu_1 + LH_2 \mu_2 \]  

(22)

\[ \mu_0 - \mu_1 \leq 0 \]  

(23)

\[ \mu_1 - \mu_1 - \mu_2 \leq 0 \]  

(24)

\[ \mu_2 - \mu_2 \leq 0 \]  

(25)

\[ \mu_0 + \mu_1 + \mu_2 = 1 \]  

(26)

\[ \eta_1 + \eta_2 = 2 \]  

(27)

Batch-level activity constraints:

\[ x_i \leq \sigma_i B_{ij}, \quad i = 1, 2, \ldots, n; \quad j \in B \]  

(28)

\[ \sum_{i=1}^{n} \delta_{ij} B_{ij} \leq T_j, \quad j \in B \]  

Product-level constraints:

\[ x_i \leq D_i R_i, \quad i = 1, 2, \ldots, n \]  

(30)

Machine hour constraints:

\[ \sum_{i=1}^{n} \lambda_i x_i - \sum_{k=0}^{r} MH_k \theta_k \leq 0 \]  

(32)

\[ \sum_{k=0}^{r} \theta_k = 1 \]  

(33)

VOC emissions constraints:

\[ TVOC = VOQ_1 \alpha_1 + VOQ_2 \alpha_2 + VOQ_3 \alpha_3 \]  

(34)

\[ \alpha_0 - \alpha_1 \leq 0 \]  

(35)

\[ \alpha_1 - \alpha_1 - \alpha_2 \leq 0 \]  

(36)

\[ \alpha_2 - \alpha_2 - \alpha_3 \leq 0 \]  

(37)

\[ \alpha_3 - \alpha_3 \leq 0 \]  

(38)

\[ \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 1 \]  

(39)

\[ \omega_1 + \omega_2 + \omega_3 = 1 \]  

(40)

\[ x_i \geq 0, \quad i = 1, 2, \ldots, n \]  

(41)

\[ B_{ij} \geq 0, \quad i = 1, 2, \ldots, n; \quad j \in B \]  

where

\[ \pi \]  

the company's profit; \[ x_i \]  

the production quantity of product \( i \); \[ P_i \]  

the unit selling price of product \( i \); \[ C_m \]  

the unit cost of the \( m \)th material; \[ a_{im} \]  

the requirement of the \( m \)th material for one unit of product \( i \); \[ LH_1 \]  

the available normal direct labor hours; \[ LC_1 \]  

total direct labor cost in \( LH_1 \); \[ LC_2 \]  

total direct labor cost in \( LH_2 \); \[ d_j \]  

the actual running direct activity cost per activity driver for activity \( j \); \[ \lambda_j \]  

the requirement of the activity driver of unit-level activity \( j \) \( (j \in U) \) for one unit of product \( i \); \[ \delta_{ij} \]  

the requirement of the activity driver of batch-level activity \( j \) \( (j \in B) \) for product \( i \); \[ B_{ij} \]  

the number of batches of batch-level activity \( j \) \( (j \in B) \) for product \( i \); \[ R_i \]  

the indicator for producing product \( i \) \( (R_i = 1) \) or not producing product \( i \) \( (R_i = 0) \); \[ TL \]  

total direct labor hours; \[ MH_k \]  

the available direct machine hours; \[ VOP_i \]  

total VOC emissions cost in \( VOQ_i \); \[ VOP_1 \]  

total VOC emissions cost in \( VOQ_1 \); \[ VOP_2 \]  

total VOC emissions cost in \( VOQ_2 \); \[ VOP_3 \]  

total VOC emissions cost in \( VOQ_3 \); \[ TVOC \]  

the total quantity of VOC emissions; \[ VOQ_1 \]  

the total quantity of VOC emissions from product execution (must increase the carbon tax rate); \[ VOQ_2 \]  

the total quantity of VOC emissions from product execution (must increase the carbon tax rate); \[ VOQ_3 \]  

the total quantity of VOC emissions from product execution (must increase the carbon tax rate); \[ \theta_k \]  

an SOS1 (special ordered set of type 1) set of \( 0 \) \( \sim \) \( 0 \) variables, within which exactly one variable must be non-zero \( (Beale \) and \( Tomlin, 1970; Williams, 1985)) \( \theta_k = 1 \) \( (k \neq 0) \) means that the capacity needs to be expanded to the \( k \)th level, i.e., \( MH_k \) machine hours; \[ \eta_1, \eta_2, \omega_1, \omega_2, \omega_3 \]  

an SOS1 (special ordered set of type 1) set of \( 0 \) \( \sim \) \( 0 \) variables, within which exactly one variable must be non-zero \( (Beale \) and \( Tomlin, 1970; Williams, 1985)) \( \eta_1, \eta_2 \) \( \sim \) \( s \) variables. \( \omega_1, \omega_2, \omega_3 \) \( \sim \) \( s \) variables.

