Market Panics, Frenzies, and Informational Efficiency: Theory and Experiment

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Abstract

In a market rush, the fear of future adverse price movements causes traders to trade before they become well-informed, reducing the informational efficiency of the market. I derive theoretical conditions under which market rushes are equilibrium behavior and study how well these conditions organize trading behavior in a laboratory implementation of the model. Market rushes, including both panics and frenzies, occur more frequently when predicted by theory. However, subjects use commonly-discussed, momentum-like strategies that lead to informational losses not predicted by theory, suggesting that these strategies may exacerbate both the occurrence and consequences of panics and frenzies.

1 Introduction

‘Panics’ and ‘frenzies’ have been a fixture of interest in financial markets for centuries (stretching back at least to Charles Mackay’s 1841 book, ‘Extraordinary Popular Delusions and the Madness of Crowds’). In these episodes, people worry about getting out of an asset before it falls further (a panic) or about getting in before it’s too late (a frenzy). An intuitive, but understudied, feature of these phenomena is that, in a rush to get into or out of an asset, people may spend little time acquiring information about the asset they are trading. The consequences may be detrimental not only to the traders themselves, but, through informationally inefficient market prices, also impact the real economy (Bond, Edmans, and Goldstein (2012)).

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When do panics and frenzies (collectively, *market rushes*) lead to poor information aggregation? In this paper, I study this question using two tools. First, in the spirit of papers which have provided rational explanations for the related phenomena of price crashes and bubbles (Allen, Morris, and Postlewaite (1993), Romer (1993), Bulow and Klemperer (1994), Lee (1998), Barlevy and Veronesi (2003), Moinas and Pouget (2013), and Brunnermeier and Pedersen (2005)), I construct a theoretical model of trade timing based on Glosten and Milgrom (1985). In the main theoretical mechanism, traders trade off the benefit of better private information with the cost of adverse prices movements due to being preempted by other traders. I show that, indeed, ‘rational’ market rushes can lead to informational losses in equilibrium, and I provide conditions under which these losses (i) must occur, and (ii) should never occur.

Second, acknowledging the fact that market phenomena have long been described by irrational, affective terms like ‘panics’, ‘frenzies’, and ‘manias’, I ask whether market rushes lead to informational losses only because of rational motives. To do so, I precisely implement the model in a laboratory experiment. In one treatment (called Rush), I parameterize the experiment such that a market rush should always occur in equilibrium: subjects should trade at the first opportunity, foregoing valuable information about asset values. In another (called Wait), I parameterize the experiment such that subjects should instead delay trade until receiving full information. Although market rushes are far more common and severe in Rush (where they are equilibrium behavior) than Wait (where they are not), frequent trades prior to full information occur in the Wait treatment. I show that this behavior is driven by subjects using a simple price-chasing heuristic commonly ascribed to momentum traders. The results therefore suggest that a well-documented trading heuristic, popular in naturally-occurring settings, may exacerbate informational losses in settings conducive to panics or frenzies.

I begin by introducing a simple model of trade timing in the spirit of Glosten and Milgrom (1985). Multiple risk-neutral traders receive binary signals and trade a binary-valued, common-value asset with a market maker. In the main departure from standard models, each trader chooses when to trade, receiving private information over time. Specifically, each simultaneously receives initial (poor quality) information upon arrival to the market, and is then given several opportunities to trade before receiving additional information just

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1Here, a note on terminology is useful. In popular usage, the terms ‘panic’ and ‘frenzy’ typically refer to timing phenomena, which is the focus of my work. Panics are often associated with bad states of the world, so much of the theoretical literature on market ‘panics’ focuses on (discontinuous) price crashes rather than timing per se. Price crashes can (but need not) result from these types of episodes. Frenzies, in common usage, often imply excess demand, and hence rising prices or bubbles. For the purposes of this paper, panics and frenzies - which I refer to collectively as ‘market rushes’ - are timing phenomena associated with falling and rising prices, respectively.
prior to the final trading period. Additional information generates a benefit to waiting, but waiting is costly due to the presence of other traders - should another trader trade, information is revealed to the market which reduces uncertainty in the asset value and thus potential profits. In order to make this tradeoff simple and to be able to easily define and identify early trades (trades prior to full information), I restrict each trader to a single trade.²

In the theoretical analysis, I first show the standard result that traders can profit from trading according to their private information (i.e. buying with favorable information and selling otherwise). In the main theoretical result, I prove a pair of sufficient conditions, one of which ensures that a market rush occurs in equilibrium (all traders trade in the first period), and the other of which ensures that traders instead wait for better information (trade in the final period). Due to the symmetric nature of the model, market rushes can take the form of either panics (early sales generating falling prices) or frenzies (early purchases generating rising prices).

I then conduct a laboratory experiment which is designed to ask whether market rushes occur only when they can be sustained as an equilibrium or if, instead, they also arise due to non-equilibrium behavior. Experimental evidence on this question is important because it is difficult to evaluate the source of panics and frenzies and their informational consequences using naturally-occurring data, where the key determinants of equilibrium (particularly the timing and quality of information) are generally unobservable. However, existing experiments on trade timing (discussed below) do not implement models with precise theoretical predictions, making it difficult to pinpoint the source and severity of market rushes.

In the experiment, eight subjects are endowed with a low quality signal of the value of an asset and allowed to trade once (with an automated market maker) in any one of eight periods. Subjects are also informed that, just prior to the eighth trading period, each will learn the value of the asset perfectly. I study two treatments: in the Wait treatment, the early signal is very noisy while in the Rush treatment it is of significantly higher quality. Using equilibrium predictions from the theory, I identify two nested hypotheses. The weaker comparative static hypothesis is that trades occur significantly earlier in the Rush treatment than in the Wait treatment. The stronger point predictions are that trades occur in the first period in the Rush treatment and the final period in the Wait treatment. In order to study learning and assess the behavior of experienced subjects, I allow subjects to play this eight-period game thirty times.

The results reveal strong support for the comparative static hypothesis. After a few

²With multiple trades, speculative motives for trade complicate the theoretical analysis, as well as the interpretation of experimental results. I discuss the possibility of such extensions in future work in the Conclusion.
repetitions of the game, market rushes robustly emerge in the Rush treatment: subjects forgo perfect information about the asset value so that market prices are able to aggregate only weak signals. Subjects in the Wait treatment, by contrast, delay trade significantly even after thirty repetitions of play. Thus, I observe a large difference in the degree of early trades across treatments, as predicted. The point prediction hypothesis is, however, partially rejected by the data. Subjects in Rush almost universally trade in the first period as hypothesized, but subjects in Wait rarely wait until the final period.

Why do subjects behave according to theory in the Rush treatment but not the Wait treatment? A natural alternative hypothesis is that the price trends that form a salient part of panics and frenzies play a role. In fact, a closer examination of the data reveals that subjects follow momentum-like strategies: rather than best responding to beliefs about future price movements as in an equilibrium model, subjects instead respond to backward-looking price trends, waiting until sufficiently certain about the asset value and then trading in the direction most likely to pay off. Subjects adapt their behavior over multiple repetitions of the game in a way that suggests that they fine tune the threshold of certainty they require in order to trade. Simulations of a simple, parsimonious learning model (à la the win-stay-lose-shift model of Nowak and Sigmund (1993) or learning direction theory of Selten and Buchta (1998)) show that this heuristic, adapted over time, converges to outcomes that look almost identical to the data: perfect market rushes emerge in simulations of the Rush treatment and intermediate and heterogeneous trading times emerge in simulations of the Wait treatment. Furthermore, this momentum-like behavior, a trading heuristic familiar to students of technical analysis and commonly discussed in the behavioral finance literature (Grinblatt, Titman, and Wermers (1995), Hong and Stein (1999), Grinblatt and Keloharju (2000), Brozynski et al. (2003), Baltzer, Jank, and Smajlbegovic (2015), and Grinblatt et al. (2016))), causes short-term positive correlations in returns to emerge in the experimental data, a common artifact of trade observed in financial markets.3

The theoretical model builds upon that of Kendall (2018a). There, in a model with only two traders, I demonstrate the main theoretical force that operates here: public information revealed either directly via news, or inferred from others’ trades, imposes a cost of waiting to acquire better information. In that paper, I solve for equilibrium behavior in the standard environment (Glosten and Milgrom (1985)) in which market makers condition prices on the history of trades and the current order, therefore posting separate bid and ask prices. In the

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3For a description of momentum trading in the context of technical analysis, see http://www.investopedia.com/articles/trading/02/090302.asp. For a review of the empirical evidence of correlations in returns, see Daniel, Hirshleifer, and Subrahmanyam (1998). These correlations are often given behavioral explanations, including momentum strategies (Hong and Stein (1999)), but I’m unaware of any other direct evidence linking momentum strategies to correlations in returns.
resulting equilibrium, traders mix between acquiring better information and not in an attempt to disguise the fact that they have information (similar to Kyle (1985)). These mixed strategies complicate the analysis and would also create difficulties in interpreting experimental data. Here, therefore, I instead assume the market maker posts only a single price equal to the expected value of the asset (Lee (1998) makes the same simplifying assumption). This assumption does not change the main force of interest - the strategic interaction between the traders - but considerably simplifies the analysis, allowing me to extend the model to multiple traders as is necessary to study market rushes. The contemporaneous papers of Bouvard and Lee (2016) and Dugast and Foucault (2017) also theoretically study the tradeoff between better information and the effects of competition. Bouvard and Lee (2016) study firm’s decisions to conduct time-consuming risk management. Dugast and Foucault (2017) study informational efficiency in a model in which rumors take time to verify. In both of these papers, competition produces informational efficiencies, but in perfectly competitive environments less amenable to identifying market panics in a laboratory setting.4

On the experimental side, I’m the first to study the informational efficiency consequences of market rushes. Park and Sgroi (2012) study herding and contrarian behavior in the setup of Park and Sabourian (2011). Their experiment confirms qualitative predictions about when trades should occur, but their environment differs in that private information does not improve with time and it is not amenable to precise theoretical predictions. The environment of Shachat and Srinivasan (2011) features these same two differences. They study trading in continuous double auctions with the sequential arrival of private or public information. Asparouhova, Bossaerts, and Tran (2016) study a credit rollover game in which subjects must decide when to convert an asset to cash. Their environment does not feature private information. Brunnermeier and Morgan (2010) study clock games theoretically and experimentally. Their game features something akin to a market rush but absent informational inefficiencies. Finally, several experiments consider timing decisions with precise theoretical timing predictions, but in non-market settings. Sgroi (2003) and Ziegelmeyer et al. (2005) both implement the irreversible investment model of Chamley and Gale (1994). Ivanov, Levin, and Peck (2009,2013) and Çelen and Hyndham (2012) study similar environments. Incentives to wait are quite different in these papers as observing others’ decisions can be beneficial when their information is not incorporated into prices.

The paper is organized as follows. In Section 2, I develop the model and derive the conditions under which we expect to see market rushes as equilibrium behavior. In Section 3, I describe the experimental design, develop hypotheses, and provide the experimental

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4Smith (2000) also studies trade timing theoretically, but in a setting in which private information doesn’t improve with time.
results. I then show that subjects use momentum-like strategies and develop a simple learning model to explain differences across treatments. In Section 4, I conclude by discussing the relationship between the experimental findings and behavior in naturally-occurring markets, as well as hypothesize about the underlying psychological foundations of momentum-like behavior.

2 Theory
2.1 Model

The model is meant to capture the idea that producing private information about asset values requires time, either to acquire the information, or to process it into a trading decision (or both). To give a concrete example, consider an unexpected news release of a firm deciding to acquire another. Scanning the news provides a very rapid, but likely very noisy, indication of the change in the acquirer’s fundamental value. More detailed analysis (developing new valuation models, interviewing managers, etc.) improves estimates of the firm’s value, but takes time. During this time, any information others act on gets imputed into prices, reducing potential profits and creating a tradeoff between acting quickly and producing better information.\(^5\)

The model is set in discrete time with trading periods at \(t = 1, 2, \ldots, T\). In each trading period, each of \(n\) risk-neutral traders may trade a single, common-value asset, \(V \in \{0, 1\}\), at a price established by a market maker. Each trader may trade only a single unit (buy or sell) in any of the \(T\) trading periods. The initial prior that the asset is worth \(V = 1\) is \(p_1 = \frac{1}{2}\). When the asset value is realized at \(T\), those who purchased the asset at time \(t\) receive a payoff of \(V - p_t\) and those who sold (short) receive a payoff of \(p_t - V\). There is no discounting.

