Market Panics, Frenzies, and Informational Efficiency: Theory and Experiment

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Abstract

In a market rush, the fear of future adverse price movements causes traders to trade before they become well-informed, reducing the informational efficiency of the market. I derive theoretical conditions under which market rushes are equilibrium behavior and study how well these conditions organize trading behavior in a laboratory implementation of the model. Market rushes, including both panics and frenzies, occur more frequently and severely when predicted by theory. However, because subjects use momentum-like strategies, rushing and positive correlations in returns not predicted by theory also occur. The results therefore suggest that commonly-discussed momentum strategies may exacerbate panics and frenzies.

1 Introduction

In financial markets, traders decide not only how to trade but also when to trade assets. Delaying trade benefits a trader (and the market) by giving her time to learn more about the value of an asset, improving the quality of her trades. However, delaying trade comes at a cost: it allows for preemption by other traders, generating adverse price movements that reduce potential earnings. In a market rush, fear of these adverse price movements causes traders to trade before becoming fully informed, leading to long-run informational inefficiencies which can have significant impacts on the real economy (Bond, Edmans, and Goldstein (2012)). These market rushes - which can take the form of panics (rushed sales)
or frenzies (rushed purchases) - have been a fixture of the study of financial markets for centuries.

Why do market rushes (inefficiently early trade due to fear of future price movements) occur? One possibility is that they arise from irrational decision-making: observers have long used affective terms like ‘panics’, ‘frenzies’, and ‘manias’ to describe these episodes (stretching back at least to Charles Mackay’s 1841 book, ‘Extraordinary Popular Delusions and the Madness of Crowds’). Another is that they arise instead due to individually rational assessments of the tradeoffs between the benefits of information and the costs of preemption. After all, theoretical models (Allen, Morris, and Postlewaite (1993), Romer (1993), Bulow and Klemperer (1994), Lee (1998), Barlevy and Veronesi (2003), Moinas and Pouget (2013), and Brunnermeier and Pedersen (2005)) have shown that related phenomena, such as price crashes and bubbles, which have long been viewed as irrational outcomes, can arise as standard, ‘rational’ equilibrium behavior.\(^1\)

In this paper, I address this question using two tools. First, I ask whether market rushes (including both panics and frenzies) in which traders forgo better private information can ever arise as a neoclassical, equilibrium outcome by building a theoretical model of trade timing in the spirit of Glosten and Milgrom (1985). I show that, indeed, ‘rational’ market rushes (both panics and frenzies) can occur in equilibrium, and I provide conditions under which (i) they must occur, and (ii) should never occur. Second, I ask whether market rushes can only arise due to rational motives by precisely implementing the model in a laboratory experiment. In one treatment (called Rush), I parameterize the experiment so that rushing should always occur in equilibrium: subjects should trade at the first opportunity, foregoing valuable information about asset values. In another (called Wait), I parameterize the experiment so that rushing cannot be equilibrium behavior and subjects should delay trade until receiving better information. Although market rushes are far more common and severe in Rush (where they are equilibrium behavior) than Wait (where they are not), frequent rushing occurs in the Wait treatment as well. I show that this behavior is driven by subjects using a simple price-chasing heuristic commonly ascribed to momentum traders. The results therefore suggest that rushing behavior is more common than standard theory predicts due to a well-documented trading heuristic popular in naturally-occurring settings.

I begin by introducing a simple model of trade timing with two crucial features. First,

\(^1\)Here, a note on terminology is useful. In popular usage, the terms ‘panic’ and ‘frenzy’ typically refer to timing phenomena, which is the focus of my work. Panics are often associated with bad states of the world, so much of the theoretical literature on market ‘panics’ focuses on (discontinuous) price crashes rather than timing per se. Price crashes can (but need not) result from these types of episodes. Frenzies, in common usage, often imply excess demand, and hence rising prices or bubbles. For the purposes of this paper, panics and frenzies - which I collectively refer to as ‘market rushes’ - are timing phenomena associated with falling and rising prices, respectively.
traders receive private information over time, generating a benefit to waiting. Specifically, agents simultaneously receive initial (poor quality) information when they arrive to the market, and are then given several opportunities to trade before receiving additional information just prior to the final trading period. Second, although each trader in the model has the opportunity to trade in many trading periods, I restrict them to a single trade. This restriction makes the tradeoff between acquiring better information and the desire to trade before others as simple as possible, allowing me to develop benchmark theoretical predictions and to easily define and identify rushing behavior.

The structure of the model is in the spirit of Glosten and Milgrom (1985). Risk-neutral traders receive binary signals and trade a binary-valued, common-value asset with a market maker. As is standard, traders can profit from trading according to their private information: buying with favorable information and selling otherwise. I prove a pair of sufficient conditions, one of which ensures that rushing occurs in equilibrium (all traders trade in the first period), and the other of which ensures that traders instead wait for better information (trade in the final period). Due to the symmetric nature of the model, market rushes can take the form of either panics (rushed sales generating falling prices) or frenzies (rushed purchases generating rising prices).

I then conduct a laboratory experiment based upon the model which is designed to ask whether market rushes occur only when they can be sustained as an equilibrium or if, instead, they also arise due to non-equilibrium behavior on the part of subjects. Experimental evidence on this question is important because it is difficult to evaluate the source of panics and frenzies using naturally-occurring data where the key determinants of equilibrium (particularly the timing and quality of information) are generally unobservable. However, existing experiments on trade timing (discussed below) do not implement models with precise theoretical predictions, making it difficult to pinpoint the source and severity of market rushes.

In the experiment, eight subjects are endowed with a low quality signal of the value of an asset and allowed to trade once (with an automated market maker) in any one of eight periods. Subjects are also informed that, just prior to the eighth trading period, each will receive a high quality signal. I study two treatments: in the Wait treatment, the early signal is very noisy while in the Rush treatment it is of significantly higher quality. Using the equilibrium predictions from the theory, I identify two nested hypotheses for these treatments. The weaker comparative static hypothesis is that trades occur significantly

\[Methods of inferring information from trades do exist. For example, Hasbrouck (1991) uses the persistence of price impacts and Easley et al. (1996) construct a structural model based on the arrival of news. Kendall (2017a) discusses the observable implications of traders rushing to trade on weak information.\]
earlier in the Rush treatment than in the Wait treatment. The stronger point predictions are that trades occur in the first period in the Rush treatment and the final period in the Wait treatment. In order to study learning and assess the behavior of experienced subjects, I allow subjects to play this eight-period game dozens of times.

The results reveal strong support for the comparative static hypothesis. After a few repetitions of the game, market rushes emerge in the Rush treatment. Subjects in the Wait treatment, by contrast, delay trade significantly even after dozens of repetitions of play. Thus, I observe a large difference in the degree of rushing across treatments, as predicted. The point prediction hypothesis is, however, partially rejected by the data. Subjects in Rush almost universally trade in the first period as hypothesized, but subjects in Wait rarely wait until the final period. Instead, I observe subjects in Wait rushing and delaying to various and heterogeneous degrees.

Why do subjects robustly rush in the Rush treatment but fail to fully delay trade in the Wait treatment? A closer examination of the data reveals that subjects follow momentum-like strategies, a trading heuristic familiar to students of technical analysis and commonly discussed in the behavioral finance literature (Grinblatt, Titman, and Wermers (1995), Hong and Stein (1999), Grinblatt and Keloharju (2000), Brozynski et al. (2003), Baltzer, Jank, and Smajlbegovic (2015), and Grinblatt et al. (2016)).

Rather than best responding to beliefs about future price movements as in an equilibrium model, subjects instead respond to backward-looking trends, waiting until sufficiently certain about the asset value, then trading in that direction. Subjects adapt their behavior over multiple repetitions of the game in a way that suggests that they fine tune the threshold of certainty they require in order to trade. Simulations of a simple, parsimonious learning model (à la the win-stay-lose-shift model of Nowak and Sigmund (1993) or learning direction theory of Selten and Buchta (1998)) show that this heuristic, adapted over time, converges to outcomes that look almost identical to the data: perfect market rushes emerge in simulations of the Rush treatment and intermediate and heterogeneous rushing emerges in simulations of the Wait treatment. I also demonstrate that this momentum-like behavior causes short-term positive correlations in returns to emerge in the experimental data, a common artifact of trade observed in financial markets.

The contemporaneous papers of Bouvard and Lee (2016), Dugast and Foucault (2017), and Kendall (2017a) also theoretically study the tradeoff between better information and

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4 For a review of the empirical evidence of correlations in returns, see Daniel, Hirshleifer, and Subrahmanyan (1998). These correlations are often given behavioral explanations, including momentum strategies (Hong and Stein (1999)).
the effects of competition. Kendall (2017a) has a similar setup to this paper, but in a model with only two traders (which precludes market rushes). Nevertheless, much of the intuition developed there carries over. Bouvard and Lee (2016) study firm’s decisions to conduct time-consuming risk management. Dugast and Foucault (2017) study informational efficiency in a model in which rumors take time to verify. In both of these papers, competition produces informational efficiencies, but in perfectly competitive environments less amenable to identifying market panics in a laboratory setting. Thus, the main theoretical contribution of this paper is to provide conditions under which we should expect the tradeoff between better information and potential adverse price movements to result in easily identifiable market rushes.\(^5\)

On the experimental side, no paper of which I’m aware studies the informational efficiency consequences of market rushes. Park and Sgroi (2012) study herding and contrarian behavior in the setup of Park and Sabourian (2011). Their experiment confirms qualitative predictions about when trades should occur, but their environment differs in that private information does not improve with time and it is not amenable to precise theoretical predictions. The environment of Shachat and Srinivasan (2011) features these same two differences. They study trading in continuous double auctions with the sequential arrival of private or public information. Asparouhova, Bossaerts, and Tran (2016) study a credit rollover game in which subjects must decide when to convert an asset to cash. Their environment does not feature private information. Brunnermeier and Morgan (2010) study clock games theoretically and experimentally. Their game features something akin to a market rush but absent informational inefficiencies. Finally, several experiments consider timing decisions with precise theoretical timing predictions, but in non-market settings. Sgroi (2003) and Ziegelmeyer et al. (2005) both implement the irreversible investment model of Chamley and Gale (1994). Ivanov, Levin, and Peck (2009, 2013) and Çelen and Hyndham (2012) study similar environments. Incentives to wait are quite different in these papers as observing others’ decisions can be beneficial when their information is not incorporated into prices.

The paper is organized as follows. In Section 2, I develop the model and derive the conditions under which we expect to see rushing as equilibrium behavior. In Section 3, I describe the experimental design, develop hypotheses, and provide results documenting the existence of both equilibrium and non-equilibrium rushes. I then show that subjects use momentum-like strategies and develop a simple learning model to explain the differences across treatments. In Section 4, I conclude by discussing the relationship between the experimental findings and behavior in naturally-occurring markets.