\[ \eta_1, \eta_2, \theta_k (k = 0, 1, \ldots, t); \{ \omega_1, \omega_2, \omega_3 \}; \) SOS1 set of \( 0 \) \( \sim \) \( 0 \) variables. \[ \eta_1, \eta_2, \theta_k (i = 1, 2, \ldots, n); \{ \omega_1, \omega_2, \omega_3 \}; \) SOS1 set of \( 0 \) \( \sim \) \( 0 \) variables.

\[ (\eta_1, \eta_2, \theta_k (k = 0, 1, \ldots, t); \{ \omega_1, \omega_2, \omega_3 \}); \) SOS1 set of \( 0 \) \( \sim \) \( 0 \) variables. \[ (\eta_1, \eta_2, \theta_k (i = 1, 2, \ldots, n); \{ \omega_1, \omega_2, \omega_3 \}); \) SOS1 set of \( 0 \) \( \sim \) \( 0 \) variables.

Eq. (20) is the profit function \( (\pi) \) to be maximized in the model. The first term \( (\sum_{i=1}^{n} P_i x_i) \) of Eq. (20) is Total Revenue, and is described in Eq. (1). The second term \( (\sum_{i=1}^{n} \sum_{m=1}^{s} c_{mn} a_{mn} x_i) \) of Eq. (20) is Total Direct Material Cost and the associated constraints are in Eq. (21), where the available quantity of the \( m \)th material is \( Q_m \). The third term \( (LC_1 + LC_2) \) of Eq. (20) is Total Direct Labor Cost and the associated constraints are in Eqs. (22)–(27), which come from Eqs. (3)–(8).
The fourth term ($\sum_{j=0}^{p-1} \sum_{i=0}^{n} d_{ji}X_{i}$) of Eq. (20) is Total Unit-level Activity Cost, which may have specific associated constraints (not shown in the model), for a specific case problem. Strictly speaking, direct material and direct labor costs are unit-level costs which increase with the production quantity. The fifth term ($\sum_{j=0}^{p-1} \sum_{i=0}^{n} d_{ji}B_{ij}$) of Eq. (20) is Total Batch-level Activity Cost and the associated constraints are in Eqs. (28) and (29), where $T_{ij}$ is the capacity limit of the activity driver of batch-level activity $j (j \in B)$ in Eq. (29) and $\sigma_{B_{ij}}$ is the upper limit of the $i$th product’s quantity from the perspective of the $j$th batch-level activity in Eq. (28). The sixth term ($\sum_{j=0}^{p-1} d_{ji}R_{i}$) of Eq. (20) is Total Product-Level Activity Cost and the associated constraints are in Eqs. (30) and (31). If $R_{i}$ = 1, this means that the $i$th product will be produced in the period. Eq. (30) is the constraint related to the demand limit of the $i$th product and Eq. (31) is the constraint related to the capacity limit of the activity driver of product-level activity $j (j \in P) (V)$. The seventh term ($\sum_{j=0}^{p-1} FC_{ik}B_{kj}$) of Eq. (20) is Total Facility-Level Activity Cost and the associated constraints are in Eqs. (32) and (33) which correspond to Eqs. (10) and (11). The machine hours required increase proportionally with the production quantity, but the cost of this is a stepwise fixed cost function. This is the typical example of theory of constraints.