Each trader, identified by \(i \in n\), receives a private signal before the first trading period, \(s_i \in \{0, 1\}\), which has correct realization with probability \(q = Pr(s_i = 1|V = 1) = Pr(s_i = 0|V = 0) \in (\frac{1}{2}, 1)\). If a trader waits until time \(T\) to trade, she receives an additional private signal, \(\bar{s}_i \in \{0, 1\}\), immediately prior to her final trading opportunity, which has correct realization with probability \(\bar{q} = Pr(\bar{s}_i = 1|V = 1) = Pr(\bar{s}_i = 0|V = 0) \in (q, 1]\). Note that I assume signal quality improves with time, \(\bar{q} > q\), and that I allow for the second private signal to reveal the true asset value perfectly.

At the beginning of each time period other than the first, a binary public signal, \(s_{P,t} \in\)

\(^5\)Although the model is silent on the time-scale, I have in mind information that is acquired over relatively short time frames, perhaps minutes or hours, such that adverse price movements are a primary consideration.
\{0,1\}, which has correct realization with probability \(q_P = \Pr(s_{P,t} = 1|V = 1) = \Pr(s_{P,t} = 0|V = 0) \in (\frac{1}{2}, 1)\) becomes public knowledge. The public signals represent information learned by the market maker directly. They impose a cost of waiting that is independent of the strategies of other traders, which (under conditions I establish) can resolve the underlying coordination game between the traders, providing a unique equilibrium outcome. After the public signal is available, but prior to any trades, a market maker establishes a single price, \(p_t = E[V|H_t] = \Pr[V = 1|H_t]\), equal to the expected value of the asset conditional on all publicly available information (prior trades, timing decisions, public signals, and prices), \(H_t\), at which all trades occur. Having the market maker post a single price simplifies the theoretical analysis, but also, importantly, increases incentives in the laboratory setting.\(^6\) However, as discussed in the Introduction, it does not drive the qualitative nature of the results - Kendall (2018a) shows that competition can induce traders to act before receiving full information even under the assumption that the market maker additionally conditions prices on the order.

### 2.2 Analysis

The solution concept is sequential equilibrium and I focus on Markov strategies that depend only upon the payoff-relevant state. Proposition 1 begins with a characterization of the optimal trading strategy of a trader (trade direction), whether on-path or off-path. All proofs are given in part A of the Appendix.

**Proposition 1 (Equilibrium Trading Strategies):** In any equilibrium:

1. traders who trade prior to period \(T\) buy if \(\bar{s}_i = 1\) and sell if \(\bar{s}_i = 0\).
2. traders who trade in period \(T\) buy if \(\bar{s}_i = 1\) and sell if \(\bar{s}_i = 0\)

Proposition 1 states that traders optimally buy with favorable private signals and sell otherwise (and trade according to the stronger of the two signals when they trade at time \(T\) with both signals). Intuitively, a favorable signal (or a strong favorable signal and a weak unfavorable signal) ensures that a trader’s private belief exceeds the public belief (which is equal to the price) so that buying results in a positive expected profit. Conversely, an unfavorable signal leads to a private belief below the price so that selling is profitable. Proposition 1 ensures that all trades reveal public information, which, when incorporated into prices, imposes payoff externalities on other traders. These payoff externalities preclude

\(^6\)In the standard setup, the market maker also conditions the price on the order, posting separate bid and ask prices. Separate bid and ask prices reduce the expected profit a subject can earn from private information and would also necessitate having noise traders, complicating the description and implementation of the experiment.
herding behavior in the spirit of Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992): although a trader learns about the asset value from previous trades, the price fully reflects this information so that one must always trade according to one’s private information in order to earn an (expected) profit (Avery and Zemsky (1998) were the first to demonstrate this feature of market prices). More importantly, these externalities create a cost of waiting to get better information: through its impact on prices, public information reduces uncertainty in the asset value and therefore the value of private information.

Given the optimal trading strategies of Proposition 1, we can calculate the expected profit from trading in the current period and compare it to the expected profit from trading in some future period in order to determine a trader’s optimal timing strategy. Because Markov strategies can depend upon the current price, the number of traders who have yet to trade, and traders’ private signals, the strategy space is large, making equilibrium characterization potentially very challenging. In particular, the fact that the two types of traders (those with different private signal realizations) may choose to delay their trades with different probabilities means that information may be revealed by a decision not to trade, impacting the price and in turn the expected value of waiting. In a key result, however, I show (Lemma A2 of part A of the Appendix) that both types must follow the same timing strategy (trade immediately or wait) in every possible state.7

Given this result, no information is revealed by a decision to wait on the equilibrium path. However, if an equilibrium specifies that a trader trades in some period prior to T, then off-equilibrium beliefs about the type of the deviating trader become relevant in determining the profit from deviating to trade in a later period. In particular, the beliefs of the market maker are critical because they determine how the price changes upon observing a deviation.8 Consistent with the result that no information is revealed by a decision to wait on the equilibrium path, I specify that the off-equilibrium beliefs about types after a deviation are the same as before the deviation (i.e. the deviation to not trade at the specified time reveals no information). Under this specification (formalized as B1), the price does not change after a deviation, a rule which I also impose in the experimental implementation.

7This result depends upon the assumption that the market maker posts only a single price. With separate bid and ask prices, due to the strategic interaction with market maker, timing strategies generally involve mixing (Kendall (2018a)) with probabilities that depend upon a trader’s private signal. It is for this reason that having the market maker post only a single price greatly improves tractability, allowing me to focus on the strategic interactions between traders.

8Technically, the market maker is not a strategic player. He does not maximize any particular function and in fact loses money in expectation. However, it is convenient to describe the price-setting rule as if prices are determined by a market maker with the beliefs of a strategic player.
**B1 (Off-equilibrium Beliefs):** Any off-equilibrium decision to wait when a trader should have traded reveals nothing about her private information. The market maker therefore does not update the price.

With the specification of beliefs in B1, two additional intermediate results characterize the impact of public information on the value to waiting for more information. The first (Lemma A3) formalizes the intuition that this information creates a cost to delaying trade. The second (Lemma A4) is that if it is not worth waiting for more information at some particular price, then it is also not worth waiting when the price is closer to one-half (all else equal). This result is a consequence of the main theoretical contribution of Kendall (2018a): when the level of uncertainty in the asset value is highest (the price is closest to one-half), public information has the greatest impact on expected profits so that it is more costly to wait. It has an important consequence: in any equilibrium, all trades must take place at $t = 1$ or at $t = T$. If a trader doesn’t plan to always wait until $T$ to obtain better information, then she must trade in the first period where uncertainty in the asset value is maximal.

**Lemma 1 (Timing Strategies):** With off-equilibrium beliefs given by B1, in any equilibrium, all trades must take place either at $t = 1$ or $t = T$.

To complete the equilibrium characterization it remains to determine in which of the two periods ($t = 1$ or $t = T$) trades occur. Intuitively, given that others’ trades impose a cost of obtaining better information, the game is a form of coordination game with strategic complementarity between trade timing decisions. Proposition 2 characterizes the equilibrium set, establishing intuitive sufficient conditions that guarantee unique equilibrium trade timing predictions.

**Proposition 2 (Equilibrium Timing Strategies):** With off-equilibrium beliefs given by B1:

1. If the expected profit from waiting to trade at $t = T$ when all other traders trade at $t = T$ is strictly less than the expected profit from trading at $t = 1$, then the unique equilibrium outcome is that all trades occur at $t = 1$.
2. If the expected profit from waiting to trade at $t = T$ when all other traders trade at $t = 1$ is strictly greater than the expected profit from trading at $t = 1$, then the unique equilibrium outcome is that all trades occur at $t = T$.
3. Otherwise, equilibria in which all trades occur at $t = 1$ and all trades occur at $t = T$ (and, generically, a mixed strategy equilibrium) exist simultaneously.
The intuition behind the sufficiency conditions in Proposition 2 is straightforward. If the public signals themselves create a cost that makes it optimal to trade at $t = 1$ even if all of the other traders wait, then all trades must occur at $t = 1$ (part 1). On the other hand, if a trader finds it worth it to wait for better private information even if all other traders trade before her, then all traders wait for better information. Note that the initial signal quality, $q$, plays a dual role, affecting equilibrium forces in two ways. Most straightforwardly, it increases the profit from trading early by increasing the difference between a trader’s private belief and the price. But, importantly, it also increases the price impacts of others’ trades, making it more costly to wait should they trade. An increase in $q$ therefore unambiguously leads to greater incentives to trade early.

Part 1 of Proposition 2 establishes conditions under which we expect an equilibrium market rush: a situation in which all traders simultaneously trade in the first period. Traders with initial unfavorable signals expect a market panic: they put more weight on the bad state of the world and fear falling prices. Those with favorable signals instead expect a market frenzy: they believe the state of the world is good and fear missing out on purchasing the asset before prices rise. In the example of an unanticipated announcement of a merger which may be good or bad for the acquiring firm, heterogeneous private beliefs create differences in opinion, but, regardless of their opinion, traders want to act as soon as possible. In doing so, they forgo valuable information, reducing the informational efficiency of market prices.\footnote{Specifically, trades prior to full information reduce long-run informational efficiency. They, in fact, improve short-term informational efficiency by imparting some information into prices more quickly. At least over relatively short time frames (minutes or hours), short-term efficiency is presumably less important than longer-run efficiency for decisions that affect the real economy. Formally, the informational efficiency of the market price at time $T$, $p_T$, prior to $V$ being revealed is typically measured by the ex ante measure, $E(V - p_T)^2$ (or $Var(V|p_T)$). Intuitively, informational efficiency is decreasing in the quality of information traders have, but expressions for these measures are analytically cumbersome. However, in the experiment, we observe both $V$ and $p_T$, allowing me to quantify informational losses using the simpler, ex post measure, $|V - p_T|$.}

In the model, early trades (trades prior to obtaining full private information in period $T$) always take the form of a full market rush. In the experimental data, by contrast, individuals may trade early at various intermediate times without generating a market rush.

3 Experiment

3.1 Experimental Design

The experiment is a direct implementation of the model. Each of the $n = 8$ subjects in the market trades a stock with the computer in each of 30 trials (repetitions) of the game (each trial is a full run of a $T = 8$ period game). At the beginning of each trial, each subject
receives a weak private signal (the quality of which varies across treatments as described below), corresponding to a randomly drawn ball from an urn that corresponds to the true asset value. The trial then unfolds over 8 periods, each of which consists of the following sequence of events:

1. Each trader has the opportunity to (simultaneously) trade at a price determined by the (computerized) market maker (the initial price is \( p_1 = \frac{1}{2} \)), or to wait until a later period.
2. The market maker updates the price to reflect the new expected value of the asset given the information revealed by the trades (if any).
3. A public signal (also determined by an urn draw) of quality, \( q_P = \frac{17}{24} \), becomes available and the market maker again updates the price in response.

Subjects can only trade once per trial and if a subject waits in the first seven periods, she learns the asset value for certain (\( \overline{q} = 1 \)), and is forced to trade in the final period. Throughout, subjects observe the complete history of the game including the price path and others’ trades (+1 indicating a purchase and −1 a sale) on the display pictured in Figure 1.\(^{10}\) All of the details of the experiment are provided to subjects prior to play through instructions reproduced in Appendix C.

The goal of the experiment is to understand the degree to which early trades are motivated by equilibrium forces. There are two components to this question. First, do early trades occur when the theory predicts they should? Second, do early trades occur only when theory predicts they should, or do behavioral tendencies not included in a standard model cause them to occur even when not predicted as equilibrium behavior?