\(^5\)Smith (2000) also studies trade timing theoretically, but in a setting in which private information doesn’t improve with time.
2 Theory

2.1 Model

The model is meant to capture the idea that producing private information about asset values requires time, either to acquire the information, or to process it into a trading decision (or both). To give a concrete example, consider an unexpected news release of a firm deciding to acquire another. Scanning the news provides a very rapid, but likely very noisy, indication of the change in the acquirer's fundamental value. More detailed analysis (developing new valuation models, interviewing managers, etc.) improves estimates of the firm's value, but takes time. During this time, any information others act on gets imputed into prices, reducing potential profits and creating a tradeoff between acting quickly and producing better information.\(^6\)

In the model, time is discrete with \(t = 1, 2, \ldots, T\) trading periods. In each trading period, each of \(n\) risk-neutral traders may trade a single, common-value asset, \(V \in \{0, 1\}\), at a price established by a market maker. The initial prior that the asset is worth \(V = 1\) is \(p_1 = \frac{1}{2}\). When the asset value is realized at \(T\), those who purchased the asset at time \(t\) receive a payoff of \(V - p_t\) and those who sold (short) receive a payoff of \(p_t - V\). There is no discounting.

Each trader, identified by \(i \in n\), receives a private signal before the first trading period, \(s_i \in \{0, 1\}\), which has correct realization with probability \(q = Pr(s_i = 1|V = 1) = Pr(s_i = 0|V = 0) \in (\frac{1}{2}, 1)\). Each trader may trade only once in any of the \(T\) trading periods. If a trader waits until time \(T\) to trade, she receives an additional private signal, \(\bar{s}_i \in \{0, 1\}\), immediately prior to her final trading opportunity, which has correct realization with probability \(\bar{q} = Pr(\bar{s}_i = 1|V = 1) = Pr(\bar{s}_i = 0|V = 0) \in (q, 1]\). Note that I assume signal quality improves with time, \(\bar{q} > q\), and that I allow for the second private signal to reveal the true asset value perfectly.

At the beginning of each time period other than the first, a binary public signal, \(s_{P,t} \in \{0, 1\}\), which has correct realization with probability \(q_P = Pr(s_{P,t} = 1|V = 1) = Pr(s_{P,t} = 0|V = 0) \in (\frac{1}{2}, 1)\) becomes public knowledge.\(^7\) After this signal is available, but prior to any trades, a market maker establishes a single price, \(p_t = E[V|H_t] = Pr[V = 1|H_t]\), equal to the expected value of the asset conditional on all publicly available information (prior trades, timing decisions, public signals, and prices), \(H_t\), at which all trades occur. Therefore, all competition stems from traders attempting to preempt other traders, rather than from

\(^6\)Although the model is silent on the time-scale, I have in mind information that is acquired over relatively short time frames, perhaps minutes or hours, such that adverse price movements are a primary consideration.

\(^7\)The public signals represent information learned by the market maker and are necessary (but not sufficient) for equilibrium uniqueness.
competition within a period.\footnote{Having trades occur simultaneously and the market maker post a single price (rather than separate bid and ask prices) improve tractability, but are at odds with how most real markets work. However, in related work (Kendall (2017a)), I have shown that the main trade-off between better information and adverse price impacts is a primary force even when these two simplifying assumptions are relaxed. In that paper, I make progress by restricting attention to only two traders, while here simplifying the environment allows me to have many strategic traders, which is necessary for studying market rushes.}

2.2 Analysis

The solution concept is sequential equilibrium and I focus on Markov strategies that depend only upon the payoff-relevant state.\footnote{Sequential equilibrium, as opposed to Perfect Bayesian Equilibrium, helps to pin down beliefs upon observing trade in an off-equilibrium period. Markov strategies rule out coordination based upon the exact history of play. See part A of the Appendix for further details.} I begin with Proposition 1 - the standard result that traders optimally buy when their private belief is greater than the public belief (which is equal to the price), and sell otherwise. For a risk-neutral trader, this strategy leads to a positive expected profit, while doing the opposite leads to an expected loss. All proofs are given in part A of the Appendix.

**Proposition 1:** In any equilibrium:

1. traders who trade prior to period $T$ buy if $\bar{s}_i = 1$ and sell if $\bar{s}_i = 0$.
2. traders who trade in period $T$ buy if $\bar{s}_i = 1$ and sell if $\bar{s}_i = 0$

Given the optimal trading strategies of Proposition 1, we can calculate the expected profit from trading in the current period and compare it to the expected profit from trading in some future period in order to determine a trader’s optimal timing strategy. Because strategies depend upon the current price, the number of traders who have yet to trade, and traders’ private signals, the strategy space is large, making equilibrium characterization potentially very challenging. However, in part A of the Appendix I prove that a trader’s timing strategy must be independent of her private signal, such that no information can be inferred from her decision to not trade in the current period.\footnote{This result is a consequence of the assumption that the market maker posts only a single price. With separate bid and ask prices, due to the strategic interaction with market maker, timing strategies generally involve mixing (Kendall (2017a)) with probabilities that depend upon a trader’s private signal. It is for this reason that having the market maker post only a single price greatly improves tractability, allowing me to focus on the interaction between traders.} This result reduces the dimensionality of the strategy space, greatly simplifying the problem.

Two additional intermediate results characterize the impact of public information on the value to waiting for more information. The first is that any information that becomes public due to a public signal or another’s trade reduces the expected profit from trading.
in the future. This result is the main driver behind a market rush. Although signals are conditionally independent, unconditionally they are correlated such that prices move against a trader in expectation. Intuitively, if one discovers the asset is currently undervalued, others are more likely to discover the same. The second result is that the reduction in expected profits due to public information is greatest when uncertainty in the asset value is highest because of two facts: (i) expected profits are proportional to uncertainty in the asset value (private information creates a large difference between private and public beliefs when uncertainty is high), and (ii) public information reduces uncertainty more when it is initially high (see also Kendall (2017a)).

Having established that others’ trades impose a cost of obtaining better information, we see that the game is a form of coordination game: if others wait to trade until period $T$, it may also be beneficial to wait (depending upon the cost associated with the public signals), but if they trade earlier, it creates incentives to trade earlier as well. Although not conceptually difficult, a full equilibrium characterization of this coordination game is tedious to specify: whether there exists a unique equilibrium outcome or a multiplicity of equilibria depends in a non-trivial way on the three signal qualities. Rather than pursue a full characterization, in Proposition 2 I instead establish intuitive sufficient conditions to guarantee unique equilibrium trade timing predictions. In the subsequent experiment, I choose parameters such that these conditions are satisfied.

**Proposition 2:**

1. **If the expected value from waiting to trade in period $T$ when all other traders do the same is strictly less than the expected profit from trading in period 1, then, in any equilibrium, all trades occur in period 1.**

2. **If the expected value from waiting to trade in period $T$ when all other traders trade prior to $T$ is strictly greater than the expected profit from trading in period 1, then, in the unique equilibrium, all trades occur in period $T$.**

The intuition behind the sufficiency conditions in Proposition 2 is straightforward given the properties of the costs associated with public information discussed above. For the first part, if all other traders wait to trade until period $T$, then the cost of waiting is due to the public signals only. When this cost exceeds the value of obtaining better private information, a trader prefers to trade in the first period to waiting until period $T$. To also rule out waiting until an intermediate period, I show that if a trader prefers trading at an

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11 The proof of Part 1 of Proposition 2 does not specify the off-equilibrium timing strategies and therefore does not establish equilibrium uniqueness, but nevertheless provides a unique prediction as to when trades occur.
intermediate period to trading at $T$, she must then prefer trading in the first period (more public signals remain making it more costly to wait). Furthermore, in the first period, the asset value uncertainty is highest such that waiting is then most costly. Therefore, each trader prefers to trade immediately even if the others wait, so that all trades occur in the first period in equilibrium.

For the second part, if all other traders trade prior to $T$ and a trader prefers to wait in the first period, then she must also prefer to wait in all subsequent periods: there are (weakly) fewer trades by other traders, less public signals remain, and the asset value uncertainty is (weakly) reduced, all of which make waiting less costly. Therefore, in the unique equilibrium, all traders wait to trade until period $T$.

Note that the initial signal quality plays a dual role, affecting the equilibrium forces in two ways. Most straightforwardly, it increases the profit from rushing by increasing the difference between a trader’s private belief and the price. But, importantly, it also increases the price impacts of others’ trades, making it more costly to wait. An increase in $q$ therefore unambiguously leads to greater incentives to rush.

Part 1 of Proposition 2 establishes conditions under which we expect an equilibrium market rush: a situation in which all traders simultaneously rush to trade (in the first period). Traders with initial unfavorable signals expect a market panic: they put more weight on a bad state of the world and fear falling prices. Those with favorable signals instead expect a market frenzy: they believe the state of the world is good and fear missing out on purchasing the asset before prices rise. In the example of an unanticipated announcement of a merger which may be good or bad for the acquiring firm, heterogeneous private beliefs create differences in opinion, but, regardless of their opinion, traders prefer to act as soon as possible. In doing so, they forgo valuable information, reducing the informational efficiency of the market price.\footnote{Throughout, informational efficiency refers to long-run informational efficiency. Rushing produces short-run improvements in efficiency at the expense of long-run efficiency. At least over relatively short time frames (minutes or hours), short-term efficiency is presumably less important than long-run efficiency for decisions that affect the real economy.}

In the model, rushing (trading prior to obtaining full private information) always takes the form of a full market rush (everyone trading at the same time), because trading at intermediate dates cannot be optimal (when either sufficient condition holds). In the experimental data, by contrast, individual rushing need not (and does not always) result in a market rush: traders may rush to trade at different times though this, too, reduces the informational efficiency of market prices.
3 Experiment

3.1 Experimental Design

The experiment is a direct implementation of the model in the previous section. Each of the $n = 8$ subjects in the market are told that they will trade a stock with the computer in each of 30 trials (repetitions) of the game (each trial is a full run of a $T = 8$ period game). At the beginning of each trial, each subject receives a weak private signal (the quality of which varies across treatments as described below), corresponding to a randomly drawn ball from an urn that corresponds to the true asset value. The trial then unfolds over 8 periods, each of which consists of the following sequence of events:

1. Each trader has the opportunity to (simultaneously) trade at a price determined by the (computerized) market maker (the initial price is $p_1 = \frac{1}{2}$), or to wait until a later period.

2. The market maker updates the price to reflect the new expected value of the asset given the information revealed by the trades (if any).

3. A public signal (also determined by an urn draw) of quality, $q_P = \frac{17}{24}$, becomes available and the market maker again updates the price in response.

Subjects can only trade once per trial and if a subject waits in the first seven periods, she learns the asset value for certain ($\bar{q} = 1$), and is forced to trade in the final period. Throughout, subjects observe the complete history of the game including the price path and others’ trades (+1 indicating a purchase and −1 a sale) on the display pictured in Figure 1.\textsuperscript{13} All of the details of the experiment are provided to subjects prior to play through instructions reproduced in Appendix C.

The goal of the experiment is to understand the degree to which rushes are motivated by equilibrium forces. There are two components to this question. First, do rushes occur when the theory predicts they should? Second, do rushes occur only when theory predicts they should, or do behavioral tendencies not included in a standard model cause them to occur even when they cannot be called equilibrium phenomena?

To answer this question, the design varies whether market rushes can be supported as an equilibrium outcome across two treatments (with parameters summarized in Table 1). The only difference between treatments is in the value of $q$. In the first, treatment W (Wait), $q = \frac{13}{24}$, so that initial private information is very weak. In part A of the Appendix, I use Proposition 2 to show that this parameterization results in a unique equilibrium in which all

\textsuperscript{13}The software was developed using the Redwood package (Pettit and Oprea (2013)) which uses HTML5 to allow for rapid updating of the computer interface. This feature allows for many more trials than would have been possible otherwise, which is important for learning in this relatively complex environment.
traders wait to trade in period $T$. In the second treatment, treatment R (Rush), the initial signal quality is higher, $q = \frac{3}{4}$, for which theory predicts a market rush in the first period. Other parameters are held constant across treatments. I chose $\overline{q} = 1$ to provide a very salient reason to wait to obtain better private information: traders can guarantee themselves a positive profit by waiting to trade until period $T$. The public signal quality and length of the game were chosen to generate gradual revelation of the true asset value and thus scope for realistic, rich price dynamics to potentially influence behavior.