The eighth term ($VOP_{1}a_{1} + VOP_{2}a_{2} + VOP_{3}a_{3}$) of Eq. (20) is Total VOC Emissions Cost and the associated constraints are in Eqs. (34)–(40) as described earlier in this section, for Eqs. (12)–(19). TVOC in Eq. (34) is the total quantity of VOC emissions, and the function of TVOC will depend on the case problem. As for the constraints in the model, some constraints are used for constructing the specific cost functions like piecewise linear functions and stepwise linear functions. However, the labor resource constraint is hidden in the formulations. Some constraints are the real constraints for resources, like Eqs. (21), (29), (31) and (32) for setting up the resource limits of materials, of the activity drivers of batch-level activities and product-level activities, and of machine hours, respectively.

As mentioned in Section 2.2 the theory of constraints (TOC), we can relax the bottleneck resources by using the methods of capacity expansions for the specific resources. This will be done in the post-optimal analyses. However, the model proposed in this paper incorporating the possible levels of capacity expansions for the various resources. Then it can, simultaneously, consider more than one resource constraints.

4. Numerical example

A numerical example is presented to illustrate the application of a mathematical programming model to determine the optimal product-mix under activity-based costing. Company A manufactures metal component parts for the automotive industry. Numerous environmental problems are preventing the Company from getting its finished components to the market. Company A now has applied GMTs to produce their products and has overcome the following environmental problems:

(1) Solid sludge from the metal pretreatment process requires the disposal of relatively large volumes of poisonous waste.

(2) Large volumes of rinse water from the topcoating spray booth for topcoating and/or clear coating need more special treatment due to the local water treatment plant’s requirements.

(3) The finished metal component parts do not have the same gloss and the quality of the coating is unstable.

Despite using electrostatic turbo bells, the transfer efficiency is still too low for such paint spray guns. Thus, VOC emissions already come near the permitted cap and are almost double what was originally estimated when the permits were applied for.

Assume that Company A is considering producing products 1, 2 and 3 ($i = 1, 2, 3$) by using new GMTs and that these products need three different kinds of direct materials ($m = 1, 2, 3$). Company A needs to calculate the following essential costs in producing these products: (1) unit-level costs and activities: manufacturing costs include machine costs, labor costs and material costs; (2) batch-level costs: inventory handling costs, carbon absorption costs and setup costs; (3) product-level costs: design costs; (4) facility-level costs: environment regulatory costs and VOC emissions costs. The related data for this example are presented in Table 1. Environmental Regulatory Cost, which refers to the costs associated with handling regular inspections, discharging waste and ensuring that processes are in compliance with the Environmental standard value specification according to the laws and regulations of local government, is a fixed cost and so can be expressed with a constant (12,000 in this example). Company A has to decide the optimal quantity of products with its current capacity. By using Eqs. (20)–(40) and the example data in Table 1, the green product-mix decision model for the example is formulated as follows:

Maximize

$$\pi = (\text{Total Revenue}) - (\text{Total Direct Material Cost})$$

- (Total Direct Labor Cost)
- (Total Unit – Level Activity Cost)
- (Total Batch – Level Activity Cost)
- (Total Product – Level Activity Cost)
- (Total Facility – Level Activity Cost)
- (Total VOCs Emission Cost)

$$= \sum_{i=1}^{n} P_{i}X_{i} - \sum_{i=1}^{n} \sum_{m=1}^{s} C_{m}q_{im}X_{i} - (LC_{1}T_{1} + LC_{2}T_{2}) - \sum_{i=1}^{n} \sum_{j=0}^{p-1} d_{ji}R_{i} - \sum_{j=0}^{p-1} FC_{ik}B_{kj} - (VOP_{1}a_{1} + VOP_{2}a_{2} + VOP_{3}a_{3})$$

$$= (200X_{1} + 230X_{2} + 250X_{3}) - [(10^6 + 2^9 + 1^2 + 3^1)X_{1} + (10^6 + 2^9 + 1^2 + 3^1)X_{2} + (10^6 + 2^9 + 1^2 + 3^1)X_{3}] - (120,000q_{1} + 220,000q_{2}) - (2^9 + 4^6)X_{1}$$

$$+ (2^9 + 4^6)X_{2} + (2^9 + 4^6)X_{3} - (100^2)B_{13} + (100^2)B_{23} + (100^2)B_{33} - (2^9)B_{14} + (2^9)B_{24} + (2^9)B_{34} - (15^2)B_{15} + (15^2)B_{25} + (15^2)B_{35}$$

$$- (100^2)R_{1} + (100^2)R_{2} + (100^2)R_{3} - (40,000)\theta_{1} + 75,000\theta_{1} + 12,000 \theta_{2} - (100,000q_{1} + 150,000q_{2} + 195,000q_{3} + 12,000)$$

$$= 91X_{1} + 99X_{2} + 100X_{3} - 120,000q_{1} + 220,000q_{2}$$

Subject to:

Direct material constraints:
Table 1
Example data.