To answer this question, the design varies whether market rushes can be supported as an equilibrium outcome across two treatments (with parameters summarized in Table 1). The only difference between treatments is in the value of \( q \). In the first, treatment Wait (abbreviated W), \( q = \frac{13}{24} \), so that initial private information is very weak. In part A of the Appendix, I show that this parameterization satisfies the sufficient condition in part 2 of Proposition 2 so that there exists a unique equilibrium in which all traders wait to trade in period \( T \). In the second treatment, treatment Rush (abbreviated R), the initial signal quality is higher, \( q = \frac{3}{4} \), for which theory predicts a market rush in the first period (part 1 of Proposition 2). Other parameters are held constant across treatments. I chose \( \overline{q} = 1 \) to provide a very salient reason to wait to obtain better private information: traders can guarantee themselves a positive profit by waiting to trade until period \( T \). The public signal

\(^{10}\)The software was developed using the Redwood package (Pettit and Oprea (2013)) which uses HTML5 to allow for rapid updating of the computer interface. This feature allows for many more trials than would have been possible otherwise, which is important for learning in this relatively complex environment.
quality and length of the game were chosen to generate gradual revelation of the true asset value and thus scope for realistic, rich price dynamics to potentially influence behavior.

A technical question that arises in the experimental design is how prices should be set by the (computerized) market maker. The information revealed by past trades depends upon subjects’ strategies, which are not known ex ante. I follow the previous literature (Cipriani and Guarino (2005) and Drehmann, Oechssler, and Roider (2005)) by having the market maker assume subjects are following equilibrium strategies. After a trade, whether in a period on or off the equilibrium path, the price is updated assuming subjects follow the trading strategies of Proposition 1 (which are sequentially rational off-equilibrium). In the absence of a trade (a wait decision), consistent with the off-equilibrium beliefs specified in equilibrium construction (B1), and the fact that no information is revealed by such a decision it turns out, of course, that not all subjects follow equilibrium strategies. In Section 3.4.2 and Appendix B, I discuss the consequences of traders understanding that other traders may not be making equilibrium trade and timing decisions.

<table>
<thead>
<tr>
<th>Treatment Name</th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$q_P$</th>
<th>Subjects</th>
<th>Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rush (R)</td>
<td>$\frac{3}{2}$</td>
<td>1</td>
<td>$\frac{17}{24}$</td>
<td>$n = 8$</td>
<td>$T = 8$</td>
</tr>
<tr>
<td>Wait (W)</td>
<td>$\frac{3}{4}$</td>
<td>1</td>
<td>$\frac{17}{24}$</td>
<td>$n = 8$</td>
<td>$T = 8$</td>
</tr>
</tbody>
</table>
on the equilibrium path, the price is unchanged. Subjects were told that prices reflect the mathematical expected value of the value of the asset, conditional on all public information. They were also explicitly told that prices increase after buy decisions and favorable public signals, and conversely for sell decisions and unfavorable public signals. The exact sizes of the price changes (per Bayes’ rule) were not communicated ex ante, but subjects participated in many trials and observed many price movements, facilitating learning over time.

3.2 Implementation Details

I recruited subjects from the University of British Columbia student population using the experimental recruitment package Orsee (Greiner (2004)). Subjects came from a variety of majors and no subject participated in more than one session. I conducted four sessions of each treatment, for a total sample of 64 subjects \( n = 8 \) subjects in each session), with new randomizations for each session’s asset values and signals (in order to avoid the possibility of a particular set of draws influencing the results). In each session, subjects first signed consent forms and then received the instructions verbally. Subjects were encouraged to ask questions while the instructions were read and then completed a short quiz. All quiz questions had to be answered correctly by each subject before the experiment began, and this policy was common knowledge. Once the experiment began, no communication of any kind between subjects was permitted.\(^{12}\)

In each session, subjects participated in 30 paid trials preceded by two practice trials. It was emphasized that they could only trade once, and that they must trade in some period (in each trial). The asset value, \( V \), and prices were scaled by a factor of 100 currency units, and subjects were endowed with 100 units with which to trade prior to each trial, for a maximum possible earning of 200 currency units per trial. In order to induce risk-neutrality (Roth and Malouf (1979)), each currency unit represented a lottery ticket with a \( 1/200 \) chance to earn $1.00 Canadian. After all trials were completed, a computerized lottery was conducted for each paid trial and subjects were paid according to the results of the lotteries. Average earnings (including a $5.00 show-up fee) were $21.53 (minimum $12.00, maximum $30.00) with a corresponding wage rate over an hour and a half of $14.35/hour.

3.3 Hypotheses

The main question the experiment is designed to answer is whether equilibrium theory predicts when early trades and market rushes, and their corresponding informational losses, \(^{12}\)Subjects were separated by physical barriers so that they could not observe each others’ private information or decisions.
occur. To take the theoretical predictions of the model to the data, I decompose them into two nested hypotheses, one a weak test and the other stronger. My first hypothesis derives from a simple comparative static prediction of the theory: trade times should be earlier in treatment R than W.

**Hypothesis 1 (Comparative Static)** *Trades occur earlier in treatment R than in treatment W.*

My second hypothesis derives from the point predictions of the model for the two treatments and represents a more strenuous test of the theory: the R treatment should generate a full market rush while the W treatment should induce no early trades at all.

**Hypothesis 2 (Point Predictions)** *Trades occur at $t = 1$ in treatment R and at $t = 8$ in treatment W.*

### 3.4 Results

In Section 3.4.1, I begin by comparing the timing decisions across treatments, showing that subjects trade earlier in the R than the W treatment in accordance with Hypothesis 1. In contrast, the stronger point predictions of Hypothesis 2 are only partially supported: while full market rushes occur in treatment R as hypothesized, frequent early trades also occur in treatment W. I then show that early trades result in informational inefficiencies in both treatments. In Section 3.4.2, I provide evidence that the unexpected early trades in the W treatment result from subjects employing a simple heuristic often attributed to momentum traders. In Section 3.4.3, I show that simple learning rules can explain why this heuristic behavior converges to the theoretical predictions in R, but not in W. Finally, in Section 3.4.4, I show that the momentum-like heuristic subjects use in the experiment produces positive short-term correlations in returns, a phenomenon commonly observed in naturally-occurring financial markets.

When reporting results, I report ‘Initial’ behavior using data from the first five trials and ‘Late’ behavior using data from the last five trials. The qualitative nature of the results is unchanged if I instead use the first and last ten trials.

#### 3.4.1 Trade Timing and Informational Inefficiencies

Figure 2 plots the empirical cumulative distribution functions (CDFs) of the times at which trades occur across all subjects for each treatment. In each panel, I include the Late behavior
of experienced subjects (plotted as solid lines) and, for reference, Initial behavior (as dotted lines).

Figure 2: Trade Timing

Note: Cumulative fraction of trades that occur prior to each period for treatments R (left) and W (right). Solid lines are for the last five trials (Late) and dashed lines for the first five (Initial).

I begin by evaluating Hypothesis 1, focusing on the Late behavior of experienced subjects. Although subjects initially trade at similar times across the two treatments (Initial CDFs), with experience (Late CDFs) they learn to make very different timing decisions, as predicted: in treatment R subjects overwhelmingly learn to trade at the very beginning of the game (with 80% trading in period 1), while in treatment W subjects continue to delay trade significantly even after substantial experience. A Kolmogorov-Smirnov test allows me to reject the null of equal empirical CDFs in the Late data (p-value = 0.01), supporting Hypothesis 1 and providing a first finding:13

Finding 1 Subjects trade earlier in treatment R than in treatment W, supporting Hypothesis 1.

Although Figure 2 shows strong evidence in support of Hypothesis 1, it reveals decidedly mixed evidence in support of Hypothesis 2, even in the last 5 trials of the experiment. In agreement with Hypothesis 1, subjects overwhelmingly (80% of the time) rush to trade in period 1 in Late trials of all four sessions of the R treatment (the median trading period is 1 in every session in Late trials). By contrast, subjects in the W treatment rarely (15% of the time) delay trade until the final period, instead trading at heterogeneous times throughout the game. Thus, early trades occur reliably when predicted by theory, but also occur (though

13Alternatively, the median trading periods in the four sessions of R and W are {1, 1, 1, 1} and {2, 3, 4, 4}, respectively. Applying a non-parametric Mann-Whitney U test rejects the null of equal median trading periods (test statistic = 0, p-value = 0.03).
to a less extreme extent) when not predicted by equilibrium theory. I report this partial failure of Hypothesis 2 as a second finding:

Finding 2: Subjects generally trade in period 1 in treatment R, but they rarely delay trade until period 8 in treatment W. This pattern of behavior only partially supports Hypothesis 2.

Early trades generate both predicted and unpredicted informational inefficiencies in the experimental markets. In the R treatment, when a trader trades in the first period as predicted, she trades with information that is correct only 75% of the time, forgoing the opportunity to obtain perfect information. In fact, no subject in the last five trials (160 observations) ever waits to learn the asset value at \( t = 8 \), which generates informational inefficiencies as high as 38% (computed as the absolute difference between the final price in the experiment and the true asset value, \( |V - p_T| \)). In treatment W, where no inefficiencies are predicted to emerge, they are in fact even higher, as high as 93%. These informational inefficiencies are particularly surprising in the W treatment because, should any one of the eight traders wait to obtain perfect information (as they are predicted to do), prices would be fully efficient. I report these inefficiencies as a further result:

Finding 3: Early trades generate informational inefficiencies in both the R treatment (where such inefficiencies are predicted) and the W treatment (where they are not).

3.4.2 Momentum-like Strategies

What decision rules account for the early trades observed in treatment W? One hypothesis is that subjects simply make their entry time decisions randomly: after all, both Initial and Late behaviors in the W treatment (and even Initial behavior in the R treatment) are nearly uniformly distributed in Figure 2. An alternative possibility is that subjects do not condition their trading decision on time per se (as the theory predicts), but rather based on the strength of the information revealed by past price trends.

To investigate, I define the strength of a subject’s information in period \( t \) as: \( \text{strength}_t = \max \{1 - Pr[V = 1|H_t, z_i], Pr[V = 1|H_t, z_i]\} \). Intuitively, a subject has stronger information the closer her posterior is to 0 or 1. In Figure 3, I plot in black the fraction of times a subject trades in a period in which her information strength is more extreme than in any previous period of the trial (i.e. in which the price reaches a level more extreme than in any previous period). For comparison, I also plot in gray the fraction of trades that occur at an
Figure 3: Trades by Information Strength

Note: Each bar graph is the fraction of trades that occur when a subject’s information strength reaches a new threshold (black bars) and does not (gray bars).

information strength weaker than (or equal to) that previously attained. The plot shows that trades almost always occur at a new threshold level of information strength: the black bars are much larger than the gray bars for each subset of the data. This pattern is clearly inconsistent with uniformly random decision-making: in all four cases, if subjects had made decisions in a uniformly random period (given the actual, realized signals), at least 98% of the time we would expect to observe a lower fraction of trades at new threshold levels of information strength than is observed in the data.

These results are consistent with the theory that subjects employ a threshold rule to determine timing, trading at the first moment that their information provides a sufficiently high level of confidence in the asset value (a threshold that, naturally, differs from subject to subject). One source of such rules are the momentum strategies commonly observed among traders in financial markets and discussed in the behavioral finance literature (Grinblatt, Titman, and Wermers (1995), Hong and Stein (1999), Grinblatt and Keloharju (2000), Brozynski et al. (2003), Baltzer, Jank, and Smajlwegovic (2015), and Grinblatt et al. (2016)).

14I omit results from period $T$ because subjects are forced to trade in this period and their information strength is perfect at this time.

15I performed this calculation using simulations of 10,000 repetitions of the first and last 5 trials of the game using the actual public and private signal draws, and assuming trades are uniformly distributed across periods. The percentage of simulations with a lower fraction of trades at new threshold levels of information strength are 98% and 100% in Initial trials of the W and R treatments, respectively, and 99% and 100% in Late trials.
A trader employing a classical momentum strategy, buys (sells) after observing a sequence of price increases (decreases), a behavior that requires setting a threshold change in prices before trading. With private information, the natural extension to this strategy is to trade once the strength of the totality of one’s information has changed by a sufficient amount (in Section 4, I discuss this connection further), and to trade in the direction suggested by the information (buy when information suggests $V = 1$ and sell otherwise). In Late behavior, 80% and 96% of trades are in the direction of a subject’s information (conditional on trading at a new threshold in information strength), in the W and R treatments, respectively. This evidence of ‘momentum-like’ strategies provides a fourth finding:

**Finding 4:** Subject behavior is consistent with momentum-like strategies in which they wait for sufficient changes in information and then trade in the direction suggested by the information.