A technical question that arises in the experimental design is how prices should be set by the (computerized) market maker. The information revealed by past trades depends upon subjects’ strategies, which are not known ex ante. I follow the previous literature (Cipriani and Guarino (2005) and Drehmann, Oechssler, and Roider (2005)) by having the market maker assume subjects are following equilibrium strategies.\footnote{It turns out, of course, that not all subjects follow equilibrium strategies. In Section 3.4.2 and Appendix B, I discuss the consequences of traders understanding that other traders may not be making equilibrium} In equilibrium, no information
is revealed by the decision to wait so that, if no trades occur, the price is updated due to
the information contained in the public signal only. After a trade, whether in a period on or
off the equilibrium path, the price is updated assuming subjects follow the trading strategies
of Proposition 1 (which are sequentially rational off-equilibrium). Subjects were told that
prices reflect the mathematical expected value of the value of the asset, conditional on all
public information. They were also explicitly told that prices increase after buy decisions
and favorable public signals, and conversely for sell decisions and unfavorable public signals.
The exact sizes of the price changes (per Bayes’ rule) were not communicated ex ante, but
subjects participated in many trials and observed many price movements, facilitating learning
over time.

3.2 Implementation Details

I recruited subjects from the University of British Columbia student population using the
experimental recruitment package Orsee (Greiner (2004)). Subjects were from a variety of
majors and no subject participated in more than one session. I conducted four sessions of
each treatment, for a total sample of 64 subjects ($n = 8$ subjects in each session), with new
randomizations for each session’s asset values and signals (in order to avoid the possibility
of a particular set of draws influencing the results). In each session, subjects first signed
consent forms and then received the instructions (provided in part C of the Appendix)
verbally. Subjects were encouraged to ask questions while the instructions were read and
then completed a short quiz. All quiz questions had to be answered correctly by each subject
before the experiment began, and this policy was common knowledge. Once the experiment
began, no communication of any kind between subjects was permitted.\footnote{In each session, subjects participated in 30 paid trials preceded by two practice trials. It
was emphasized that they could only trade once, and that they must trade at some point
(during each trial). The asset value, $V$, and prices were scaled by a factor of 100 currency
units, and subjects were endowed with 100 units with which to trade prior to each trial,
for a maximum possible earning of 200 currency units per trial. In order to induce risk-
neutrality, each currency unit represented a lottery ticket with a $1/200$ chance to earn $\$1.00
Canadian (Roth and Malouf (1979)). After all trials were completed, a computerized lottery
was conducted for each paid trial and subjects were paid according to the results of the
lotteries. In addition, each subject was paid $5.00 as a show-up fee. Average earnings were
$21.53$ (minimum $12.00$, maximum $30.00$) with a corresponding wage rate over an hour
trade and timing decisions.}

Subjects were separated by physical barriers so that they could not observe each others’ private infor-
mation or decisions.
and a half of $14.35/hour.

### 3.3 Hypotheses

The main question the experiment is designed to answer is whether equilibrium theory predicts when market rushes occur. Do they occur only when predicted or do they also occur when they cannot be an outgrowth of equilibrium forces? To take the theoretical predictions of the model to the data, I decompose them into two nested hypotheses, one a weak test and the other stronger. My first hypothesis derives from a simple comparative static prediction of the theory: rushing should be more severe (that is, trading times should be earlier) in Rush than Wait.

**Hypothesis 1 (Comparative Static)** *Trades occur earlier in treatment R than in treatment W.*

My second hypothesis derives from the point predictions of the model for the two treatments and represents a more strenuous test of the theory: the Rush treatment should generate a full market rush while the Wait treatment should induce no rushing at all.

**Hypothesis 2 (Point Predictions)** *Trades occur predominantly at \( t = 1 \) in treatment R and at \( t = 8 \) in treatment W.*

### 3.4 Results

In Section 3.4.1, I begin by comparing the timing decisions across treatments, and show that subjects rush more severely in the R than the W treatment in accordance with Hypothesis 1. In contrast, the stronger point predictions of Hypothesis 2 are only partially supported: while full market rushes occur in treatment R as hypothesized, frequent rushing occurs even in treatment W in contrast to the hypothesis. I then show that rushes result in informational inefficiencies in both treatments. In Section 3.4.2, I provide evidence that the unpredicted rushing in the W treatment results from subjects employing a simple heuristic often attributed to momentum traders. In Section 3.4.3, I show that simple learning rules explain the patterns observed across the two treatments. Finally, in Section 3.4.4, I show that the momentum-like heuristic subjects use in the experiment produces positive short-term correlations in returns, a phenomenon commonly observed in naturally-occurring financial markets.

When reporting results, I report ‘Early’ behavior using data from the first five trials and ‘Late’ behavior using data from the last five trials. The qualitative nature of the results is
unchanged if I instead use the first and last ten trials.

### 3.4.1 Trade Timing and Informational Inefficiencies

Figure 2 plots the empirical cumulative distribution functions (CDFs) of the times at which trades occur across all subjects for each treatment. In each panel, I include the Late behavior of experienced subjects (plotted as solid lines) and, for reference, Early behavior (as dotted lines).

**Figure 2: Trade Timing**

<table>
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<tr>
<th>Period</th>
<th>R CDF</th>
<th>W CDF</th>
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</table>

Note: Cumulative fraction of trades that occur prior to each period for treatments R (left) and W (right). Solid lines are for the last five trials and dashed lines for the first five.

I begin by evaluating Hypothesis 1, focusing on the Late behavior of experienced subjects. Although subjects initially trade at similar times across the two treatments (Early CDFs), with experience (Late CDFs) they learn to make very different timing decisions, as predicted: in treatment R subjects overwhelmingly learn to trade at the very beginning of the game (with 80% moving in period 1), while in treatment W subjects continue to delay trade significantly (with the median subject delaying trade until halfway into the game) even after dozens of periods of experience. A Kolmogorov-Smirnov test allows me to reject the null of equal empirical CDFs in the Late data (p-value = 0.01), supporting Hypothesis 1 and providing a first finding:  

**Finding 1** Subjects learn to trade earlier in treatment R than in treatment W, supporting Hypothesis 1.

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16 Alternatively, the median trading periods in the four sessions of R and W are \{1, 1, 1, 1\} and \{2, 3, 4, 4\}, respectively. Applying a non-parametric Mann-Whitney U test rejects the null of equal median trading periods (test statistic = 0, p-value = 0.03).
Although Figure 2 shows strong evidence in support of Hypothesis 1, it reveals decidedly mixed evidence in support of Hypothesis 2, even in the last 5 trials of the experiment. In agreement with Hypothesis 1, subjects overwhelmingly (80% of the time) rush to trade in period 1 in Late trials of the R treatment, with subjects scrambling to be the first to trade in all four sessions of the treatment (the median trading period is 1 in every session in Late trials). By contrast, subjects in the W treatment rarely (15% of the time) delay trade until the final period in treatment W, instead trading at heterogeneous times throughout the game. Thus, rushing behavior occurs reliably when predicted by theory, but also occurs (though to a less extreme extent) when it cannot be supported by equilibrium forces. We report this partial failure of Hypothesis 2 as a second finding:

Finding 2: Subjects generally trade in period 1 in treatment R, but they rarely delay trade until period 8 in treatment W. This pattern of behavior only partially supports Hypothesis 2.

Rushing generates both predicted and unpredicted informational efficiencies in the experimental markets. In the R treatment, when a trader rushes as predicted, she trades with information that is correct only 75% of the time, forgoing the opportunity to obtain perfect information. In fact, no subject (in the last five trials - 160 observations) ever waits to learn the asset value at \( t = 8 \), which generates informational efficiencies as high as 38%. In treatment W, where no inefficiencies are predicted to emerge, efficiencies are in fact even higher, as high as 93%.\(^{17}\) These informational inefficiencies are particularly surprising in the W treatment because, should any one of the eight traders wait to obtain perfect information (as they are predicted to do), prices would be fully efficient. We report these inefficiencies as a further result:

Finding 3: Rushing behavior generates informational inefficiencies in both the R treatment (where such inefficiencies are predicted) and the W treatment (where they are not).

3.4.2 Momentum-like Strategies

What decision rules account for the excess rushing observed in treatment W? One hypothesis is that subjects simply make their entry time decisions randomly: after all, both Early and Late behavior in the W treatment (and even Early behavior in the R treatment) are nearly

\(^{17}\)Here, I report the ex post inefficiency, measured as the absolute difference between the final price in the experiment and the true asset value, \( |V - p_T| \). Theoretical measures of inefficiency are typically ex ante \( (E(V - p_T)^2) \) or conditioned on the realized price \( (Var(V|p_T)) \), but because both \( V \) and \( p_T \) are known here, ex post efficiency is the most suitable measure.
uniformly distributed in Figure 2. An alternative possibility is that subjects do not condition their trading decision on time per se (as the theory predicts), but rather based on the strength of the information they have accumulated about the asset’s value.

To investigate, I define the strength of a subject’s information in period $t$ as: $\text{strength}_t = \max \{ 1 - P_r[V = 1|H_t, \xi_t], P_r[V = 1|H_t, \xi_t] \}$. Intuitively, a subject has stronger information the closer the posterior generated by her accumulated signals is to 0 or 1. In Figure 3, I plot in black the fraction of times a subject trades in a period in which her information strength is more extreme than in any previous period of the trial. For comparison, I also plot in gray the fraction of trades that occur at an information strength weaker than (or equal to) that previously attained.\(^{18}\) The plot shows that trade almost always occurs at a new threshold level of information strength: the black bars are much larger than the gray bars for each subset of the data. This pattern is clearly inconsistent with uniformly random decision-making: in all four cases, if subjects had made decisions in a uniformly random period (given the actual, realized signals), at least 98% of the time we would expect to observe a lower fractions of trades at new threshold levels of information strength than is observed in the data.\(^{19}\)

These results are consistent with the theory that subjects employ a threshold rule to determine timing, trading at the first moment their information justifies a sufficiently high level of confidence in the asset value (a threshold that, naturally, differs from subject to subject). One source of such rules are the momentum strategies commonly observed among traders in financial markets and discussed in the behavioral finance literature (Grinblatt, Titman, and Wermers (1995), Hong and Stein (1999), Grinblatt and Keloharju (2000), Brozynski et al. (2003), Baltzer, Jank, and Smajlbeogovic (2015), and Grinblatt et al. (2016)). A trader employing a classical momentum strategy, buys (sells) after observing a sequence of price increases (decreases), a behavior that requires setting a threshold change in prices before trading. With private information, the natural extension to this strategy is to trade once the strength of the totality of one’s information has changed by a sufficient amount (in Section 4, I discuss this connection further), and to trade in the direction suggested by the information (buy when information suggests $V = 1$ and sell otherwise). In Late behavior, 80% and 96% of trades are in the direction of a subject’s information (conditional on trading at

\(^{18}\)I omit results from period $T$ because subjects are forced to trade in this period and their information strength is perfect at this time.