<table>
<thead>
<tr>
<th>Maximum demand</th>
<th>Product 1</th>
<th>Product 2</th>
<th>Product 3</th>
<th>Available capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct material (Unit-level)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 1</td>
<td>$10/unit</td>
<td>$2/unit</td>
<td>$3/unit</td>
<td>$1/unit</td>
</tr>
<tr>
<td>m = 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 4 (Disposal of hazardous material)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit-level activity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine hours</td>
<td>$2</td>
<td>$1</td>
<td>$2</td>
<td></td>
</tr>
<tr>
<td>Labor hours</td>
<td>$4</td>
<td>$2</td>
<td>$4</td>
<td></td>
</tr>
<tr>
<td>Batch-level activity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inventory handling</td>
<td>Handling hours</td>
<td>$100</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Carbon adsorption</td>
<td>Machine hours</td>
<td>$2</td>
<td>4</td>
<td></td>
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<tr>
<td>Setup</td>
<td>Setup hours</td>
<td>$15</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Product-level activity − Design (Excluding regulatory costs)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine hours</td>
<td>$20,000</td>
<td>$30,000</td>
<td>$40,000</td>
<td></td>
</tr>
<tr>
<td>Environment regulatory cost</td>
<td>Total cost</td>
<td>$12,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct labor constraint-cost</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor hours</td>
<td>$10,000</td>
<td>$10,000</td>
<td>$12,000</td>
<td></td>
</tr>
<tr>
<td>Wage rate</td>
<td>$4/h</td>
<td>$5/h</td>
<td></td>
<td></td>
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<tr>
<td>VOCs emission constraint</td>
<td>Cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emission quantities</td>
<td>VOP₁ = $100,000</td>
<td>VOP₂ = $150,000</td>
<td>VOP₃ = $195,000</td>
<td></td>
</tr>
<tr>
<td>Tax rate</td>
<td>$10/ton</td>
<td>$20/ton</td>
<td>$30/ton</td>
<td></td>
</tr>
</tbody>
</table>

$6X₁ + 7X₂ + 8X₃ ≤ 120,000$

$5X₁ + 9X₂ + 10X₃ ≤ 100,000$

$2X₁ + 2X₂ + 2X₃ ≤ 50,000$

$X₁ + X₂ + 2X₃ ≤ 40,000$

Stepwise facility-level machine hour constraints:

$5X₁ + 5X₂ + 5X₃ - 20,000θ₀ - 30,000θ₁ - 40,000θ₂ ≤ 0$

$θ₀ + θ₁ + θ₂ = 1$

Direct labor constraints:

$6X₁ + 7X₂ + 8X₃ - 30,000μ₁ - 50,000μ₂ = 0$

$μ₀ - μ₁ ≤ 0$

$μ₁ - μ₁ - μ₂ ≤ 0$

$μ₂ - μ₂ ≤ 0$

$μ₀ + μ₁ - μ₂ ≤ 1$

$η₁ + η₂ = 1$

VOC emissions constraints:

$2X₁ + X₂ + X₃ - 10,000α₁ - 12,500α₂ - 14,000α₃ = 0$

$α₀ - α₁ ≤ 0$

$α₂ - α₂ - α₃ ≤ 0$

$α₃ - α₃ ≤ 0$

$α₀ + α₁ + α₂ + α₃ = 1$

$ω₁ + ω₂ + ω₃ = 1$

Batch-level inventory handling activity constraints:

$X₁ - 10B_{13} ≤ 0$

$X₂ - 10B_{23} ≤ 0$

$X₃ - 30B_{33} ≤ 0$

$2B_{13} + 2B_{23} + 2B_{33} ≤ 900$;

Batch-level carbon adsorption activity constraints:

$X₁ - 4B_{14} ≤ 0$

$X₂ - 4B_{24} ≤ 0$

$X₃ - 8B_{34} ≤ 0$

$10B_{14} + 10B_{24} + 20B_{34} ≤ 18,000$;

Batch-level setup activity constraints:

$X₁ - 5B_{15} ≤ 0$
quantities. The total pro-

\[ X_2 - 5B_{25} \leq 0 \]
\[ X_3 - 10B_{35} \leq 0 \]
\[ 2B_{15} + 2B_{25} + 4B_{35} \leq 3000; \]

Product-level constraints:
\[ X_1 - 3000R_1 \leq 0 \]
\[ X_2 - 2500R_2 \leq 0 \]
\[ X_3 - 5000R_3 \leq 0 \]
\[ 20R_1 + 10R_2 + 20R_3 \leq 55 \]
\[ X_i \geq 0, \quad i = 1, 2, 3 \]
\[ B_{ij} \geq 0, \quad i = 1, 2, 3; \quad j = 3, 4, 5; \]

\[ (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6, \eta_7, \eta_8, \eta_9); \text{SOS1 set of 0–1 variables.} \]
\[ \eta_1, \eta_2, R_1, R_2, R_3, \eta_4, \eta_5, \eta_6, \eta_7, \eta_8, \eta_9; \text{SOS1 set of 0–1 variables.} \]

This is a mixed-integer programming (MIP) model. We solve this problem by utilizing the software, LINGO 13.0, and obtain the following optimal solutions.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td>3000</td>
<td>( X_2 )</td>
<td>0</td>
<td>( X_3 )</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>0</td>
<td>( \beta_1 )</td>
<td>0</td>
<td>( \beta_2 )</td>
</tr>
<tr>
<td>( \mu_0 )</td>
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<td>( \mu_1 )</td>
<td>0</td>
<td>( \mu_2 )</td>
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<tr>
<td>( \eta_1 )</td>
<td>0</td>
<td>( \eta_2 )</td>
<td>1</td>
<td>( \eta_3 )</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>1</td>
<td>( R_2 )</td>
<td>0</td>
<td>( R_3 )</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>1</td>
<td>( \alpha_2 )</td>
<td>0</td>
<td>( \alpha_3 )</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>1</td>
<td>( \alpha_5 )</td>
<td>0</td>
<td>( \alpha_6 )</td>
</tr>
<tr>
<td>( B_{13} )</td>
<td>300</td>
<td>( B_{14} )</td>
<td>0</td>
<td>( B_{15} )</td>
</tr>
<tr>
<td>( B_{16} )</td>
<td>750</td>
<td>( B_{17} )</td>
<td>0</td>
<td>( B_{18} )</td>
</tr>
<tr>
<td>( B_{19} )</td>
<td>600</td>
<td>( B_{20} )</td>
<td>0</td>
<td>( B_{21} )</td>
</tr>
</tbody>
</table>

According to the results, the optimal quantity of green product-mix is \( (X_1, X_2, X_3) = (3000, 0, 4000) \), which requires 50,000 units \((= 6 \times 3000 + 7 \times 0 + 8 \times 4000)\) of the first kind of material, 55,000 units \((= 5 \times 3000 + 9 \times 0 + 10 \times 4000)\) of the second kind of material, 14,000 units \((= 2 \times 3000 + 2 \times 0 + 2 \times 4000)\) of the third kind of material, 35,000 \((= 5 \times 3000 + 5 \times 0 + 5 \times 4000)\) machine hours, 50,000 \((= 6 \times 3000 + 7 \times 0 + 8 \times 4000)\) direct labor hours, and 10,000 \((= 2 \times 3000 + 1 \times 0 + 1 \times 4000)\) VOC emissions quantities. The total profit \( \pi \) is $52,200. This means that the machine capacity does not exceed 40,000 h or that the company has to rent machines, that the direct labor capacity is equal to 50,000 labor hours and that the VOC emissions are equal to 10,000 tons of emission quantity which is in the upper limit of the first range of VOC emission cost function.