Given the widespread use of momentum-like strategies, a natural question is whether or not such strategies form a best response to other subjects using these strategies. Theoretically, the answer is ‘no’: if other subjects use momentum-like strategies, then prices are more extreme than the expected value of the asset conditional on the actual signal realizations (for example, buy decisions that increase prices may actually reflect unfavorable signals). If this price discrepancy is sufficiently large, it may become optimal to trade early, but rather than trading with the price trend, a subject should optimally trade against it (see Appendix B for further discussion). Empirically, given the actual prices generated by subjects’ trades, waiting continues to be a best response in the W treatment: over all trials, the average profit generated by early trades is actually slightly negative ($-0.6\%$), while the average profit generated by trades in the final period is $15.3\%$, very close to the theoretical expected profit of $15.9\%$.

### 3.4.3 Learning

As Figure 2 shows, the distributions of trade times in the W and R treatments are very similar initially (Initial trials) ($p = 0.93$, a Kolmogorov-Smirnov test), but become dramatically different across treatments as subjects gain experience (Late trials). Why do subjects learn to behave so differently in the two treatments, converging to the theoretical predictions in R but not in W? In this section I show that simple hill-climbing rules generate patterns very

\[16\] Threshold strategies are also often employed in drift diffusion models (Ratcliff (1978)) in which one makes a decision once having accumulated a threshold level of information. Here, and perhaps more broadly, one could interpret momentum strategies as resulting from this deeper, psychological information-processing mechanism.
similar to the ones in the data, and discuss the intuition that this provides about the source of the large treatment effects we observe in the data.

I consider a simple learning model in which agents adjust behavior in response to the counterfactual earnings consequences of their recent past actions (see, for example the win-stay-lose-shift model of Nowak and Sigmund (1993) or learning direction theory of Selten and Buchta (1998)). Agents set a threshold for the strength of information that they require to trade (employing a momentum-like strategy as documented in the previous section), and adjust it from trial to trial towards thresholds that would have performed better in the previous trial. Intuitively, if a subject buys a good asset \((V = 1)\) when she uses a particular threshold, she could have done better by setting a lower threshold and buying the asset sooner. On the other hand, if she buys a bad asset \((V = 0)\), she would have typically done better by waiting for more information by setting a higher threshold. To simulate this learning process, I assume agents adjust over a small number of discrete thresholds (I use thresholds corresponding to the net number of favorable public signals). An agent updates her threshold by considering what would have happened had she used a different threshold in the previous trial, and stochastically choosing among her current threshold and any threshold that would have earned strictly more. To limit the model’s degrees of freedom, I simulate using the simplest possible rule: agents uniformly randomize over the candidate thresholds.

Figure 4 plots CDFs of the Late (final five trials) trading times from 500 simulations of 30 trials using the actual parameters from each treatment of the experiment. For reference, I also reproduce the actual CDFs from the experiment. Although the simulations are seeded with virtually identical initial first trial behaviors and identical adjustment rules, they result in comparative statics (across treatments) and distributions (within treatments) that look almost identical to the actual data. Thus, this simple learning model provides my fifth finding:

**Finding 5:** A simple hill-climbing threshold adjustment rule generates the observed difference across treatments.

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18 After buying a bad asset, a lower threshold would also have done better because it results in a smaller loss. Other possibilities also exist, but occur infrequently. For example, even after a correct decision, there is a small probability that the price is lower in period \(T\) such that a higher threshold that induces waiting until period \(T\) does better.

19 Many other variations of the model produce similar results. For example, probabilistic weights can be assigned to thresholds based on how much they would have earned. Or, one can allow for noise, placing some probability mass on thresholds that would have earned less than the current threshold. Rather than fine-tuning the results by adding degrees of freedom, I chose the most parsimonious model.
Figure 4: Simulated Trade Times

Note: Cumulative fraction of trades that occur prior to each period for treatment R (left) and treatment W (right). In each graph, the trading periods from the last five trials of the data are compared to those generated by the learning model.

Guided by the intuition from this simple learning model, we can ask what makes learning to wait in the W treatment so difficult? The reason lies in the feedback of the rewards. If a subject follows a momentum strategy, more often than not she earns a positive amount, because prices tend to move in the direction of the true asset value. Thus, frequent positive rewards tend to reinforce trading with the same threshold or perhaps lowering one’s threshold to try to earn more. It is only in the less frequent case that prices initially move in the wrong direction that a subject learns that it would have been better to wait to learn the asset value. The relative infrequency of losses that encourage waiting thus makes it difficult to recognize that a momentum strategy is not optimal, even after participating in many trials. Learning to rush in the R treatment is much easier because trading immediately leads to positive earnings very frequently (75% of the time). Furthermore, waiting even one period results in a much smaller profit due to the large amount of information aggregated by others’ trades, which provides immediate feedback that trading earlier is optimal.20

3.4.4 Correlations in Returns

One consequence of the momentum-like strategies documented above is the emergence in the data of another pattern often observed in real-world financial markets: positive short-term correlations in returns. Indeed, these correlations are among the most commonly studied

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20The reason subjects don’t learn to trade immediately in the W treatment is because of the differences in private signal quality across treatments. Waiting is not nearly so costly in the W treatment because others’ trades reveal very little information.
phenomena in financial markets (see Daniel, Hirshleifer, and Subrahmanyam (1998) for a review), and are often given a behavioral explanation, including that traders use a price-chasing momentum strategy (Hong and Stein (1999)). Documenting these correlations in the experimental data therefore further reveals the role that these strategies play.

I focus on Late behavior in the W treatment, and, in particular, on the second trading period just after the first public signal is revealed. At this time, 86% of trades are in the direction of the first public signal. Table 2 provides the Spearman correlation coefficients between the return due to the first public signal and that due to trades in each of the next seven trading periods, excluding the last where the asset value is known.

<table>
<thead>
<tr>
<th>Trading Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman Correlation Coefficient</td>
<td>0.05</td>
<td>0.85***</td>
<td>0.13</td>
<td>0.09</td>
<td>-0.27</td>
<td>0.09</td>
<td>0.57***</td>
</tr>
<tr>
<td>p-value</td>
<td>[0.83]</td>
<td>[0.00]</td>
<td>[0.59]</td>
<td>[0.72]</td>
<td>[0.25]</td>
<td>[0.70]</td>
<td>[0.01]</td>
</tr>
</tbody>
</table>

Notes: Correlations between the return due to the first public signal and those due to trades in each period. p-value of two-tailed t-test included in brackets. Each correlation is over 20 trials (last 5 trials of each of 4 sessions).

In Table 2, we observe a very strong positive short-term correlation between the return due to the first public signal and that due to the trades in the following trading period (period 2). Were traders simply trading early and according to their signals, prices would form a martingale and such a correlation would be statistically improbable. This correlation therefore provides further evidence that subjects chase price trends on average. Finding 6 summarizes the correlation result.

21In an environment with private information, chasing price trends leads to informational herding: subjects trade in the direction of the price trend independently of their private signal. Herding, broadly defined, has also been considered an explanation for correlations in returns (see Devenow and Welch (1996) and Hirshleifer and Teoh (2003) for reviews of herding in the context of financial markets). Whether one considers momentum-like strategies and herding to be distinct explanations for the behavior observed here depends upon one’s definition of herding. Rational informational herding of the sort first discussed by Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992) is precluded because subjects should rationally trade according to their private signals (Proposition 1). Herding in the sense of simple imitation could perhaps provide an alternative explanation, but, in a related setting, Kendall (2018b) provides evidence that subjects follow momentum-like strategies even when there are no previous subjects to imitate. Finally, note that informational herding only partially explains the correlations in returns in the data. The remaining part is driven by subjects whose signals oppose the price trend choosing to wait more often that those whose signal confirms the price trend, consistent with the first group being less certain about the asset value.

22Because the vast majority of trades occur in the first trading period in treatment R, any correlation due to price-chasing behavior is drastically weakened. The second period of treatment W provides the cleanest test for correlations because correlations due to the use of higher thresholds are spread across multiple trading periods.

23The results are very similar if the Pearson correlation coefficient is used.

4 Conclusion

Pathologies in financial markets, such as panics and frenzies, are of perennial concern to both social scientists and policy makers. I theoretically show that these types of episodes can arise as equilibrium outcomes by rational traders in a variation of a standard, workhorse model of trade timing (Glosten and Milgrom (1985)), leading to informationally inefficient market prices. I then report a theoretically-structured experiment designed to study whether market rushes can also arise due to less rational motives (in settings where they are not rational), as market observers have long suggested (Mackay (1841)). In the laboratory experiment, I study a parameterization (Rush) in which market rushes are the only equilibrium outcome and another (Wait) in which market rushes are never an equilibrium outcome. As the theory predicts, subjects trade much earlier in the Rush treatment than they do in the Wait treatment, generating market rushes consistent with the finding of Dufour and Engle (2000) that informed trades tend to cluster together, and the fact that volumes spike around earnings announcements (Frazzini and Lamont (2006)). However, in contrast to the theoretical predictions, I observe significant trades prior to full information, and their accompanying informational efficiencies, even in the Wait treatment where it cannot be a consequence of individually rational behavior.

The experimental data reveals that excessive early trading is caused by the widespread use of momentum-like strategies by subjects: instead of choosing when to trade based upon forward-looking concerns about future information flows, subjects instead trade after their accumulated, past information makes them sufficiently certain of the asset value. This suboptimal strategy is remarkably resistant to learning, persisting even after significant experience. Subjects do, however, adjust the level of the information threshold they require for trade, and by following simple hill-climbing rules, eventually learn to make significantly different timing decisions across treatments. In both treatments, momentum-like strategies lead most subjects to trade before becoming fully informed, suggesting that such strategies may exacerbate panics and frenzies, and their informational consequences.

Momentum strategies have been documented in financial markets both for mutual funds

\textsuperscript{24}For the latter, the assumption is that heterogeneous abilities to process information turn public announcements into private information (Holthausen and Verrecchia (1990), Indjejikian (1990), and Kandel and Pearson (1995)).
(Grinblatt, Titman, and Wermers (1995), Brozynski et al. (2003), Baltzer, Jank, and Smajlbegovic (2015), and Grinblatt et al. (2016)) and individual investors (Grinblatt and Kehlharju (2000)). (Indeed, momentum strategies cause short-run correlations in returns in the experimental data, just as has been suggested for real markets (De Long et al. (1990) and Hong and Stein (1999)).) One potential implication of the experimental results is that momentum strategies may not (only) arise from the use of technical analysis, but may also derive from a deeper behavioral source - after all, unlike practitioners in the field, few of my undergraduate subjects are likely to be formally employing technical analysis when making trading decisions.

One potential such ‘behavioral’ source of these strategies is systematic mistakes in the formation of beliefs (as in Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998)). For instance, subjects may be overextrapolating from recent trends (Jin (2015), Barberis, Greenwood, Jin, and Shleifer (2015,2016)), for which survey evidence exists (Greenwood and Shleifer (2014)). Here, such overextrapolation would cause a trader to form too-certain beliefs about the asset value, inducing trade in the direction of accumulated information, in line with the data. Another possible source of momentum-like strategies is non-expected utility preferences. Kendall (2018b) considers a simpler (Glosten-Milgrom) environment in which traders have private information but trade timing is exogenous and sequential. There, I show theoretically that prospect theory preferences (Kahneman and Tversky (1979,1992)) cause traders to buy when prices rise and sell when prices fall, and subsequently confirm this hypothesis in an experiment designed to rule out belief-based explanations. Other common explanations for departures from standard predictions are unlikely to be the source of momentum strategies. In Appendix B, I discuss why risk aversion and popular behavioral game-theoretic models including quantal response equilibrium (McKelvey and Palfrey (1995,1998)), level k reasoning (Stahl and Wilson (1994) and Nagel (1995)), cognitive hierarchy (Camerer, Ho, and Chong (2004)), and cursed equilibrium (Eyster and Rabin (2005)) are inconsistent with the results reported here.