\(^{19}\)I performed this calculation using simulations of 10,000 repetitions of the first and last 5 trials of the game using the actual public and private signal draws, and assuming trades are uniformly distributed across periods. The percentage of simulations with a lower fraction of trades at new threshold levels of information strength are 98% and 100% in Early trials of the W and R treatments, respectively, and 99% and 100% in Late trials.
Figure 3: Trades by Information Strength

Note: Each bar graph is the fraction of trades that occur when a subject’s information strength reaches a new threshold (black bars) and does not (gray bars).

a new threshold in information strength), in the W and R treatments, respectively. This evidence of ‘momentum-like’ strategies provides a fourth finding:

**Finding 4:** Subjects use momentum-like strategies consistently, waiting for sufficient changes in information and then trading in the direction suggested by the information.

Given the widespread use of momentum-like strategies, a natural question is whether or not such strategies form a best response to other subjects using these strategies. The answer is no. Given the empirical prices, waiting continues to be a best response: over all trials, the average profit generated by rushed trades is actually slightly negative (−0.6%), while the average profit generated by trades in the final period is 15.3%, very close to the theoretical expected profit of 15.9%.

3.4.3 Learning

As Figure 2 shows, the distributions of entry times in the W and R treatments are very similar initially (Early trials) \( (p = 0.93, \text{ a Kolmogorov-Smirnov test}) \), but become dramatically

---

\(^{20}\)Threshold strategies are also often employed in drift diffusion models (Ratcliff (1978)) in which one makes a decision once having accumulated a threshold level of information. Here, and perhaps more broadly, one could interpret momentum strategies as resulting from this deeper, psychological information-processing mechanism.
different across treatments as subjects gain experience (Late trials). Why do subjects learn to behave so differently in the two treatments? Why do subjects learn to respect theoretical predictions in R but not in W? In this section I show that simple hill-climbing rules generate patterns very similar to the ones in the data, and discuss the intuition that this provides about the source of the large treatment effects we observe in the data.

I consider a simple learning model in which agents adjust behavior in response to the counterfactual earnings consequences of their recent past actions (see, for example the win-stay-lose-shift model of Nowak and Sigmund (1993) or learning direction theory of Selten and Buchta (1998)). Agents set a threshold for the strength of information that they require to trade (employing a momentum-like strategy as documented in the previous section), and adjust it from trial to trial towards thresholds that would have performed better in the previous trial. Intuitively, if a subject buys a good asset ($V = 1$) when she uses a particular threshold, she could have done better by setting a lower threshold and buying the asset sooner. On the other hand, if she buys a bad asset ($V = 0$), she would have have typically done better by waiting for more information by setting a higher threshold. To simulate this learning process, I assume agents adjust over a small number of discrete thresholds (I use thresholds corresponding to the net number of favorable public signals). An agent updates her threshold by considering what would have happened had she used a different threshold in the previous trial, and stochastically choosing among her current threshold and any threshold that would have earned strictly more. To limit the model’s degrees of freedom, I simulate using the simplest possible rule: agents uniformly randomize over the candidate thresholds.

Figure 4 plots CDFs of the Late (final five trials) trading times from 500 simulations of 30 trials using the actual parameters from each treatment of the experiment. For reference, I also reproduce the actual CDFs from the experiment. Although the simulations are seeded with virtually identical initial first trial behaviors and identical adjustment rules, they result in comparative statics (across treatments) and distributions (within treatments) that look almost identical to the actual data. Thus, this simple learning model provides my fifth finding:

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22 After buying a bad asset, a lower threshold would also have done better because it results in a smaller loss. Other possibilities also exist, but occur infrequently. For example, even after a correct decision, there is a small probability that the price is lower in period $T$ such that a higher threshold that induces waiting until period $T$ does better.

23 Many other variations of the model produce similar results. For example, probabilistic weights can be assigned to thresholds based on how much they would have earned. Or, one can allow for noise, placing some probability mass on thresholds that would have earned less than the current threshold. Rather than fine-tuning the results by adding degrees of freedom, I chose the most parsimonious model.
Finding 5: A simple hill-climbing threshold adjustment rule generates the difference across treatments.

Guided by the intuition from this simple learning model, we can ask what makes learning to wait in the W treatment so difficult? The reason lies in the feedback of the rewards. If a subject follows a momentum strategy, more often than not she earns a positive amount because prices tend to move in the direction of the true asset value. Thus, frequent positive rewards tend to reinforce trading with the same threshold or perhaps lowering one’s threshold to try to earn more. It is only in the less frequent case that prices initially move in the wrong direction that a subject learns that it would have been better to wait to learn the asset value. The relative infrequency of losses that encourage waiting thus makes it difficult to recognize that a momentum strategy is not optimal, even after participating in dozens of trials. Learning to rush in the R treatment is much easier because trading immediately leads to positive earnings very frequently (75% of the time). And, waiting even one period results in a much smaller profit due to the large amount of information aggregated by others’ trades, which provides immediate feedback that trading earlier is optimal.²⁴

²⁴The reason subjects don’t learn to trade immediately in the W treatment is because of the differences in private signal quality across treatments. Waiting is not nearly so costly in the W treatment because others’ trades reveal very little information.
3.4.4 Correlations in Returns

One consequence of the momentum-like strategies documented above is the emergence in the data of another pattern often observed in real-world financial markets: positive short-term correlations in returns. Indeed, these correlations are among the most commonly studied phenomena in financial markets (see Daniel, Hirshleifer, and Subrahmanyum (1998) for a review), and are often given a behavioral explanation, including that traders use a price-chasing momentum strategy (Hong and Stein (1999)). Documenting these correlations in the experimental data therefore further reveals the role that these strategies play.

I focus on Late behavior in the W treatment, and, in particular, on the second trading period just after the first public signal is revealed. At this time, 86% of trades are in the direction of the first public signal. Table 2 provides the Spearman correlation coefficients between the return due to the first public signal and that due to trades in each of the next seven trading periods, excluding the last where the asset value is known.

Table 2: Correlation of Trading Returns in the W Treatment

<table>
<thead>
<tr>
<th>Trading Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman Correlation Coefficient</td>
<td>0.05</td>
<td>0.85***</td>
<td>0.13</td>
<td>0.09</td>
<td>-0.27</td>
<td>0.09</td>
<td>0.57***</td>
</tr>
<tr>
<td>[0.83]</td>
<td>[0.00]</td>
<td>[0.59]</td>
<td>[0.72]</td>
<td>[0.25]</td>
<td>[0.70]</td>
<td>[0.01]</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Correlations between the return due to the first public signal and those due to trade in subsequent periods. p-value of two-tailed t-test included in brackets. Each correlation is over 20 trials (last 5 trials of each of 4 sessions).

In Table 2, we observe a very strong positive short-term correlation between the return

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25In an environment with private information, chasing price trends leads to informational herding: subjects trade in the direction of the price trend independently of their private signal. Herding, broadly defined, has also been considered an explanation for correlations in returns (see Devenow and Welch (1996) and Hirshleifer and Teoh (2003) for reviews of herding in the context of financial markets). Whether one considers momentum-like strategies and herding to be distinct explanations for the behavior observed here depends upon one’s definition of herding. Rational informational herding of the sort first discussed by Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992) is precluded because subjects should rationally trade according to their private signals (Proposition 1). Herding in the sense of simple imitation could perhaps provide an alternative explanation, but, in a related setting, Kendall (2017b) provides evidence that subjects follow momentum-like strategies even when there are no previous subjects to imitate. Note finally that informational herding only partially explains the correlations in returns in the data. The remaining part is driven by subjects whose signals oppose the price trend choosing to wait more often that those whose signal confirms the price trend, consistent with the first group being less certain about the asset value.

26Because the vast majority of trades occur in the first trading period in treatment R, any correlation due to price-chasing behavior is drastically weakened. The second period of treatment W provides the cleanest test for correlations because correlations due to the use of higher thresholds are spread over multiple trading periods.

27The results are very similar if the Pearson correlation coefficient is used, but there is no a priori reason to expect the correlations to be linear.
due to the first public signal and that due to the trades in the following trading period. Were traders simply rushing and trading according to their signals, prices would form a martingale and such a correlation would be statistically improbable. This correlation therefore provides further evidence that subjects chase price trends on average. Finding 6 summarizes the correlation result.


4 Conclusion

Rushed trade in financial markets is of perennial concern to both social scientists and policy makers, generating pathologies like panics and frenzies that can lead to serious losses in informational efficiency. I theoretically show that these types of episodes can arise as equilibrium outcomes by rational traders in a variation of a standard, workhorse model of trade timing (Glosten and Milgrom (1985)). I then report a theoretically-structured experiment designed to study whether market rushes can also arise due to less rational motives (in settings where they are not rational), as market observers have long suggested (Mackay (1841)). In the laboratory experiment, I study a parameterization (Rush) in which market rushes are the only equilibrium outcome and another (Wait) in which market rushes are never an equilibrium outcome. As the theory predicts, subjects trade much earlier in the Rush treatment than they do in the Wait treatment, generating market rushes consistent with the finding of Dufour and Engle (2000) that informed trades tend to cluster together, and the fact that volumes spike around earnings announcements (Frazzini and Lamont (2006)). However, in contrast to the theoretical predictions, I observe significant rushing, and its accompanying informational efficiencies, even in the Wait condition where it cannot be a consequence of individually rational behavior.

The experimental data reveals that excessive rushing is caused by the widespread use of momentum-like strategies by subjects: instead of choosing when to trade based upon forward-looking concerns about future information flows, subjects instead trade after their accumulated, past information makes them sufficiently certain of the asset value. In both treatments, this momentum-like threshold rule leads most subjects to rush to trade before becoming fully informed. This suboptimal strategy is remarkably resistant to learning, per-

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28 For the latter, the assumption is that heterogeneous abilities to process information turn public announcements into private information (Holthausen and Verrecchia (1990), Indjejikian (1990), and Kandel and Pearson (1995)).
sisting in subjects even after dozens of games of experience. Subjects do, however, adjust the level of the information threshold they require for trade, and by following simple hill-climbing rules, eventually learn to make significantly different timing decisions across treatments.

These information-based threshold rules appear to be a variation of the familiar momentum strategies that have long been documented in financial markets both for mutual funds (Grinblatt, Titman, and Wermers (1995), Brozynski et al. (2003), Baltzer, Jank, and Smajlbegovic (2015), and Grinblatt et al. (2016)) and individual investors (Grinblatt and Keloharju (2000)). (Indeed, momentum strategies cause short-run correlations in returns in the experimental data, just as has been suggested for real markets (De Long et al. (1990) and Hong and Stein (1999)).) One potential implication of the experimental results is that momentum strategies may not (only) arise from the use of technical analysis, but may also derive from a deeper behavioral source - after all, unlike practitioners in the field, few of our undergraduate subjects are likely to be formally employing technical analysis when making trading decisions.