Company A has adopted ABC. Therefore, some related ABC data can be obtained directly from the accounting department of Company A. We can also ask about the related business units to provide related data required in the product-mix decision models. For example, we can ask the production department to provide information concerning maximum demand, the sales department to provide information concerning selling price, the purchasing department to provide cost data for direct materials, and the accounting department to provide information on unit-level, batch-level, product-level and facility-level activity costs obtained from the ABC study. As for the Environmental Regulatory cost and VOC emissions, we can invite the HSE management department to provide information, and can also ask the human resources department to provide related direct labor cost information. However, the most direct, and fastest, method is to invite the decision maker to request business units to submit related data to the accounting department for collection, and then provide this information for the study.

In addition, VOCs can also be obtained from the emission cost analysis of the government unit or the company. Furthermore, additional overtime situations and related operation activities, caused by production capacity expansion, can also be provided by the production department of Company A.

Using the TOC procedure, the constraints are identified and used so that new GMTs are used most profitably. Company A is able to choose products that produce the least amount of VOC emissions since the TOC approach fosters the selection of optimal product-mix, which emits fewer VOC emissions. In effect, when the optimal product-mix is chosen, the output volumes of product 2 are zero. This means that there is an unsatisfied demand for product 2 in the market, and Company A may increase the price of product 2 to reflect its environmental costs. Although investments in new GMTs can be very expensive, by adopting the TOC procedure, a sound environmental investment by Company A can maximize their profits.

5. Discussion

It seems that the model proposed in this paper will select a product mix with higher pollution with the only objective of maximizing the profit of a product mix based on the most constrained resources. Although the model does not explicitly select an optimal product mix that emits fewer VOC emissions, we can use the related constraints to restrain VOC emission quantity within certain limits. As a matter of fact, it is impractical to aim to minimize VOC emission quantity or cost. Alternatively, we can set up a goal programming model to maximize the operating profit, to minimize deviation from the VOC emission quantity target, and meet other objectives. In the numerical example presented in Section 4, it was assumed that the total VOC emissions cost function would be a piecewise linear function. The unit cost for VOC emissions (i.e. tax rate) will increase with increasing VOC emissions from $10/ton, $20/ton to $30/ton within the different ranges of emission quantity [0, 10,000], [10,000, 12,500] and [12,500, 14,000], respectively. This constraint will restrain the product-mix solution to emit the VOCs within the upper limit of the first range. Otherwise, the tax rate will increase double. Managers may want to search for new GMTs with fewer VOC emissions from the top coat spray processes to simultaneously increase operating profits and to maintain corporate social responsibility.

6. Conclusion

In this paper, we considered the capacity expansions for machine hours and direct labor hours. We also used piecewise and stepwise linear functions to approximate the non-linear direct labor costs and machine costs. Specifically, we further extended the piecewise linear function to quantize the VOC emissions so that we could depict the total VOC emissions cost function, which in turn helps to build a product-mix model using the green, multi-product primer-topcoat system under ABC. Limited by cost considerations, the profits combination also undergoes certain changes along with adjustment, such that a maximum profit combination is calculated according to the conditions. Compared with earlier studies and early articles that only use figures for description, such as Kee (1995), this study has explored a detailed mathematical
programming model. Furthermore, consideration is given to the carbon factor and costs have been discussed.

The integration of ABC and TOC and the application of an MIP model in our research can improve the efficiency and effect of the product-mix decision model for the green, multi-product primer-topp coat system. By integrating ABC and TOC approaches, traditional drawbacks and problems can be solved (e.g., ignoring resource constraints, relying heavily on the assumption of proportional cost structures, short-term focus of TOC, and so on). In addition, these techniques can be readily utilized to improve product-mix decisions in the context of the current regulatory environment, while producing a better mix of environmentally-friendly products.

As indicated in the research background, factory operators need accurate information from their management accounting systems to make profitable choices, especially with regard to environmental spending. This MIP model identifies and incorporates the costs of VOC emissions. The optimal product-mix from ABC combined with the TOC approach will be more accurate than traditional accounting approaches. This paper specifically focused on the priming and topcoating system and metal component parts for the automotive industry, and so is limited by only considering single objectives when evaluating the product-mix decision. Future research may attempt to extend these techniques to different industries and diverse activities that may produce certain kinds of emissions. Additionally, incorporating various unit selling prices, unit direct material costs and other kinds of emissions into the product-mix decision model to reflect the dynamics of the real world may better evaluate environmental impact. Besides, we can consider multiple objectives to make the model more realistic.

Acknowledgment

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References