One promising avenue for future work is to study richer environments to better understand the consequences of momentum-like strategies. In order to clearly identify rushing behavior and the strategies generating it, the model and the experiment reported in this paper restrict traders to a single trade. This constraint could be relaxed to examine whether relatively sophisticated subjects exploit momentum strategies to their advantage. For example, in settings in which multiple trades are possible, traders have speculative motives to

25These results strongly suggest that prospect theory preferences may explain the momentum-like strategies observed here, but analyzing equilibrium outcomes under prospect theory preferences in this more complex environment is challenging.
trade in addition to informational motives. In particular, rational traders may buy in an attempt to induce purchases by those that follow momentum strategies, which, if successful, would allow them to sell at a higher price. In this way, the strategies observed here may help to explain price bubbles, both in the field and in laboratory experiments (e.g. Smith, Suchanek, and Williams (1988)).

References


Appendix

A: Theoretical Analysis

A1. Preliminary Results

I denote the equilibrium probability that a trader of type \( s_i = x \) for \( x \in \{0,1\} \) waits in the current period, \( \hat{\beta}_x \). Other traders and the market maker have common beliefs about these strategies denoted \( \beta_x \). Assume that the timing strategies of the two types of traders (and hence beliefs about these strategies) do not differ in any period other than perhaps the current one (Lemma A2 shows that this assumption holds in any equilibrium). Then, for arbitrary beliefs, Lemma A1 provides the optimal trading strategies in the current period (a period prior to \( T \)) and in any future state on or off of the equilibrium path. \( \hat{q} \) is used to denote the quality of private information received in the future period. When no such information is received, we can model it as a hypothetical signal, \( \bar{s}_i \), with quality \( \frac{1}{2} \) so that \( \hat{q} = \bar{q} \) in period \( T \) and \( \hat{q} = \frac{1}{2} \) otherwise.

Lemma A1: If the market maker’s beliefs about a trader’s timing strategy in the current period are given by \( \beta_0 \) and \( \beta_1 \), then she trades as follows:

1. if she trades in the current period (< \( T \)), she buys if \( \hat{s}_i = 1 \) and sells if \( \hat{s}_i = 0 \).
2. if she trades in a future period, she:
   (a) buys if \( \hat{s}_i = 1 \) and \( \bar{s}_i = 1 \); sells if \( \hat{s}_i = 0 \) and \( \bar{s}_i = 0 \)
   (b) buys (sells) if \( \hat{s}_i = 0 \), \( \bar{s}_i = 1 \), and \( g_0(q, \hat{q}) \equiv (1 - q)\hat{q} \hat{NT}_0 - q(1 - \hat{q})\hat{NT}_1 \geq (<) 0 \)
   (c) buys (sells) if \( \hat{s}_i = 1 \), \( \bar{s}_i = 0 \), and \( g_1(q, \hat{q}) \equiv q(1 - \hat{q})\hat{NT}_0 - (1 - q)\hat{q}\hat{NT}_1 \geq (<) 0 \)

where \( \hat{NT}_0 = (1 - q)\beta_1 + q\beta_0 \), \( \hat{NT}_1 = q\beta_1 + (1 - q)\beta_0 \) are shorthand for the probabilities of observing no trade in the current period conditional on \( V = 0 \) and \( V = 1 \), respectively.

Proof of Lemma A1:

The proof when a trader trades prior to \( T \) is similar to that when she trades at \( T \), only simpler, so I provide the latter. Let \( \hat{e} \in E \) denote the set of realizations of public events that may occur during the time a trader waits (due to public signals or trades by other traders) other than from trader \( i \) herself not trading (denoted \( NT \)), and denote \( Pr(\hat{e}|V = y) \) as \( \hat{e}_y \), for \( y \in \{0,1\} \). A trader who waits until \( T \) buys if the expected value of the asset conditional
on her information at that time exceeds the price at that time. Denoting the current history, $H_t$, the current price $p_t$, and the future price, $Pr(V = 1|\hat{e}, NT, H_t)$, a trader buys if

$$Pr(V = 1|s_i, \overline{s}_i, \hat{e}, H_t) > Pr(V = 1|\hat{e}, NT, H_t)$$

$$\iff \frac{p_tPr(s_i|V = 1)Pr(\overline{s}_i|V = 1)\hat{e}_1}{p_tPr(s_i|V = 1)Pr(\overline{s}_i|V = 1)\hat{e}_1 + (1 - p_t)Pr(s_i|V = 0)Pr(\overline{s}_i|V = 0)\hat{e}_0} > \frac{p_t\hat{e}_1Pr(NT|V = 1)}{p_t\hat{e}_1Pr(NT|V = 1) + (1 - p_t)\hat{e}_0Pr(NT|V = 0)}$$

$$\iff Pr(s_i|V = 1)Pr(\overline{s}_i|V = 1)NT_0 > Pr(s_i|V = 0)Pr(\overline{s}_i|V = 0)NT_1$$

(1)

where the first equivalence follows from applying Bayes’ rule to each side of the inequality and using the fact that the public belief is the initial price, $Pr(V = 1|H_t) = p_t$. Using (1), a trader with $s_i = 1$ and $\overline{s}_i = 1$ buys if

$$q\hat{q}NT_0 > (1 - q)(1 - \hat{q})NT_1$$

$$\iff q(1 - q)(2\hat{q} - 1)\beta_1 + (\hat{q}^2\hat{q} - (1 - \hat{q})^2(1 - \hat{q})) \beta_1 > 0$$

which is true for all parameterizations. Similarly, a trader with $s_i = 0$ and $\overline{s}_i = 0$ always sells. Finally, the conditions in Lemma A1 under which traders with $s_i = 1$ and $\overline{s}_i = 0$, or $s_i = 0$ and $\overline{s}_i = 1$, buy or sell are easily obtained by substituting the appropriate values for the probabilities in (1). □

Given Lemma A1, I derive a general function for the benefit from waiting (the expected profit from following some future plan of action, less the profit from trading in the current period). When this benefit is positive (negative), a trader optimally waits (trades immediately). The benefit depends upon the market maker’s beliefs about the probability that each type of trader waits in the current period ($\beta_0$ and $\beta_1$) because information is revealed if these probabilities differ across types. In equilibrium, these probabilities (and the corresponding beliefs) must be consistent with the sign of the benefit for the corresponding trader. I denote the benefit from waiting, $B_x(p_t, \beta_0, \beta_1)$, for traders with $s_i = x$.

Consider first the expected profit of a trader who trades in the current period, $t$, with $s_i = 1$. From Lemma A1, she buys so that her profit is given by

$$Pr(V = 1|s_i = 1, H_t) - p_t \iff \frac{p_tq}{p_tq + (1 - p_t)(1 - q)} - p_t \iff \frac{p_t(1 - p_t)(2q - 1)}{p_tq + (1 - p_t)(1 - q)}$$

The profit for a trader with $s_i = 0$ is calculated similarly, using the fact that she sells. The current profit for both types of traders can be written $\frac{p_t(1 - p_t)(2\hat{q} - 1)}{Pr(s_i)}$.

I calculate the expected profit from a planned set of future trades for a trader with $s_i = 1$ who buys when receiving $\overline{s}_i = 1$ and sells when $\overline{s}_i = 0$ ($g_1(q, \hat{q}) \leq 0$). The other cases are
calculated similarly and are omitted for brevity. The expected profit is

\[
\sum_{\hat{e} \in E} \{ Pr(\bar{s}_i = 1, \hat{e} | \bar{s}_i = 1) (Pr(V = 1 | \bar{s}_i = 1, \bar{s}_i = 1, \hat{e}) - Pr(V = 1 | NT, \hat{e})) + Pr(\bar{s}_i = 0, \hat{e} | \bar{s}_i = 1) (Pr(V = 1 | NT, \hat{e}) - Pr(V = 1 | \bar{s}_i = 1, \bar{s}_i = 0, \hat{e})) \} \quad (2)
\]

where all probabilities are also conditional on \( H_t \). The first term corresponds to the profit from buying the asset after receiving \( \bar{s}_i = 1 \) and the second term from selling after receiving \( \bar{s}_i = 0 \). The sum is over all possible realizations of the public events, which may depend upon the trader’s future course of action.\(^{26}\) Using Bayes’ rule and the independence of signals, (2) becomes

\[
\sum_{\hat{e} \in E} \left\{ \frac{Pr(\bar{s}_i = 1, \hat{e} = 1)}{Pr(\bar{s}_i = 1)} \left( \frac{p_t q \hat{q} Pr(\hat{e} | V = 1)}{Pr(\bar{s}_i = 1, \hat{e} = 1)} - \frac{p_t NT_1 Pr(\hat{e} | V = 1)}{Pr(NT, \hat{e})} \right) + \frac{Pr(\bar{s}_i = 0, \hat{e} = 1, \hat{a})}{Pr(\bar{s}_i = 1)} \left( \frac{p_t NT_1 Pr(\hat{e} | V = 1)}{Pr(NT, \hat{e})} - \frac{p_t (1 - \hat{q}) Pr(\hat{e} | V = 1)}{Pr(\bar{s}_i = 0, \hat{e} = 1, \hat{e})} \right) \right\}
\]

with \( NT_0 \) and \( NT_1 \) as in Lemma A1. Simple algebra results in

\[
\frac{p_t (1 - p_t)}{Pr(\bar{s}_i = 1)} \sum_{\hat{e} \in E} \frac{\hat{e}_0 \hat{e}_1}{Pr(NT, \hat{e})} \left( (2\hat{q} - 1)(qNT_0 + (1 - \hat{q})NT_1) \right)
\]

Finally, subtracting the profit at \( t \) from the expected profit and factoring out common terms gives the benefit. For each type of trader and every possible strategy, the benefit can be written

\[
B_x(p_t, \beta_0, \beta_1) = \frac{p_t (1 - p_t)}{Pr(\bar{s}_i = x)} \left[ \sum_{\hat{e} \in E} \frac{\hat{e}_0 \hat{e}_1 f(q, \hat{q}, \beta_0, \beta_1)}{Pr(\hat{e}, NT)} - (2\hat{q} - 1) \right] \quad (3)
\]

where

\[
f(q, \hat{q}, \beta_0, \beta_1) = \begin{cases} 
(2\hat{q} - 1)(qNT_0 + (1 - \hat{q})NT_1) & \text{if } \hat{s}_i = 1, g_1(q, \hat{q}) \leq 0 \\
qNT_0 - (1 - q)NT_1 & \text{if } \hat{s}_i = 1, g_1(q, \hat{q}) > 0 \\
qNT_1 - (1 - q)NT_0 & \text{if } \hat{s}_i = 0, g_0(q, \hat{q}) < 0 \\
(2\hat{q} - 1)(qNT_1 + (1 - \hat{q})NT_0) & \text{if } \hat{s}_i = 0, g_0(q, \hat{q}) \geq 0
\end{cases}
\]

Note that \( \hat{q} \) in (3) depends upon the state reached (and potentially, therefore, on the realization, \( \hat{e} \), that occurs), but I suppress this dependence in the notation for simplicity. Given the generic formula for the benefit, Lemma A2 establishes that both types of traders must follow the same timing strategy at every state in any equilibrium. Importantly, this result holds without any assumption on off-equilibrium beliefs.

\(^{26}\)For example, the trader could plan to trade after a favorable public signal, but wait one more period after an unfavorable public signal. In this case, the set of public events due to the public signals is \( E = \{ s_p, t = 1, s_p, t = 0, s_{p, t+1} = 0 \}, \{ s_p, t = 0, s_{p, t+1} = 1 \} \)
Lemma A2: In any equilibrium, traders with $s_t = 0$ and $s_t = 1$ must follow the same timing strategy at every history ($\hat{\beta}_0 = \hat{\beta}_1$ for all $t$ and $H_t$).

Proof of Lemma A2:

The proof uses backwards induction. In particular, I show that the claim holds in period $T - 1$ and then in an arbitrary period, provided that it holds for all subsequent periods.