One potential such ‘behavioral’ source of these strategies is systematic mistakes in the formation of beliefs (as in Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, and Subrahmaniam (1998)). For instance, subjects may be overextrapolating from recent trends (Jin (2015), Barberis, Greenwood, Jin, and Shleifer (2015,2016)), for which survey evidence exists (Greenwood and Shleifer (2014)). Here, such overextrapolation would cause a trader to form too-certain beliefs about the asset value, inducing trade in the direction of accumulated information, in line with the data. Another possible source of momentum-like strategies is non-expected utility preferences. Kendall (2017b) considers a simpler (Glosten-Milgrom) environment in which traders have private information but trade sequentially. There, I show theoretically that prospect theory preferences (Kahneman and Tversky (1979,1992)) cause traders to buy when prices rise and sell when prices fall, and subsequently confirm this hypothesis in an experiment designed to rule out belief-based explanations.29

One promising avenue for future work is the development of experiments to better understand whether momentum-like strategies derive from preferences or from beliefs. Another promising extension is to study richer environments to better understand the consequences of these strategies. In order to clearly identify rushing behavior and the strategies generating it, the model and the experiment reported in this paper both restrict traders to a single trade. This constraint could be relaxed to examine whether relatively sophisticated subjects exploit

29In Appendix B, I show that common explanations for departures from standard predictions such as risk aversion and popular behavioral game-theoretic models such as quantal response equilibrium (McKelvey and Palfrey (1995,1998)), level k reasoning (Stahl and Wilson (1994) and Nagel (1995)), cognitive hierarchy (Camerer, Ho, and Chong (2004)), and cursed equilibrium (Eyster and Rabin (2005)) are inconsistent with the results reported here.
momentum strategies to their advantage. For example, in settings in which multiple trades are possible, rational traders may buy in an attempt to induce purchases by those that follow momentum strategies. If successful, the rational traders can then profitably sell at a higher price. In this way, the strategies observed here may help to explain price bubbles, both in the field and in laboratory experiments (e.g. Smith, Suchanek, and Williams (1988)).

References


Appendix

A: Theoretical Analysis and Omitted Proofs

A1. Preliminaries

The solution concept is sequential equilibrium and I restrict attention to Markov strategies which are a function only of the payoff-relevant state.\(^{28}\) To determine her optimal timing decision, a trader compares the expected profit from trading in the current period to the expected profit from following some future plan. The difference in expected profits, the benefit to waiting, in turn depends upon the optimal trading strategies that the trader follows conditional on reaching each particular information set in the future. I denote the equilibrium probability that a trader of type \(s_i = x\) for \(x \in \{0, 1\}\) is believed to not trade in the current period, \(\beta_x\). For arbitrary beliefs, Lemma A1 provides the optimal trading strategies in the current period and the sequentially rational trading strategies at any future information set on or off of the equilibrium path.\(^{31}\) \(\hat{q}\) is used to denote the quality of private information received in the future period. When no such information is received, we can model it as a hypothetical signal, \(\pi_i\), with quality \(\frac{1}{2}\) so that \(\hat{q} = \bar{q}\) in period \(T\) and \(\hat{q} = \frac{1}{2}\) otherwise.

\(^{28}\)Technically, the market maker is not a strategic player but I describe the price formation process as if the market maker has beliefs about the strategies of the players. After an off-equilibrium wait decision, I assume the price is unchanged (see the text following Lemma A2) until the trade is observed, at which point it is updated according to the sequentially rational trading decision (Lemma A1).

\(^{31}\)I implicitly assume that the timing strategies of the two types of traders differ only in the current period such that, when \(\beta_0\) and \(\beta_1\) differ, the same information is revealed to the market on every future path. I show below using a backwards induction argument that this assumption is without loss of generality.
Lemma A1: Optimal trading decisions:

1. any trader who trades in the current period buys if \( s_i = 1 \) and sells if \( s_i = 0 \).
2. any trader who trades in a future period, trades as follows:
   (a) buys if \( \bar{s}_i = 1 \) and \( \bar{\pi}_i = 1 \); sells if \( \bar{s}_i = 0 \) and \( \bar{\pi}_i = 0 \).
   (b) buys (sells) if \( \bar{s}_i = 0, \bar{\pi}_i = 1 \), and \( q_0(q, \hat{q}) \equiv (1 - \hat{q})\hat{q}NT_0 - \hat{q}(1 - \hat{q})NT_1 \geq (\leq) 0 \).
   (c) buys (sells) if \( \bar{s}_i = 1, \bar{\pi}_i = 0 \), and \( q_1(q, \hat{q}) \equiv \hat{q}(1 - \hat{q})NT_0 - (1 - \hat{q})\hat{q}NT_1 \geq (\leq) 0 \)

where \( NT_0 = (1 - \hat{q})\beta_1 + \hat{q}\beta_0 \), \( NT_1 = \hat{q}\beta_1 + (1 - \hat{q})\beta_0 \) are shorthand for the probabilities, from the market maker's perspective, of observing no trade in the current period conditional on \( V = 0 \) and \( V = 1 \), respectively.

Proof of Lemma A1:

The proof when a trader trades prior to \( T \) is similar to that she trades at \( T \), only simpler, so I provide only the latter. Let \( \hat{e} \in E \) denote the set of realizations of public events that may occur during the time a trader waits (due to public signals or trades by other traders) other than from trader \( i \) herself not trading (denoted \( NT \)), and abbreviate \( Pr(\hat{e}|V = y) \) as \( \hat{e}_y \), for \( y \in \{0, 1\} \). A trader who waits buys if her expected value of the asset conditional on her information in the future period is greater than the future price. Denoting the current history, \( H_t \), the current price \( p_t \), and the future price, \( Pr(V = 1|\hat{e}, NT, H_t) \), a trader buys if

\[
Pr(V = 1|\bar{s}_i, \bar{\pi}_i, \hat{e}, H_t) > Pr(V = 1|\bar{e}, NT, H_t)
\]

\[
\iff p_tPr(\bar{s}_i|V = 1)Pr(\bar{\pi}_i|V = 1)\hat{e}_1
\]

\[
> p_t\hat{e}_1Pr(NT|V = 1) + (1 - p_t)\hat{e}_0Pr(NT|V = 0)
\]

\[
\iff Pr(\bar{s}_i|V = 1)Pr(\bar{\pi}_i|V = 1)NT_0 > Pr(\bar{s}_i|V = 0)Pr(\bar{\pi}_i|V = 0)NT_1
\]

(1)

where the first equivalence follows from applying Bayes' rule to each side of the inequality and using the fact that the public belief is the initial price, \( Pr(V = 1|H_t) = p_t \). Using (1), a trader with \( s_i = 1 \) and \( \pi_i = 1 \) buys if

\[
\hat{q}\hat{q}NT_0 > (1 - \hat{q})(1 - \hat{q})NT_1
\]

\[
\iff \hat{q}(1 - \hat{q})(2\hat{q} - 1)\beta_1 + (\hat{q}^2\hat{q} - (1 - \hat{q})^2(1 - \hat{q}))\beta_1 > 0
\]

which is true for all parameterizations. Similarly, a trader with \( s_i = 0 \) and \( \pi_i = 0 \) always sells. Finally, the conditions in Lemma A1 under which traders with \( s_i = 1 \) and \( \pi_i = 0 \), or \( s_i = 0 \) and \( \pi_i = 1 \), buy or sell are easily obtained by substituting for the appropriate probabilities into (1).□

Given Lemma A1, I derive a general function for the benefit from waiting (the expected profit from trading at some subset of future paths, less the profit from trading in the current period). When this benefit is positive (negative), a trader optimally waits (rushes). The benefit depends upon the market maker’s beliefs about the probability each type of trader
waits because information is revealed if these probabilities differ across types. In equilibrium, the two probabilities must be consistent with the sign of the benefit for each type of trader. I denote the benefit from waiting, \( B_x(p_t, \beta_0, \beta_1) \), for traders with \( s_x = x \).

Consider first the expected profit of a trader who trades in the current period, \( t \), with \( s_t = 1 \). From Lemma A1, she buys so that her profit is given by

\[
Pr(V = 1|s_t = 1, H_t) - p_t \iff \frac{p_t q}{p_t q + (1 - p_t)(1 - q)} - p_t \iff \frac{p_t(1 - p_t)(2q - 1)}{p_t q + (1 - p_t)(1 - q)}
\]

The profit for a trader with \( s_t = 0 \) is calculated similarly, using the fact that she sells. The current profit for both types of traders can be written \( \frac{p_t(1 - p_t)(2q - 1)}{Pr(s_t)} \).

I calculate the expected profit from a planned set of future trades for a trader with \( s_t = 1 \) who buys when receiving \( \bar{s}_t = 1 \) and sells when \( \bar{s}_t = 0 \) \((g_t(q, \bar{q}) \leq 0)\). The other cases are calculated similarly and are omitted for brevity. The expected profit is

\[
\sum_{\hat{e} \in E} \left\{ Pr(\bar{s}_t = 1|\hat{e}|s_t = 1) \left[ Pr(V = 1|\bar{s}_t = 1, \bar{s}_t = 1, \hat{e}) - Pr(V = 1|NT, \hat{e}) \right] + Pr(\bar{s}_t = 0|\hat{e}|s_t = 1) \left[ Pr(V = 1|NT, \hat{e}) - Pr(V = 1|s_t = 1, \bar{s}_t = 0, \hat{e}) \right] \right\}
\]

where all probabilities are also conditional on \( H_t \). The first term corresponds to the profit from buying the asset after receiving \( \bar{s}_t = 1 \) and the second term from selling after receiving \( \bar{s}_t = 0 \). The sum is over all possible realizations of the public events, which may depend upon the trader’s future course of action.\(^{32}\) Using Bayes’ rule and the independence of signals, the above becomes

\[
\sum_{\hat{e} \in E} \left\{ \frac{Pr(\bar{s}_t = 1|\bar{s}_t = 1, \hat{e})}{Pr(\bar{s}_t = 1)} \left( \frac{p_t q \hat{q} Pr(\hat{e}|V = 1)}{Pr(\bar{s}_t = 1, \bar{s}_t = 1, \hat{e})} - \frac{p_t NT_1 Pr(\hat{e}|V = 1)}{Pr(NT \& \hat{e})} \right) + \frac{Pr(\bar{s}_t = 0|\bar{s}_t = 1, \hat{e})}{Pr(\bar{s}_t = 1)} \left( \frac{p_t NT_1 Pr(\hat{e}|V = 1)}{Pr(NT \& \hat{e})} - \frac{p_t q (1 - \hat{q}) Pr(\hat{e}|V = 1)}{Pr(\bar{s}_t = 0, \bar{s}_t = 1, \hat{e})} \right) \right\}
\]

with \( NT_0 \) and \( NT_1 \) as in Lemma A1. Simple algebra results in

\[
\frac{p_t(1 - p_t)}{Pr(s_t = 1)} \sum_{\hat{e} \in E} \frac{\hat{e}_0 \hat{e}_1}{Pr(NT \& \hat{e})} \left( (2\hat{q} - 1)(q NT_0 + (1 - q)NT_1) \right)
\]

Finally, subtracting the profit at \( t \) from the expected profit and factoring out common terms gives the benefit.