For a given period, the proof is by contradiction. Assume that there exists an equilibrium in which the two types of traders wait with different probabilities in period $T - 1$. Let $\beta_t$ be the probability of a type that waits when $s_t = 0$, and $\hat{\beta}_t$ be the probability of a type that waits when $s_t = 1$. Then

$$\beta_t \neq \hat{\beta}_t$$

I must prove the proof for $\beta_t > \hat{\beta}_t$. The proof for the opposite case is identical. For beliefs to be consistent with strategies, we must then have $\beta_t > \hat{\beta}_t$. Consider period $T - 1$. There, a timing strategy simply consists of $\beta_t$ and $\hat{\beta}_t$ because $T$ is the final trading period. With $\beta_t > \beta_0$, we must have $\beta_t \in (0, 1]$ and therefore $B_1(p_t, \beta_0, \beta_1) \geq 0$ such that the type 1 trader waits with positive probability consistent with this belief. I establish below that $\beta_t > \beta_0$ and $B_1(p_t, \beta_0, \beta_1) \geq 0$ together imply $B_0(p_t, \beta_0, \beta_1) > 0$ (i.e. if a trader with a favorable signal weakly benefits from waiting when she is believed to wait more often, then the trader with an unfavorable signal strictly benefits). However, $\beta_t > \beta_0$ also implies $\beta_0 \in [0, 1)$ and therefore $B_0(p_t, \beta_0, \beta_1) \leq 0$, a contradiction. Therefore, we must have $\beta_t = \hat{\beta}_t$ in any equilibrium in period $T - 1$.

Now, consider period $t$ and assume that in all future periods, the timing strategies of the two types of traders are the same. Then, the timing strategies in the current period pin down beliefs about the trader’s type (and therefore prices) on all future paths, so that the benefit functions of the two types are determined by (3). But, in this case, the above argument directly applies again.

To see $\beta_t > \beta_0$ and $B_1(p_t, \beta_0, \beta_1) \geq 0$ together imply $B_0(p_t, \beta_0, \beta_1) > 0$, note from (3) that the sign of $B_2(p_t, \beta_0, \beta_1)$ is determined by the term in square brackets and that the only difference across types is due to differences in the functions, $f(q, \hat{q}, \beta_0, \beta_1)$ which depend upon whether or not new private information is received. I show that, in each case, $f(q, \hat{q}, \beta_0, \beta_1)$ is strictly greater for type 0 than for type 1 whenever $\beta_t > \beta_0$.

First, consider an information set in which no new information is received ($\hat{q} = \frac{1}{2}$). In this case, $g_0(q, \hat{q}) < 0$ and $g_1(q, \hat{q}) > 0$. Comparing $f(q, \hat{q}, \beta_0, \beta_1)$ in $B_0(p_t, \beta_0, \beta_1)$ relative to $B_1(p_t, \beta_0, \beta_1)$ gives $qNT_1 - (1 - q)NT_0 > qNT_0 + (1 - q)NT_1 \iff NT_1 - NT_0 > 0 \iff (\beta_t - \hat{\beta}_0)(2q - 1) > 0$ so $f(q, \hat{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$.

Now consider an information set in which new information is received ($\hat{q} = \overline{q}$). There are four possible cases depending upon the optimal strategies of buying and selling from Lemma A1.

1. $g_1(q, \overline{q}) \leq 0$ and $g_0(q, \overline{q}) \geq 0$. The comparison of $f(q, \overline{q}, \beta_0, \beta_1)$ in $B_0(p_t, \beta_0, \beta_1)$ relative to $B_1(p_t, \beta_0, \beta_1)$ is $qNT_1 + (1 - q)NT_0 > qNT_0 + (1 - q)NT_1 \iff (NT_1 - NT_0)(2\overline{q} - 1) > 0 \iff (\beta_t - \hat{\beta}_0)(2\overline{q} - 1)^2 > 0$ so $f(q, \overline{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$.

2. $g_1(q, \overline{q}) > 0$ and $g_0(q, \overline{q}) < 0$. This case is identical to the $\hat{q} = \frac{1}{2}$ case.

3. $g_1(q, \overline{q}) \leq 0$ and $g_0(q, \overline{q}) < 0$. Note that $g_0(q, \overline{q}) < 0 \iff qNT_1 - (1 - q)NT_0 > (2\overline{q} - 1)(qNT_1 + (1 - q)NT_0)$. This can be seen algebraically or simply by noting that $B_0(p_t, \beta_0, \beta_1)$ must be larger when $t$ follows her optimal trading strategy instead of the
optimal strategy for the other case, \( g_0(q, \bar{q}) \geq 0 \). But, then we have, 
\[
qNT_1 - (1 - q)NT_0 > (2\bar{q} - 1)(qNT_1 + (1 - q)NT_0) > (2\bar{q} - 1)(qNT_0 + (1 - q)NT_1)
\]
where the second inequality was shown in the first case above, so \( f(q, \bar{q}, \beta_0, \beta_1) \) is strictly greater in \( B_0(p_t, \beta_0, \beta_1) \).

4. \( g_1(q, \bar{q}) > 0 \) and \( g_0(q, \bar{q}) \geq 0 \). \( g_0(q, \bar{q}) \geq 0 \) implies \((2\bar{q} - 1)(qNT_1 + (1 - q)NT_0) > \frac{qNT_1 - (1 - q)NT_0}{\bar{q}NT_0 - (1 - q)NT_1} \) as shown for the \( \hat{q} = \frac{1}{2} \) case. Thus, \( f(q, \bar{q}, \beta_0, \beta_1) \) is strictly greater in \( B_0(p_t, \beta_0, \beta_1) \).

This completes the claim that \( \beta_1 > \beta_0 \) and \( B_1(p_t, \beta_0, \beta_1) \geq 0 \) together imply \( B_0(p_t, \beta_0, \beta_1) > 0 \). \( \square \)

If, on the equilibrium path, a trade is supposed to occur in a period prior to \( T \), then it is necessary to specify the off-equilibrium beliefs after observing a trader deviating to not trading at that time (if a trader instead deviates by trading too early, beliefs about the trader’s type are pinned down by sequential rationality of the trade direction (Lemma A1)). Consistent with the result of Lemma A2 and the implementation of the experiment, I assume that the off-equilibrium beliefs are given by specification B1 so that the price does not change after a deviation from trading at the specified time.

Under B1, the benefit function in (3) simplifies. Defining \( \beta = \beta_0 = \beta_1 \), \( f(q, \hat{q}, \beta_0, \beta_1) \) simplifies to \( \beta(2\hat{q} - 1) \) when \( \hat{q} > q \) and \( \beta(2\bar{q} - 1) \) when \( \hat{q} < q \), reflecting the fact that a trader follows her better quality signal when they are contradictory. The denominator simplifies to

\[
Pr(\hat{e}, q = NT) = \beta Pr(\hat{e})
\]

so that \( \beta \) cancels in the numerator and denominator. Denoting a trader’s better quality signal at the state corresponding to event \( \hat{e} \), \( q \equiv \max(q, \hat{q}) \) (again, I suppress the dependence of \( q \) on \( \hat{e} \) for notational ease), we can then write the simplified benefit function,

\[
B_x(p_t) = \frac{p_t(1 - p_t)}{Pr(\hat{e} = x)} \left[ \sum_{\hat{e} \in E} \hat{e}\hat{e}_1 (2q - 1) Pr(\hat{e}) - (2\bar{q} - 1) \right] \tag{4}
\]

The benefit from waiting from period \( t \) until period \( T \) in all possible future states plays a key role, so I denote it \( \overline{B}_t(p_t) \). It depends upon a trader’s type, \( x \in \{0, 1\} \), but since only the sign of the benefit matters and the type simply scales the benefit, I drop the type from the notation. This benefit is given by (4) with \( q = \bar{q} \). It is also convenient to denote the expected profit from this strategy as \( \overline{\Pi}_t(p_t) \). We can write this expected profit as the expectation over realizations of the public events of the expected profits from the trades in period \( T \), \( \overline{\Pi}_t(p_t) = \sum_{\hat{e} \in E} Pr(\hat{e})\pi(p_t, \hat{e}) \). Here, \( p_\hat{e} \equiv Pr(V = 1|\hat{e}) = \frac{p_\hat{e}\hat{e}}{p_\hat{e}\hat{e} + (1 - p_t)e_0} \) is the price conditional on the realization, \( \hat{e} \), and \( \pi(p_t, q) = \frac{p_t(1 - p_t)}{Pr(\hat{e} = x)} (2q - 1) \) is the expected profit from a trade with a private signal of quality \( q \) at a price of \( p_t \). Finally, note that \( \pi(p_t, q) \) and \( \overline{\Pi}_t(p_t) \) are strictly concave in \( p_t \) (peaking at \( p_t = \frac{1}{2} \)), which can be proven in an identical manner to Lemma 2 of Kendall (2018a).

Lemma A3 establishes that public information revealed through a public signal or another’s trade reduces a trader’s expected profit, provided it occurs before at least one of her planned trades.
Lemma A3: At any price, \( p_t \in (0, 1) \) and time, \( t \in (1, T-1) \), fix the contingent plan of a trader. The expected profit that this plan produces strictly decreases with each additional (i) public signal, or (ii) trade by another trader that occurs with positive probability, provided that the event occurs prior to at least one planned trade.

Proof of Lemma A3:
First note that public signals are informative events, as are trades by other traders (by Proposition 1). Let the expected profit of a trader’s contingent plan in the absence of the additional informative event be \( \Pi_t(p_t) \). Write this expected profit as the expectation of trading profits over all possible other events resulting from public signals or trades by others: \( \hat{\Pi}_t(p_t) = \sum_{\hat{e} \in E} Pr(\hat{e})p(\hat{e}, q) \) (where \( q \equiv \max(q, \hat{q}) \) as above). With the additional informative event, we can decompose the trader’s new expected profit, \( \hat{\Pi}_t(p_t) \) into the trading profits that are unaffected by the new event (trades that occur before the new event occurs) and those that are affected (occur after). Let \( E_1 \) be the set of event realizations that lead to trades before the new event and \( E_2 \) be those that lead to trades after. Finally, let the additional informative event occur with probability \( \alpha > 0 \), let \( \hat{e} \in E_3 \) be the set of realizations of the new informative event, and define \( p_{\hat{e}, \hat{e}} = \text{Pr}(V = 1|\hat{e}, \hat{e}) \) as the price conditional on the realizations of events \( \hat{e} \) and \( \hat{e} \). We have

\[
\hat{\Pi}_t(p_t) = \sum_{\hat{e} \in E_1} Pr(\hat{e})p(\hat{e}, q) + \alpha \sum_{\hat{e} \in E_2} \sum_{\hat{e} \in E_3} Pr(\hat{e})p(\hat{e}, \hat{e}, q) + (1 - \alpha) \sum_{\hat{e} \in E_2} Pr(\hat{e})p(\hat{e}, q)
\]

\[
< \sum_{\hat{e} \in E_1} Pr(\hat{e})p(\hat{e}, q) + \alpha \sum_{\hat{e} \in E_2} Pr(\hat{e})p(\hat{e}, \hat{e}, q) + (1 - \alpha) \sum_{\hat{e} \in E_2} Pr(\hat{e})p(\hat{e}, q)
\]

\[
= \sum_{\hat{e} \in E_1} Pr(\hat{e})p(\hat{e}, q) + \alpha \sum_{\hat{e} \in E_2} Pr(\hat{e})p(\hat{e}, q) + (1 - \alpha) \sum_{\hat{e} \in E_2} Pr(\hat{e})p(\hat{e}, q)
\]

\[
= \Pi_t(p_t)
\]

where the inequality follows from Jensen’s inequality, using the fact that \( \pi(p_t, q) \) is strictly concave and that the additional event is assumed to be informative so that \( p_{\hat{e}, \hat{e}} \neq p_{\hat{e}} \) for some realization. The second equality follows from the martingale property of prices - the expected price after the additional event is equal to that in its absence:

\[
\sum_{\hat{e} \in E_3} Pr(\hat{e}, \hat{e})p_{\hat{e}, \hat{e}} = \sum_{\hat{e} \in E_3} (p_{\hat{e}}\hat{e}_1 + (1 - p_{\hat{e}})\hat{e}_0) \frac{p_{\hat{e}}\hat{e}_1}{p_{\hat{e}}\hat{e}_1 + (1 - p_{\hat{e}})\hat{e}_0} = \sum_{\hat{e} \in E_3} p_{\hat{e}}\hat{e}_1 = p_{\hat{e}}
\]

Lemma A4 establishes an important property of the sign of the benefit from waiting until period T. Intuitively, as studied in detail in Kendall (2018a), public information reduces the benefit most when \( p_t = \frac{1}{2} \), because it reduces uncertainty in the asset value, on which profits depend, by the greatest amount at this time. Therefore, if the benefit in the presence of a public event is negative at some price, \( p_t \neq \frac{1}{2} \), it must be negative at all prices closer to one-half.
Lemma A4: For any $t \in (1, T - 1)$ if the benefit from waiting until period $T$, $\mathcal{B}_t(\hat{p}_t) \leq 0$ for some $\hat{p}_t > \frac{1}{2}$ ($\hat{p}_t < \frac{1}{2}$), we must have $\mathcal{B}_t(p_t) < 0$ for all $p_t \in \left[\frac{1}{2}, \hat{p}_t\right)$ ($p_t \in (\hat{p}_t, \frac{1}{2}]$).

Proof of Lemma A4:
Consider the derivative of the benefit function with respect to $p_t$:

$$
\frac{\partial \mathcal{B}_t(p_t)}{\partial p_t} = \frac{1 - 2p_t}{p_t(1 - p_t)} \mathcal{B}_t(p_t) - \frac{p_t(1 - p_t)}{\Pr(s_i = x)} \sum_{\hat{e} \in E} \hat{e}_0 \hat{e}_1 (2\hat{q} - 1) (\hat{e}_1 - \hat{e}_0) \Pr(\hat{e})
$$

Because of the symmetry property of signals, for each informative realization (trade or public signal), $\hat{e}_i \in E$, there is another realization, $\hat{e}_j$, such that $\hat{e}_{i,0} = \hat{e}_{j,1}$ and $\hat{e}_{i,1} = \hat{e}_{j,0}$. Therefore, we can write the expression for the derivative in terms of the sum over only those events for which $\hat{e}_{i,1} > \frac{1}{2}$. Denoting this set, $E'$, we have

$$
\frac{\partial \mathcal{B}_t(p_t)}{\partial p_t} = \frac{1 - 2p_t}{p_t(1 - p_t)} \mathcal{B}_t(p_t) - \frac{p_t(1 - p_t)(2\hat{q} - 1)}{\Pr(s_i = x)} \left[ \sum_{\hat{e} \in E'} \hat{e}_i \hat{e}_i (\hat{e}_{i,1} - \hat{e}_{i,0}) \Pr(\hat{e}_i) + \hat{e}_j \hat{e}_j (\hat{e}_{j,1} - \hat{e}_{j,0}) \Pr(\hat{e}_j)^2 \right]^{27}
$$

(5)

where $\Sigma = \sum_{\hat{e} \in E'} \hat{e}_i \hat{e}_i (\hat{e}_{i,1} - \hat{e}_{i,0}) \Pr(\hat{e}_i)^2 \Pr(\hat{e}_j)^2$ is strictly positive because $\hat{e}_{i,1} > \hat{e}_{i,0}$ for all realizations in $E'$.

Now, assume that there exists a $\hat{p}_t < \frac{1}{2}$ such that $\mathcal{B}_t(\hat{p}_t) \leq 0$. In this case, $\frac{\partial \mathcal{B}_t(p_t)}{\partial p_t} < 0$, because the first term in (5) is weakly negative and the second term strictly so. Therefore, the benefit must strictly decrease as $p_t$ increases towards $p_t = \frac{1}{2}$, and, therefore must be strictly negative over $p_t \in (\hat{p}_t, \frac{1}{2}]$. A symmetric argument applies if there exists a $\hat{p}_t > \frac{1}{2}$ such that $\mathcal{B}_t(p_t) \leq 0$. □

A2. Omitted Proofs

Proof of Lemma 1:
Suppose not: there exists at least one trader, trader $i$, whose candidate equilibrium strategy involves trade with positive probability in a period $1 < t < T$. Let $1 < \hat{t} < T$ denote the latest period in which she trades with positive probability under this candidate strategy. By Lemma A3, her expected profit for arbitrary strategies of the other traders is weakly less than if every other trader always waits to trade until $T$ with probability one (strictly so unless her plan always involves trade before the other traders). I show that her expected profit even when all other traders always wait is strictly less than her profit from

27Uninformative realizations ($\hat{e}_{i,0} = \hat{e}_{i,1}$) contribute zero to the summation so can be ignored.
trading at $t = 1$ so that no matter what the strategies of the other traders are, we have a contradiction.

We can decompose trader $i$’s expected profit at $t = 1$ according to two sets of public signal realizations for the public signals prior to $t$ (since no trades occur until $T$). Let the first set, $E_R$, consist of all realizations such that trader $i$ trades with positive probability prior to $T$ (including at $t$ or earlier). Let the second set, $E_W$, consist of all realizations such that she trades at $T$. We can write her expected profit as the (probability weighted) sum of the profits from early trades and the expected profits from waiting from $t$ until $T$:

$$
\Pi_1(\frac{1}{2}) = \sum_{\hat{e} \in E_R} Pr(\hat{e})\pi(p, q) + \sum_{\hat{e} \in E_W} Pr(\hat{e})\Pi_1(p) < \sum_{\hat{e} \in E_R} Pr(\hat{e})\pi(\frac{1}{2}, q) + \sum_{\hat{e} \in E_W} Pr(\hat{e})\Pi_1(\frac{1}{2}) \leq \sum_{\hat{e} \in E_R} Pr(\hat{e})\pi(\frac{1}{2}, q) + \sum_{\hat{e} \in E_W} Pr(\hat{e})\pi(\frac{1}{2}, q) = \pi(\frac{1}{2}, q)
$$

The first inequality follows from the fact that both $\pi(p, q)$ and $\Pi_1(p)$ are strictly concave in $p$, reaching maxima at $p_t = \frac{1}{2}$ (the inequality is strict because at least one set of public signal realizations leads to a price different from one half). The second inequality follows from Lemma A4: if the trader plans to trade with positive probability at some price at $\hat{t}$, then she must also plan to trade at $\hat{t}$ if the price is one-half (implying $\pi(\frac{1}{2}, q) \geq \Pi_1(\frac{1}{2})$). (6) then implies a contradiction: the expected profit from this contingent plan is strictly less than the profit from trading at $t = 1$, so trader $i$ should instead do the latter. Because this argument applies to all traders, all trades must occur at $t = 1$ or $t = T$ in any equilibrium. □

**Proof of Proposition 1:**

The result of Lemma A2 is that, in equilibrium, both types of traders must follow the same timing strategy. The market maker’s beliefs must be consistent with this fact so that we must have $\beta_0 = \beta_1$. Applying this equality to the formulas in Lemma A1 results in Proposition 1. □

**Proof of Proposition 2:**

Impose B1 so that the benefit from waiting is given by the simplified expression, (4). Define the benefit from waiting from $t = 1$ until $t = T$ when all other traders also wait until $t = T$ as $B_1^W(\frac{1}{2})$ where the initial price, $p_1 = \frac{1}{2}$. Similarly, define the benefit from waiting from $t = 1$ until $t = T$ when all other traders trade prior to $T$, $B_1^R(\frac{1}{2})$. By Lemma A3, $B_1^W(\frac{1}{2}) > B_1^R(\frac{1}{2})$: the benefit from waiting to trade at $t = T$ when all other traders also wait is strictly greater than that when the other traders trade at $t = 1$ (because in the latter case, the expected profit from waiting until $T$ is strictly reduced by the others’ trades).

Thus, any parameterization of the model results in one of three cases: $B_1^W(\frac{1}{2}) > B_1^R(\frac{1}{2}) > 0$, $B_1^W(\frac{1}{2}) > 0 > B_1^R(\frac{1}{2})$, or $0 > B_1^W(\frac{1}{2}) > B_1^R(\frac{1}{2})$.

Part 1. $B_1^W(\frac{1}{2}) < 0$ implies $0 > B_1^W(\frac{1}{2}) > B_1^R(\frac{1}{2})$. Thus, whether the other traders
trade at \( t = 1 \) or \( t = T \), each trader finds it optimal to trade at \( t = 1 \), ensuring a unique equilibrium outcome in which all trades occur at \( t = 1 \). Off-equilibrium, generally each trader’s equilibrium strategy involves a combination of early and late trades (not all trades need occur prior to \( T \)) which makes a full specification of the equilibrium strategies (as opposed to outcomes) tedious.

Part 2. \( B^R_1(\frac{1}{2}) > 0 \) implies \( B^W_1(\frac{1}{2}) > B^R_1(\frac{1}{2}) > 0 \). Thus, whether the other traders trade at \( t = 1 \) or \( t = T \), each trader finds it optimal to trade at \( t = T \), ensuring a unique equilibrium outcome in which all trades occur at \( t = T \). As such, the unique equilibrium strategy for each trader is to wait in every possible state.

Part 3. If neither \( B^W_1(\frac{1}{2}) < 0 \) nor \( B^R_1(\frac{1}{2}) > 0 \) hold, then we must have \( B^W_1(\frac{1}{2}) \geq 0 > B^R_1(\frac{1}{2}) \) or \( B^W_1(\frac{1}{2}) > 0 > B^R_1(\frac{1}{2}) \). In either case, each trader finds it optimal to wait to trade until \( t \) when all other traders do the same, such that there exists an equilibrium in which all trades occur at \( t = T \). In addition, each trader finds it optimal to trade at \( t = 1 \) when all other traders do the same, such that there exists another equilibrium in which all trades occur at \( t = 1 \).

Lastly, provided \( B^W_1(\frac{1}{2}) \neq 0 \) and \( B^R_1(\frac{1}{2}) \neq 0 \), which is generically the case, an equilibrium in mixed strategies exists as well (if \( B^W_1(\frac{1}{2}) = 0 \) or \( B^R_1(\frac{1}{2}) = 0 \) the mixed strategy equilibrium is degenerate, corresponding to one of the pure strategy equilibria). Let \( r \) denote the probability with which an individual trader rushes (waiting to \( t = T \) with the remaining probability). Using the fact that, by Lemma A3, a trader’s expected profit from waiting until \( T \) is strictly lower when another trader trades at \( t = 1 \) instead of \( t = T \), it is a simple exercise to show that each trader’s expected profit monotonically decreases in the probability that a single other trader trades at \( t = 1 \) (holding other strategies fixed). Then, a trader’s expected profit from waiting until \( T \) must monotonically decrease in \( r \), because the total decrease is the sum of the decreases due to each other trader trading at \( t = 1 \) more often. At \( r = 0 \), we have \( B^W_1(\frac{1}{2}) > 0 \) so that a trader’s best response is to trade at \( t = T \), and at \( r = 1 \), we have \( B^R_1(\frac{1}{2}) < 0 \), so that her best response is to trade at \( t = 1 \). Given continuity and monotonicity of a trader’s expected profit in \( r \), there must exist a value, \( r^* \), such that the trader is indifferent between trading at \( t = 1 \) and \( t = T \) so that she is herself willing to mix with probability \( r^* \). Given each trader’s problem is identical, we therefore have a symmetric mixed strategy equilibrium in which each trader trades at \( t = 1 \) with probability \( r^* \) and at \( t = T \) with probability \( 1 - r^* \).

A3. Experimental Parameterizations

Using the sufficient conditions established in the proof of Proposition 2, consider the parameters of the two treatments of the experiment. Because only public signals affect \( B^W_x(\frac{1}{2}) \), we can rewrite it as

\[
B^W_x(\frac{1}{2}) = \frac{1}{4P_1(\frac{1}{2})} \left[ \sum_{k=0}^{T-1} \frac{2C_0(k)C_1(2q - 1)}{C_1(k) + C_0(k)} - (2q - 1) \right]
\]

(7)

where \( C_0(k) = \frac{(T-1)!}{k!(T-1-k)!} (1 - q)^k q^{T-1-k} \) and \( C_1(k) = \frac{(T-1)!}{k!(T-1-k)!} q^k (1 - q)^{T-1-k} \) are the probabilities of observing \( k \) public signal realizations equal to 1, conditional on \( V = 0 \) and
For the parameters in treatment R ($q = \frac{3}{4}$, $\bar{q} = 1$, $q_p = \frac{17}{24}$, and $T = 8$), we have $B^W_1(\frac{1}{2}) = B^W_0(\frac{1}{2}) \approx -0.075 < 0$. Thus, the sufficient condition to ensure all traders trade immediately is satisfied.