For each type of trader and each possible future signal and trading strategy, the benefit

\(^{32}\)For example, the trader could trade after a favorable public signal, but wait again after an unfavorable public signal. In this case, the set of public events due to the public signals is \( E = \{s_{P,t} = 1, s_{P,t} = 0\&s_{P,t+1} = 0, s_{P,t} = 0\&s_{P,t+1} = 1\} \)
can be written

\[ B_x(p_t, \beta_0, \beta_1) = \frac{p_t(1 - p_t)}{Pr(s_t = x)} \left[ \sum_{e \in E} \hat{e}_0 \hat{e}_1 f(q, \hat{q}, \beta_0, \beta_1) \cdot Pr(e & NT) - (2q - 1) \right] \]  

(2)

where

\[ f(q, \hat{q}, \beta_0, \beta_1) = \begin{cases} 
(2\hat{q} - 1)(qNT_0 + (1 - q)NT_1) & \text{if } s_t = 1 \& g_1(q, \hat{q}) \leq 0 \\
qNT_0 - (1 - q)NT_1 & \text{if } s_t = 1 \& g_1(q, \hat{q}) > 0 \\
qNT_1 - (1 - q)NT_0 & \text{if } s_t = 0 \& g_0(q, \hat{q}) < 0 \\
(2\hat{q} - 1)(qNT_1 + (1 - q)NT_0) & \text{if } s_t = 0 \& g_0(q, \hat{q}) \geq 0 
\end{cases} \]

Note that \( \hat{q} \) in (2) depends on the information set reached (and perhaps, therefore, on the realization, \( \hat{e} \), that occurs), but I suppress this dependence in the notation for simplicity. Given the generic formula for the benefit, Lemma A2 establishes that both types of traders must follow the same timing strategies in any equilibrium. Importantly, this result holds without any assumption on off-equilibrium beliefs.

**Lemma A2:** In any equilibrium, traders with \( s_t = 0 \) and \( s_t = 1 \) must follow the same timing strategy at every history.

**Proof of Lemma A2:**

The proof uses backwards induction. In particular, I show that the claim holds in an arbitrary period provided it holds for all subsequent periods, and also that it must hold in period \( T - 1 \).

For a given period, the proof is by contradiction. Assume that there exists an equilibrium in which the two types of traders are believed to wait with different probabilities (\( \beta_1 \neq \beta_0 \)) in period \( t \) after some history, \( H_t \). If the period is period \( T - 1 \), the two types of traders are believed to arrive in period \( T \) with the corresponding probabilities. In periods prior to \( T - 1 \), the two types of traders are also believed to arrive in all future periods with these same probabilities, once we have established that they use the same timing strategy in all periods greater than \( t \).

Without loss of generality, assume \( \beta_1 > \beta_0 \). We must have \( \beta_1 \in (0, 1) \) and therefore \( B_1(p_t, \beta_0, \beta_1) \geq 0 \) such that the type 1 trader waits with positive probability consistent with the belief. I establish below that \( \beta_1 > \beta_0 \) and \( B_1(p_t, \beta_0, \beta_1) \geq 0 \) together imply \( B_0(p_t, \beta_0, \beta_1) > 0 \). However, \( \beta_1 > \beta_0 \) also implies \( \beta_0 \in [0, 1) \) and therefore \( B_0(p_t, \beta_0, \beta_1) \leq 0 \), a contradiction. Therefore, we must have \( \beta_1 = \beta_0 \) in any equilibrium.

To see \( \beta_1 > \beta_0 \) and \( B_1(p_t, \beta_0, \beta_1) \geq 0 \) together imply \( B_0(p_t, \beta_0, \beta_1) > 0 \), note from (2) that the sign of \( B_x(p_t, \beta_0, \beta_1) \) is determined by the term in square brackets and that the only difference across types is due to differences in the functions, \( f(q, \hat{q}, \beta_0, \beta_1) \) which depend upon whether or not new private information is received. I show that, in each case, \( f(q, \hat{q}, \beta_0, \beta_1) \) is strictly greater for type 0 than for type 1 whenever \( \beta_1 > \beta_0 \).

First, consider an information set in which no new information is received (\( \hat{q} = \frac{1}{2} \)). In this case, \( g_0(q, \hat{q}) < 0 \) and \( g_1(q, \hat{q}) > 0 \). Comparing \( f(q, \hat{q}, \beta_0, \beta_1) \) in \( B_0(p_t, \beta_0, \beta_1) \) relative
to $B_1(p_t, \beta_0, \beta_1)$ gives $qNT_1 - (1 - q)NT_0 > qNT_0 - (1 - q)NT_1 \iff NT_1 - NT_0 > 0 \iff (\beta_1 - \beta_0)(2q - 1) > 0$ so $f(q, \bar{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$.

Now consider an information set in which new information is received ($\hat{q} = \bar{q}$). There are four possible cases depending upon the optimal strategies of buying and selling from Lemma A1.

1. $g_1(q, \bar{q}) \leq 0$ and $g_0(q, \bar{q}) \geq 0$. The comparison of $f(q, \bar{q}, \beta_0, \beta_1)$ in $B_0(p_t, \beta_0, \beta_1)$ relative to $B_1(p_t, \beta_0, \beta_1)$ is $qNT_1 + (1-q)NT_0 > qNT_0 + (1-q)NT_1 \iff (NT_1 - NT_0)(2q - 1) > 0 \iff (\beta_1 - \beta_0)(2\bar{q} - 1)^2 > 0$ so $f(q, \bar{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$.

2. $g_1(q, \bar{q}) > 0$ and $g_0(q, \bar{q}) < 0$. This case is identical to the $\hat{q} = \frac{1}{2}$ case.

3. $g_1(q, \bar{q}) \leq 0$ and $g_0(q, \bar{q}) < 0$. Note that $g_0(q, \bar{q}) < 0 \implies qNT_1 - (1-q)NT_0 > (2\bar{q} - 1)(qNT_1 + (1-q)NT_0).$ This can be seen algebraically or simply by noting that $B_0(p_t, \beta_0, \beta_1)$ must be larger when $t$ follows her optimal trading strategy instead of the optimal strategy for the other case, $g_0(q, \bar{q}) \geq 0$. But, then we have, $qNT_1 - (1-q)NT_0 > (2\bar{q} - 1)(qNT_1 + (1-q)NT_0) > (2\bar{q} - 1)(qNT_0 + (1-q)NT_1)$ where the second inequality was shown in the first case above, so $f(q, \bar{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$.

4. $g_1(q, \bar{q}) > 0$ and $g_0(q, \bar{q}) \geq 0$. $g_0(q, \bar{q}) \geq 0$ implies $(2\bar{q} - 1)(qNT_1 + (1-q)NT_0) > qNT_1 - (1-q)NT_0$ by optimality of the future trading decision and $qNT_1 - (1-q)NT_0 > qNT_0 - (1-q)NT_1$ as shown for the $\hat{q} = \frac{1}{2}$ case. Thus, $f(q, \bar{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$.

This completes the claim that $\beta_1 > \beta_0$ and $B_1(p_t, \beta_0, \beta_1) \geq 0$ together imply $B_0(p_t, \beta_0, \beta_1) > 0$ and therefore completes the proof.\]

Given that, in equilibrium, both types of traders must follow the same timing strategy, the equilibrium trading strategies of Lemma A1 simplify (setting $\beta_0 = \beta_1$ in the formulas), resulting in Proposition 1.

The benefit function simplifies as well. $f(q, \hat{q}, \beta_0, \beta_1)$ simplifies to $\beta(2\hat{q} - 1)$ when $\hat{q} > q$ and $\beta(2q - 1)$ when $\hat{q} < q$, reflecting the fact that a trader follows her better quality signal when they are contradictory. The denominator simplifies to $Pr(\hat{e} \neq q_t = NT) = \beta Pr(\hat{e})$ so that $\beta$ cancels in the numerator and denominator. Denote a trader’s better quality signal at the information set corresponding to event $\hat{e}$, $q \equiv max(q, \hat{q})$ (again, I suppress the dependence of $q$ on $\hat{e}$ for notational ease). We can then write the general benefit function,

$$B_x(p_t) = \frac{p_t(1 - p_t)}{Pr(\hat{e} = x)} \left[ \sum_{\hat{e} \in E} \hat{e}_0 \hat{e}_1 \frac{(2q - 1)}{Pr(\hat{e})} - (2\hat{q} - 1) \right] \tag{3}$$

Thus far, I have not made any assumption about off-equilibrium beliefs, which are relevant to pin down the benefit a trader gets from deviating when she rushes in equilibrium. Given the result of Lemma A2, a natural assumption is that any unexpected wait decision is equally likely to be from either type of trader (as it must be on the equilibrium path). I impose this assumption in the remainder of the analysis, but note that it is relevant only in the construction of the equilibrium in which all traders rush in the first period. There, the beliefs of other traders are irrelevant because the sufficient condition ensures it is a dominant strategy for a trader to trade in the first period. The belief of the market maker
is, however, relevant for setting the price. This assumption ensures prices remain unchanged after an unexpected wait decision, consistent with the price-setting rule in the experiment. Given this assumption, the optimal trading strategies after a deviation and the benefit from deviation are given by Lemma A1 and (3), respectively.

The benefit from waiting from the current period until period \( T \) at all possible future information events plays an important role, so denote this benefit, \( B_t(p_t) \). This benefit still depends upon a trader’s type, \( x \in \{0, 1\} \), but since only the sign of the benefit matters and the type simply scales the benefit, I drop the type from the notation for simplicity. This benefit is given by (3) with \( q = \bar{q} \). It is also convenient to denote the expected profit from this strategy as \( \Pi_t(p_t) \). Note that we can write this expected profit as the expectation over the expected profits from the trades in period \( T \) \( \Pi_t(p_t) = \sum_{\hat{e} \in E} Pr(\hat{e})\pi(p_t, \bar{q}) \). Here, \( p_e \equiv Pr(V = 1|\hat{e}) = \frac{p_{\hat{e}_1}}{p_{\hat{e}_1} + (1 - p_{\hat{e}_1})e_0} \) is the price conditional on the event, \( \hat{e} \), and \( \pi(p_t, q) = \frac{p_t(1 - p_t)}{Pr(\hat{e} = x)} (2q - 1) \) is the expected profit from a trade with a private signal of quality \( q \) and a price of \( p_t \). Finally, note that \( \pi(p_t, q) \) and \( \Pi_t(p_t) \) are strictly concave in \( p_t \) (peaking at \( p_t = \frac{1}{2} \)), which can be proven in an identical manner to Lemma 2 of Kendall (2017a).

Lemma A3 establishes that public information revealed through a public signal or another’s trade always reduces the benefit to waiting until \( T \).

**Lemma A3:** For any \( p_t \in (0, 1) \) and any \( t \in (1, T - 1) \), the benefit to waiting, \( B_t(p_t) \), for a particular trader strictly decreases with each additional (i) public signal, or (ii) trade by another trader.

**Proof of Lemma A3:**

First note that public signals are informative events, as are trades by other traders (by Proposition 1). I show that the expected profit from waiting strictly decreases with any informative event which implies that the benefit does as well (since the expected profit from trading immediately is unchanged). With either a trade by another trader or an additional public signal, the expected profit from waiting in period \( t \) can be written as the expectation over the expected profit in the absence of the event, which I denote, \( \Pi_t'(p_e) \). We have

\[
\Pi_t(p_t) = \sum_{\hat{e} \in E} Pr(\hat{e})\Pi_t'(p_e) < \Pi_t'(\sum_{\hat{e} \in E} Pr(\hat{e})p_e) = \Pi_t'(p_t)
\]

where the first inequality follows from Jensen’s inequality using the fact that \( \Pi_t'(p_t) \) is strictly concave and an informative event implies \( p_e \neq p_t \) from some realization, \( \hat{e} \). The second equality follows from the martingale property of prices, \( \sum_{\hat{e} \in E} Pr(\hat{e})p_e = \sum_{\hat{e} \in E} Pr(\hat{e})\frac{p_{\hat{e}_1}}{p_{\hat{e}_1} + (1 - p_{\hat{e}_1})e_0} = p_t \sum_{\hat{e} \in E} \hat{e}_1 = p_t \), where \( \sum_{\hat{e} \in E} \hat{e}_1 = 1 \) because \( E \) is the exhaustive set of all realizations of the informative event. □

Lemma A4 establishes an important property of the sign of the benefit from waiting until
period $T$. Intuitively, as studied in detail in Kendall (2017a), public information reduces the benefit most when $p_t = \frac{1}{2}$, because it reduces uncertainty in the asset value, on which profits depend, by the greatest amount at this time. Therefore, if the benefit in the presence of a public event is positive at $p_t = \frac{1}{2}$, it must be positive at all prices.