To evaluate $B^R_x(\frac{1}{2})$, I rewrite the benefit when $n-1$ trades and $T-1$ public signals occur while waiting as

$$B^R_x(p_1) = \frac{1}{4 \Pr(V = 0)} \left[ \sum_{k=0}^{T-1} \sum_{j=0}^{n-1} C_0(k) C_1(k) D_0(j) D_1(j) (2\bar{q} - 1) + (1 - p_1) C_0(k) D_0(j) - (2q - 1) \right]$$

where $D_0(k) = \frac{(n-1)!}{j!(n-1-j)!}(1-q)^j q^{n-1-j}$ and $D_1(j) = \frac{(n-1)!}{j!(n-1-j)!}(1-q)^j q^{n-1-j}$ are the probabilities of observing $j$ buys by the $n-1$ other traders, conditional on $V = 0$ and $V = 1$ respectively, and $C_0(k), C_1(k)$ are as above. For the parameters of treatment W ($q = \frac{13}{24}$, $\bar{q} = 1$, $q_p = \frac{17}{24}$, and $T = 8$), we have $B^R_x(\frac{1}{2}) = B^R_x(\frac{1}{2}) \approx 0.128 > 0$, which ensures all traders trade in period $T$.

B: Alternative Models

B.1 Risk Preferences

First, note that I pay subjects in lottery tickets which theoretically induces risk-neutral behavior (Roth and Malouf (1979). However, it’s possible subjects behave as if risk-averse over lottery tickets themselves. In this case, risk-averse individuals may prefer to wait to learn about the asset value, potentially even through public information, which is consistent with one aspect of a momentum-like strategy. However, given that risk-aversion creates a desire to wait for information, it fails to produce the price-chasing behavior characterized by waiting for several periods and then trading. Instead, risk-averse subjects would wait to trade until period $T$ in the W treatment. In addition, a risk-averse subject with a signal that opposes the price trend would never trade against her private signal to follow the trend, as observed in the data.

Numerical simulations, assuming power utility, confirm the above intuition for risk-averse subjects. They also demonstrate that risk-seeking behavior can generate early trades in the W treatment, provided the coefficient of relative risk aversion is very large. However, when risk-seeking produces early trades, it predicts that a subject will trade immediately in the first period, rather than at an intermediate period, which is again not consistent with a momentum-like strategy.

B.2 Behavioral Game Theory

Behavioral game theoretic explanations, such as quantal response equilibrium (McKelvey and Palfrey (1995,1998)), level k reasoning (Stahl and Wilson (1994) and Nagel (1995)), cognitive hierarchy (Camerer, Ho, and Chong (2004)), and cursed equilibrium (Eyster and Rabin (2005)) have been extremely successful in explaining behavior in experimental games. However, because these theories each rely on subjects believing that other subjects’ actions do not always correspond to their private information, they generate predictions inconsistent
with momentum-like behavior.

First, if a subject believes other subjects make timing mistakes, but trade according to their private information, then the parameters of the experiment are such that the equilibrium timing strategy is dominant (see the proof of Proposition 2). If, in addition to timing mistakes, a subject believes other subjects make trading mistakes (as is the case if they follow momentum-like strategies), then she would think that the observed price is too extreme on average. For example, if other subjects’ trades are completely unrelated to their signals, then the true expected value of the asset is one half. If prices are too extreme, it may be optimal for a subject to rush when she should otherwise wait, but she should then trade against the price trend, rather than with it, contrary to the data. For this same reason, momentum-like strategies are not a best response to other subjects using momentum-like strategies.28

C: Instructions

I provide the instructions for the W treatment. Those for the R treatment are identical except for the difference in parameters.

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28 In a related environment (but without timing decisions), both Cipriani and Guarino (2005) and Drehmann, Oechssler, and Roider (2005) formalize the argument that only trading against price trends is consistent with believing other subjects make mistakes.
Instructions

This is a research experiment designed to understand how people make stock trading decisions in a simple trading environment. To assist with our research, we would greatly appreciate your full attention during the experiment. Please do not communicate with other participants in any way and please raise your hand if you have a question.

You will participate in a series of 30 trials. In each trial, you and the 7 other participants will each trade a stock with the computer. You will be paid $5.00 for completing all trials. In addition, in each trial you will earn lottery tickets as described next. The more lottery tickets you have, the more you will earn on average.

Trading and Profits

Before each trial, the computer will randomly select whether the stock's value, V, is 100 tickets or 0 tickets. Each is equally likely to be selected. At the start of each trial, you will be given 100 tickets. You may choose either to buy or sell the stock in any one of 8 trading periods. You can only trade ONCE and MUST trade in one of the periods. If you buy the stock, you will gain its actual value minus the price, P, you pay for it. If you want to sell the stock, you must first 'borrow' it from the computer and later pay back its actual value. So, if you sell, you will gain the price you receive minus its value. In summary, your total profit is:

\[ 100+V-P \text{ lottery tickets if you buy} \]
\[ 100+P-V \text{ lottery tickets if you sell} \]

So, for example, if you sell the stock at a price of 50 and it turns out to be worth 100, you would earn 100+(50-100)=50 tickets for that trial. But if it turns out to be worth 0, you would earn 100+(50-0)=150 tickets.

Clues about Value

To help you guess the value of the stock the computer chose, you will get a single Private Clue at the start of the trial. This Private Clue will be known only by you. Specifically, there will be two possible bins: one that the computer will use if the value is 100 and another that the computer will use if the value is 0. Bins contain some number of blue and green marbles as shown below. The computer will draw a marble randomly from the bin and show it to you as your Private Clue. Each marble in the bin is equally likely to be drawn. The color of the ball you see can give you a hint as to the stock's value.

After observing your Private Clue, you can choose to trade immediately in the first trading period. Alternatively, you can choose to wait and trade in one of the following 7 trading periods. Between trading periods, there are public announcement periods. In each announcement period, a Public Clue will become available. The Public Clue, unlike your Private Clue, is seen by everyone. For the Public Clue, the computer will draw a marble from another bin. The bin used will again depend on the stock's true value but the possible bins used for the Public Clue are different from the bins used for the Private Clue, as shown below. Note that the bins used for both clues are fixed throughout the trial - they depend only on the initially chosen random value of the stock. Also, marbles for both types of clues are always replaced before another is drawn.

<table>
<thead>
<tr>
<th>Clue</th>
<th>Contents of bin if value = 100</th>
<th>Contents of bin if value = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Clue</td>
<td>13 blue 11 green</td>
<td>11 blue 13 green</td>
</tr>
<tr>
<td>Public Clue</td>
<td>17 blue 7 green</td>
<td>7 blue 17 green</td>
</tr>
</tbody>
</table>

If you decide to wait to trade until the last trading period, the true value of the asset will be revealed to you before you trade. Otherwise, you will only learn the true asset value after the trial is complete. Importantly, however, each time you choose to wait, the price is likely to change before your next chance to trade, as described next.

Prices

The price of the stock is set by a Price Setter played by the computer. The Price Setter's job is to set the price equal to the stock's mathematical expected value given all of the public information available. Therefore, the price will always be between 0 and 100. The Price Setter can observe the trades made by you and the other participants and the Public Clues. However, she can not observe any of the Private Clues nor the true asset value (even when it is revealed in the final period).

Because the price changes with the available information, if you decide to wait, the price at which you can trade is likely to change. The initial price of the stock is 50 tickets, reflecting the fact that it is equally likely to be worth 100 tickets or 0 tickets. After a participant buys, the Price Setter will increase the price and after a participant sells, she'll decrease the price. After a Public Clue is revealed, the price will increase if it suggests (on its own) that the stock is more likely to be worth 100 and decrease if it suggests it is more likely to be worth 0.

Trading Screen

The trading screen you will use to trade is as shown below. The eight trading periods are indicated by the numbers 1-8. Public Clues are revealed between trading periods at the times indicated by the megaphone symbol (the clues are not shown in the figure - they appear elsewhere on your trading screen). All past trading decisions and prices are displayed. Trading periods in which one or more trades occur are indicated by a solid dot. The number of buys is indicated by a "+" and then a number and the number of sells by a "-" and then a number. The current price at which you can trade (74.16 in this example) is displayed at the current time which is indicated by the dashed red line. The dashed red line will progress to the right...
as we move through the periods.

In this example, one participant bought in the first trading period, so the price increased. In the second trading period, one participant bought and one sold, so the price did not change. In the third trading period, no one traded. The first and third Public Clues suggested the stock's value is 100 and the second that the stock's value is 0. It is currently the fourth trading period. Note that this is an example only and is not meant to suggest when you should trade.

In each trading period in which you haven't already traded, you must choose buy or sell or wait and then press the 'confirm' button. If you choose to wait, the red arc points to the next trading period in which you can trade. In trading periods after you have traded, you do not have to do anything - you will simply be notified that you have already traded. In the periods with Public Clues, you must press "OK" to acknowledge having seen the clue.

**Summary**
1. At the beginning of each trial, the computer randomly selects the stock's value: 0 or 100.
2. Each participant is shown their Private Clue from the same bin. Marbles are replaced so each participant may see the same (or different) marbles.
3. Each participant chooses to buy, sell, or wait in the first trading period. After all participants have made their decisions, a Public Clue is revealed and we move to the second trading period.
4. Step 3 is repeated until all 8 trading periods are complete. You must trade in one of the eight trading periods and may only trade once.
5. Just before the 8th trading period, the true value of the stock will be revealed to you if you have not already traded.
6. At each point in time, all past prices and trading decisions are available to use to help guess the value of the stock (in addition to the Clues).

After all participants have traded, the trial is complete. The true value of the asset will be revealed to all participants and you will be told how many tickets you earned for the trial. You will press 'next trial' to participate in the next trial. It is important to remember that, in each trial, the value of the stock is independently randomly selected by the computer -- there is no relationship between the value selected in one trial and another.

After all trials are complete, one lottery will be conducted for each trial. For each lottery, a random number less than 200 will be chosen by the computer. If the number is smaller than the number of lottery tickets you earned for the trial, you will get $1.00. Therefore, the more lottery tickets you earn in each trial, the more you can expect to make (partial tickets are possible and count as well). For example, if you earn 100 tickets in each trial, you can expect to make 0.65*30*1=$19.50 over the 30 trials. But, if you earn 130 tickets in each trial, you can expect to make 0.65*30*1=$19.50.

Please try to make each trading period decision within 15 seconds so that the experiment can finish on time. A timer counts down from 15 to help you keep track of time. Note, however, that if the timer hits zero, you can still enter your trading or wait decision and will still have the same chance to earn money. Before beginning the paid trials, we will have two practice trials for which you will not be paid. These trials are otherwise identical to the paid trials.

**Quiz**

Please answer the following questions and press the 'Check answers' button to see whether or not you answered all questions correctly. To ensure all participants understand the instructions, everyone must answer all of the questions correctly before we begin the experiment.

1. Your Private Clue is a blue marble. Based only on this information, What is the most likely value of the stock?
2. It is the second trading period. You observe another participant sold the stock in the first trading period. What color marble is their Private Clue most likely to be?  
- green  
- blue

3. Your Private Clue is a blue marble. What color marble is another participant's Private Clue likely to be?  
- green  
- blue

4. If you choose to sell the stock at a price of 80 and its value turns out to be 100, how many total tickets would you get for that trial?  
- 80  
- 20  
- 180

5. If you choose to buy the stock at a price of 25 and its value turns out to be 100, how many total tickets would you get for that trial?  
- 25  
- 75  
- 175

6. The stock's current price is 80. Which value of the stock is more likely?  
- 100  
- 0

Once you have completed the quiz, please press 'Check answers'.

Check answers