**Lemma A4:** For any $t \in (1, T-1)$ if the benefit from waiting until period $T$, $\mathcal{B}_t(\hat{p}_t) \leq 0$ for some $\hat{p}_t > \frac{1}{2}$, we must have $\mathcal{B}_t(p_t) < 0$ for all $p_t \in [\frac{1}{2}, \hat{p}_t)$. Similarly, if $\mathcal{B}_t(p_t) \leq 0$ for some $\hat{p}_t < \frac{1}{2}$, we must have $\mathcal{B}_t(p_t) < 0$ for all $p_t \in (\hat{p}_t, \frac{1}{2}]$.

**Proof of Lemma A4:**
Consider the derivative of the benefit function with respect to $p_t$:

$$\frac{\partial \mathcal{B}_t(p_t)}{\partial p_t} = \frac{1 - 2p_t}{p_t(1-p_t)} \mathcal{B}_t(p_t) - \frac{p_t(1-p_t)}{Pr(\hat{\epsilon}_i = x)} \sum_{\epsilon_i \in E} \hat{\epsilon}_0 \hat{\epsilon}_1 (2\hat{\epsilon}_1 - 1)(\hat{\epsilon}_1 - \hat{\epsilon}_0) Pr(\hat{\epsilon}_i)$$

Because of the symmetry property of signals, for each informative realization (trade or public signal), $\hat{\epsilon}_i \in E$, there is another realization, $\hat{\epsilon}_j$, such that $\hat{\epsilon}_{i,0} = \hat{\epsilon}_{j,1}$ and $\hat{\epsilon}_{i,1} = \hat{\epsilon}_{j,0}$. Therefore, we can write the expression for the derivative in terms of the sum over only those events for which $\hat{\epsilon}_{i,1} > \frac{1}{2}$. Denoting this set, $E'$, we have

$$\frac{\partial \mathcal{B}_t(p_t)}{\partial p_t} = \frac{1 - 2p_t}{p_t(1-p_t)} \mathcal{B}_t(p_t) - \frac{p_t(1-p_t)(2\hat{\epsilon}_1 - 1)}{Pr(\hat{\epsilon}_i = x)} \left[ \sum_{\epsilon_i \in E'} \hat{\epsilon}_{i,0} \hat{\epsilon}_{i,1} (\hat{\epsilon}_{i,1} - \hat{\epsilon}_{i,0}) + \hat{\epsilon}_{j,0} \hat{\epsilon}_{j,1} (\hat{\epsilon}_{j,1} - \hat{\epsilon}_{j,0}) \right]$$

where $\Sigma \equiv \sum_{i \in E'} \hat{\epsilon}_{i,0} \hat{\epsilon}_{i,1} (\hat{\epsilon}_{i,1} - \hat{\epsilon}_{i,0})$ is strictly positive because $\hat{\epsilon}_{i,1} > \hat{\epsilon}_{i,0}$ for all realizations in $E'$.

Now, assume there exists a $\hat{p}_t < \frac{1}{2}$ such that $\mathcal{B}_t(p_t) \leq 0$. In this case, $\frac{\partial \mathcal{B}_t(p_t)}{\partial p_t} < 0$, because the first term in (4) is weakly negative and the second term strictly so. Therefore, the benefit must strictly decrease as $p_t$ increases towards $p_t = \frac{1}{2}$, and, therefore must be strictly negative over $p_t \in (\hat{p}_t, \frac{1}{2}]$. A symmetric argument applies if there exists a $\hat{p}_t > \frac{1}{2}$ such that $\mathcal{B}_t(p_t) \leq 0$. \(\square\)

**A2. Equilibrium**

I now prove Proposition 2 (Proposition 1 was proven in Section A1). Define the benefit from waiting from $t = 1$ until $t = T$ when all other traders also wait until $t = T$ as $\mathcal{B}_1^W(\frac{1}{2})$.

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33 Uninformative realizations ($\hat{\epsilon}_{i,0} = \hat{\epsilon}_{i,1}$) contribute zero to the summation so can be ignored.
where the initial price, \( p_1 = \frac{1}{2} \). Similarly, define the benefit from waiting from \( t = 1 \) until \( t = T \) when all other traders trade prior to \( T \), \( B_1^R(\frac{1}{2}) \).

Part 1. The claim is that if \( B_1^W(\frac{1}{2}) < 0 \) for \( x \in \{0, 1\} \), in any equilibrium, all traders trade at \( t = 1 \). Suppose not. That is, suppose there exists an equilibrium in which at least one trader, \( i \), trades in period \( t > 1 \) on the equilibrium path. To be the case, trader \( i \) must plan to trade at some information set in a period \( t < T \), because if she instead plans to wait at all possible information sets, then her benefit from waiting at \( t = 1 \) is \( B_1^W(\frac{1}{2}) < 0 \) (or less if other traders also trade prior to \( T \), by Lemma A3) so that she trades in the first period, a contradiction. Therefore, let \( 1 < t < T \) denote the latest period in which trader \( i \) plans to rush (for any given strategies of the other traders). By Lemma A4, if trader \( i \) plans to rush at some price in this period (because after this period she waits at all information sets), then she must also plan to rush at \( p_i = \frac{1}{2} \).

Now, consider the expected profit from waiting at \( t = 1 \) given trader \( i \)'s strategy. We can decompose this expected profit into two sets of events prior to \( 1 \): those that lead to her rushing in period \( 1 \) or a prior period, \( E_R \), and those that have her wait until \( T \) to trade, \( E_W \). Her expected profit is

\[
\Pi_{1}(\frac{1}{2}) = \sum_{\tilde{e} \in E_R} Pr(\tilde{e})\pi(p_{\tilde{e}}, q) + \sum_{\tilde{e} \in E_W} Pr(\tilde{e})\Pi_{1}(p_{\tilde{e}})
\]

\[
< \sum_{\tilde{e} \in E_R} Pr(\tilde{e})\pi(\frac{1}{2}, q) + \sum_{\tilde{e} \in E_W} Pr(\tilde{e})\Pi_{1}(\frac{1}{2}) \tag{5}
\]

\[
\leq \sum_{\tilde{e} \in E_R} Pr(\tilde{e})\pi(\frac{1}{2}, q) + \sum_{\tilde{e} \in E_W} Pr(\tilde{e})\pi(\frac{1}{2}, q)
\]

\[
= \pi(\frac{1}{2}, q)
\]

The first inequality follows from the fact that both \( \pi(p_{\tilde{e}}, q) \) and \( \Pi_{1}(p_{\tilde{e}}) \) are strictly concave in \( p_{\tilde{e}} \), reaching maxima at \( p_{\tilde{e}} = \frac{1}{2} \) (it is strict because at least one event must lead to a price different from one half). The second inequality follows from the fact established above that the trader must plan to rush in period \( 1 \) when the price is one half: \( \pi(\frac{1}{2}, q) \geq \Pi_{1}(\frac{1}{2}) \). But, (5) implies a contradiction: the benefit from waiting under \( i \)'s strategy is strictly negative, so she must trade at \( t = 1 \). Because this argument applies for any strategy of the other traders, it is a dominant strategy for each trader \( i \) to trade in the first period, ensuring a unique equilibrium outcome (off-equilibrium, generally the sequentially rational strategies will involve both rushing and waiting, depending upon the price and number of remaining periods).

Part 2. The claim is that if \( B_1^R(\frac{1}{2}) > 0 \) for \( x \in \{0, 1\} \), then in the unique equilibrium, each trader waits at all histories in periods \( t = 1, 2, \ldots, T - 1 \), and trades in period \( T \). The argument is by backwards induction. At \( T - 1 \), less public signals (and possibly less trades by other traders) remain than at \( t = 1 \) so that, by Lemma A3, the benefit from waiting until \( T \) with \( p_{T-1} = \frac{1}{2} \), \( B_{T-1}(\frac{1}{2}) \), must be strictly greater than \( B_1^R(\frac{1}{2}) \) and therefore strictly greater than zero. Then, from the contrapositive of Lemma A4, the benefit of waiting until \( T \) must be strictly positive for all \( p_{T-1} \). Therefore, no matter how many trades have taken

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place or the current price, each trader must wait at $T - 1$. Given this, consider the benefit from waiting at $T - 2$ knowing one waits at $T - 1$. The same arguments apply as at $T - 2$ so that a trader must again wait at every possible information set. Iterating backwards in time, we arrive at $t = 1$ knowing that a trader waits in all future periods for all possible events. Then, because $B_1^R(\frac{1}{2}) > 0$ the trader must wait at $t = 1$: it is a dominant strategy, independent of the trade times of the other traders. This argument applies to each trader so that the unique equilibrium is for all traders to wait to trade until $T$. □

A3. Experiment Parameterizations

Using the sufficient conditions established in the proof of Proposition 2, I now consider the parameters of the two treatments of the experiment. Because only public signals affect $B_x^W(\frac{1}{2})$, we can rewrite it as

$$B_x^W(\frac{1}{2}) = \frac{1}{4Pr(\bar{z}_i = x)} \left[ \sum_{k=0}^{T-1} \frac{2C_0(k)C_1(k)(2\bar{q} - 1)}{C_1(k) + C_0(k)} - (2\bar{q} - 1) \right]$$

(6)

where $C_0(k) = \frac{(T-1)!}{k!(T-1-k)!} (1 - \bar{q})^k \bar{q}^{T-1-k}$ and $C_1(k) = \frac{(T-1)!}{k!(T-1-k)!} q_p^k (1 - q_p)^{T-1-k}$ are the probabilities of observing $k$ public signal realizations equal to 1, conditional on $V = 0$ and $V = 1$, respectively. In general, the sets of parameters $(\bar{q}, \bar{q}, q_p, T)$ that satisfy $B_x^W(\frac{1}{2}) < 0$ can be quite wide and, given the complexity of this benefit function, it is difficult to provide a simple characterization. However, for the parameters in treatment R: $\bar{q} = \frac{3}{4}$, $\bar{q} = 1$, $q_p = \frac{17}{24}$, and $T = 8$, we have $B_1^W(\frac{1}{2}) = B_0^W(\frac{1}{2}) \approx -0.075 < 0$. Thus, the sufficient condition to ensure in any equilibrium all traders trade immediately is satisfied.

As with $B_x^W(\frac{1}{2}) < 0$, $B_x^R(\frac{1}{2}) > 0$ is satisfied for many combinations of parameters. To evaluate $B_x^R(\frac{1}{2})$ for the parameters of treatment W ($\bar{q} = \frac{13}{24}$, $\bar{q} = 1$, $q_p = \frac{17}{24}$, and $T = 8$), I first rewrite the benefit when $n - 1$ trades and $T - 1$ public signals occur while waiting as

$$B_x^R(p_1) = \frac{1}{4Pr(\bar{z}_i = x)} \left[ \sum_{k=0}^{n-1} \sum_{j=0}^{T-1} \frac{C_0(k)C_1(k)D_0(j)D_1(j)(2\bar{q} - 1)}{p_1 C_1(k)D_1(j) + (1 - p_1) C_0(k)D_0(j)} - (2\bar{q} - 1) \right]$$

(7)

where $D_0(k) = \frac{(n-1)!}{j!(n-1-j)!} (1 - \bar{q})^j \bar{q}^{n-1-j}$ and $D_1(j) = \frac{(n-1)!}{j!(n-1-j)!} q_j (1 - q_j)^{n-1-j}$ are the probabilities of observing $j$ buys by the $n - 1$ other traders, conditional on $V = 0$ and $V = 1$ respectively, and $C_0(k), C_1(k)$ are as above. For the parameters of treatment W, we have $B_x^R(\frac{1}{2}) = B_0^R(\frac{1}{2}) \approx 0.128 > 0$, which satisfies the sufficient condition that ensures the unique equilibrium is to wait to trade at period $T$.

B: Risk Preferences and Behavioral Game Theory

B.1 Risk Preferences

Intuitively, risk-averse individuals prefer to wait to learn about the asset value, potentially even through public information, which is consistent with one aspect of the momentum-like strategy subjects use. However, given that it creates a desire to wait for information, it fails
to produce the price-chasing behavior characterized by waiting for several periods and then trading. Instead, risk-averse subjects would wait to trade until period $T$ in the W treatment. In addition, a risk-averse subject with a signal that opposes the price trend would never trade against her private signal to follow the trend as observed in the data.

Numerical simulations, assuming power utility, demonstrate that risk-seeking behavior can generate rushing the W treatment, provided the coefficient of relative risk aversion is very large. However, when risk-seeking produces rushing, it predicts that a subject will trade immediately in the first period, rather than at an intermediate period, which is again not consistent with a momentum-like strategy.

B.2 Behavioral Game Theory

Behavioral game theoretic explanations, such as quantal response equilibrium (McKelvey and Palfrey (1995,1998)), level k reasoning (Stahl and Wilson (1994) and Nagel (1995)), cognitive hierarchy (Camerer, Ho, and Chong (2004)), and cursed equilibrium (Eyster and Rabin (2005)) have been extremely successful in explaining behavior in experimental games. However, because these theories each rely on subjects understanding that other subjects make mistakes, they generate predictions inconsistent with momentum-like behavior.

First, if a subject believes other subjects make timing mistakes, but trade according to their private information, then the parameters of the experiment are such that the equilibrium timing strategy is dominant (see the proof of Proposition 2). If, in addition to timing mistakes, a subject believes other subjects make trading mistakes (as in the case in which they follow momentum-like strategies), then she would think that the observed price is too extreme on average: in the extreme, if other subjects’ trades are completely unrelated to their signals, then the true expected value of the asset is one half. If prices are too extreme, it may be optimal for a subject to rush when she should otherwise wait, but she should then trade against the price trend, rather than with it, contrary to the data. For the same reason, momentum-like strategies are not a best response to other subjects using momentum-like strategies.34

C: Instructions

I provide the instructions for the W treatment. Those for the R treatment are identical except for the difference in parameters.

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34In a related environment (but without timing decisions), both Cipriani and Guarino (2005) and Drehmann, Oechssler, and Roeder (2005) formalize the argument that only trading against price trends is consistent with believing other subjects make mistakes.
Instructions

This is a research experiment designed to understand how people make stock trading decisions in a simple trading environment. To assist with our research, we would greatly appreciate your full attention during the experiment. Please do not communicate with other participants in any way and please raise your hand if you have a question.

You will participate in a series of 30 trials. In each trial, you and the 7 other participants will each trade a stock with the computer. You will be paid $5.00 for completing all trials. In addition, in each trial you will earn lottery tickets as described next. The more lottery tickets you have, the more you will earn on average.

Trading and Profits

Before each trial, the computer will randomly select whether the stock’s value, \( V \), is 100 tickets or 0 tickets. Each is equally likely to be selected. At the start of each trial, you will be given 100 tickets. You may choose either to buy or sell the stock in any one of 8 trading periods. You can only trade ONCE and MUST trade in one of the periods. If you buy the stock, you will gain its actual value minus the price, \( P \), you pay for it. If you want to sell the stock, you must first “borrow” it from the computer and later pay back its actual value. So, if you sell, you will gain the price you receive minus its value. In summary, your total profit is:

- \( 100+V-P \) lottery tickets if you buy
- \( 100+P-V \) lottery tickets if you sell

So, for example, if you sell the stock at a price of 50 and it turns out to be worth 100, you would earn \( 100+(50-100)=50 \) tickets for that trial. But if it turns out to be worth 0, you would earn \( 100+(50-0)=150 \) tickets.

Clues about Value

To help you guess the value of the stock the computer chose, you will get a single Private Clue at the start of the trial. This Private Clue will be known only by you. Specifically, there will be two possible bins: one that the computer will use if the value is 100 and another that the computer will use if the value is 0. Bins contain some number of blue and green marbles as shown below. The computer will draw a marble randomly from the bin and show it to you as your Private Clue. Each marble in the bin is equally likely to be drawn. The color of the ball you see can give you a hint as to the stock’s value.

After observing your Private Clue, you can choose to trade immediately in the first trading period. Alternatively, you can choose to wait and trade in one of the following 7 trading periods. Between trading periods, there are public announcement periods. In each announcement period, a Public Clue will become available. The Public Clue, unlike your Private Clue, is seen by everyone. For the Public Clue, the computer will draw a marble from another bin. The bin used will again depend on the stock’s true value but the possible bins used for the Public Clue are different from the bins used for the Private Clue, as shown below. Note that the bins used for both clues are fixed throughout the trial · they depend only on the initially chosen random value of the stock. Also, marbles for both types of clues are always replaced before another is drawn.

<table>
<thead>
<tr>
<th>Clue</th>
<th>Contents of bin if value = 100</th>
<th>Contents of bin if value = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Clue</td>
<td>13 blue 11 green</td>
<td>11 blue 13 green</td>
</tr>
<tr>
<td>Public Clue</td>
<td>17 blue 7 green</td>
<td>7 blue 17 green</td>
</tr>
</tbody>
</table>

If you decide to wait to trade until the last trading period, the true value of the asset will be revealed to you before you trade. Otherwise, you will only learn the true asset value after the trial is complete. Importantly, however, each time you choose to wait, the price is likely to change before your next chance to trade, as described next.

Prices

The price of the stock is set by a Price Setter played by the computer. The Price Setter’s job is to set the price equal to the stock’s mathematical expected value given all of the public information available. Therefore, the price will always be between 0 and 100. The Price Setter can observe the trades made by you and the other participants and the Public Clues. However, she can not observe any of the Private Clues nor the true asset value (even when it is revealed in the final period).

Because the price changes with the available information, if you decide to wait, the price at which you can trade is likely to change. The initial price of the stock is 50 tickets, reflecting the fact that it is equally likely to be worth 100 tickets or 0 tickets. After a participant buys, the Price Setter will increase the price and after a participant sells, she’ll decrease the price. After a Public Clue is revealed, the price will increase if it suggests (on its own) that the stock is more likely to be worth 100 and decrease if it suggests it is more likely to be worth 0.

Trading Screen

The trading screen you will use to trade is as shown below. The eight trading periods are indicated by the numbers 1-8. Public Clues are revealed between trading periods at the times indicated by the megaphone symbol (the clues are not shown in the figure · they appear elsewhere on your trading screen). All past trading decisions and prices are displayed. Trading periods in which one or more trades occur are indicated by a solid dot. The number of buys is indicated by a “+” and then a number and the number of sells by a “−” and then a number. The current price at which you can trade (74.16 in this example) is displayed at the current time which is indicated by the dashed red line. The dashed red line will progress to the right...
as we move through the periods.

In this example, one participant bought in the first trading period, so the price increased. In the second trading period, one participant bought and one sold, so the price did not change. In the third trading period, no one traded. The first and third Public Clues suggested the stock's value is 100 and the second that the stock's value is 0. It is currently the fourth trading period. Note that this is an example only and is not meant to suggest when you should trade.

In each trading period in which you haven't already traded, you must choose buy or sell or wait and then press the 'confirm' button. If you choose to wait, the red arc points to the next trading period in which you can trade. In trading periods after you have traded, you do not have to do anything - you will simply be notified that you have already traded. In the periods with Public Clues, you must press "OK" to acknowledge having seen the clue.

Summary
1. At the beginning of each trial, the computer randomly selects the stock's value: 0 or 100.
2. Each participant is shown their Private Clue from the same bin. Marbles are replaced so each participant may see the same (or different) marbles.
3. Each participant chooses to buy, sell, or wait in the first trading period. After all participants have made their decisions, a Public Clue is revealed and we move to the second trading period.
4. Step 3 is repeated until all 8 trading periods are complete. You must trade in one of the eight trading periods and may only trade once.
5. Just before the 8th trading period, the true value of the stock will be revealed to you if you have not already traded.
6. At each point in time, all past prices and trading decisions are available to use to help guess the value of the stock (in addition to the Clues).

After all participants have traded, the trial is complete. The true value of the asset will be revealed to all participants and you will be told how many tickets you earned for the trial. You will press 'next trial' to participate in the next trial. It is important to remember that, in each trial, the value of the stock is independently randomly selected by the computer -- there is no relationship between the value selected in one trial and another.

After all trials are complete, one lottery will be conducted for each trial. For each lottery, a random number less than 200 will be chosen by the computer. If the number is smaller than the number of lottery tickets you earned for the trial, you will get $1.00. Therefore, the more lottery tickets you earn in each trial, the more you can expect to make (partial tickets are possible and count as well). For example, if you earn 100 tickets in each trial, you can expect to make $0.65*30*1=$19.50 over the 30 trials. But, if you earn 130 tickets in each trial, you can expect to make $0.65*30*1=$19.50.

Please try to make each trading period decision within 15 seconds so that the experiment can finish on time. A timer counts down from 15 to help you keep track of time. Note, however, that if the timer hits zero, you can still enter your trading or wait decision and will still have the same chance to earn money. Before beginning the paid trials, we will have two practice trials for which you will not be paid. These trials are otherwise identical to the paid trials.

Quiz

Please answer the following questions and press the 'Check answers' button to see whether or not you answered all questions correctly. To ensure all participants understand the instructions, everyone must answer all of the questions correctly before we begin the experiment.

1. Your Private Clue is a blue marble. Based only on this information, What is the most likely value of the stock? 100
2. It is the second trading period. You observe another participant sold the stock in the first trading period. What color marble is their Private Clue most likely to be?  
- green
- blue

3. Your Private Clue is a blue marble. What color marble is another participant's Private Clue likely to be?  
- green
- blue

4. If you choose to sell the stock at a price of 80 and its value turns out to be 100, how many total tickets would you get for that trial?  
- 80
- 20
- 180

5. If you choose to buy the stock at a price of 25 and its value turns out to be 100, how many total tickets would you get for that trial?  
- 25
- 75
- 175

6. The stock's current price is 80. Which value of the stock is more likely?  
- 100
- 0

Once you have completed the quiz, please press 'Check answers'.