2007

Choosing What to Protect: Strategic Defensive Allocation Against an Unknown Attacker

Vicki M. Bier
University of Wisconsin–Madison, bier@engr.wisc.edu

Santiago Oliveros
University of Wisconsin- Madison, Soliveros@wisc.edu

Larry Samuelson
University of Wisconsin- Madison, Larry.Samuelson@yale.edu

Choosing What to Protect: Strategic Defensive Allocation against an Unknown Attacker

VICKI BIER, SANTIAGO OLIVEROS, and LARRY SAMUELSON
University of Wisconsin

Abstract

We study a strategic model in which a defender must allocate defensive resources to a collection of locations and an attacker must choose a location to attack. In equilibrium, the defender sometimes optimally leaves a location undefended and sometimes prefers a higher vulnerability at a particular location even if a lower risk could be achieved at zero cost. The defender prefers to allocate resources in a centralized (rather than decentralized) manner, the optimal allocation of resources can be non-monotonic in the value of the attacker's outside option, and the defender prefers her defensive allocation to be public rather than secret.

1. Introduction

Recent events have directed attention to the question of how one might defend against an attacker whose choice of target is unknown. This paper examines the strategic interaction between a defender and attacker.

Vicki Bier, Department of Industrial Engineering, University of Wisconsin, 1550 Engineering Drive, Madison, WI 53706 (Bier@engr.wisc.edu). Santiago Oliveros and Larry Samuelson, Department of Economics, University of Wisconsin, 1180 Observatory Drive, Madison, WI 53706 (Soliveros@wisc.edu), (LarrySam@ssc.wisc.edu).

We thank the editor, associate editor, and two referees for comments. This research was supported by the United States Department of Homeland Security through the Center for Risk and Economic Analysis of Terrorism Events (CREATE) under grant number EMW-2004-GR-0112, the U. S. Army Research Laboratory and the U. S. Army Research Office under grant number DAAD19-01-1-0502, and the National Science Foundation under grant number SES-0241506. Any opinions, findings, and conclusions or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the sponsors.

Received June 9, 2005; Accepted November 21, 2005.

Journal of Public Economic Theory, 9 (4), 2007, pp. 563–587.

We study a model in which a defender must allocate defensive resources to locations and an attacker must choose a location to attack. The defender does not know the attacker's preferences, while the attacker observes the defender's resource allocation. Though the model admits a number of interpretations (e.g., the attack might be the launch of a new consumer good, and the defense an attempt to isolate a segment of a market), we hope that the paper will in particular provide insight into questions having to do with the threat of terrorist attacks.

Our model gives rise to negative externalities across locations (or, equivalently, across types of attacks and targets). Strengthening one location makes it more attractive to attack another. A centralized defender can exploit these negative externalities to advantageously affect the attacker's behavior. As a result, a defender faced with the ability to costlessly reduce the success probability of an attack on a given location may nevertheless choose not to do so, and may optimally leave some locations undefended, even if they are subject to a positive probability of attack.

We use this model to examine a number of policy questions. Are there reasons to prefer a centralized resource allocation? There are: decentralized defenders fail to internalize the costs that their defensive precautions impose on other locations and hence overinvest in defensive measures. How does the defender's optimal strategy differ if the attacker incurs an opportunity cost in launching an attack? It may be optimal to defend relatively lightly against opponents who have quite lucrative alternatives and hence are unlikely to attack, or against opponents who have quite bleak alternatives and are quite likely to attack, while expending more resources in response to intermediate cases. Finally, should the defender make her allocation public, or is it best kept a secret? We find that first-mover advantages are sufficient to ensure that the defender prefers a public allocation.

Section 2 presents our model. Section 3 characterizes the equilibrium behavior. Section 4 considers extensions of the model that allow us to develop policy implications. Sections 5 and 6 discuss related literature and conclude.

2. The Model

2.1. The Players

There are two agents in our model, a defender (D, or she) and an attacker (A, or he). The attacker can choose to attack one (and only one) of location 1 or 2 (perhaps because an attack on one location would exhaust the attacker's resources, or would lead to the attacker being detected and disabled). These alternatives may literally represent different locations against which a given type of attack may be launched (such as different cities that may be targeted with a nuclear device), or may represent different types of attacks (such as nuclear and biological attacks).

An attack may be either a success or failure. The attacker receives a payoff of $a_i \in \mathbb{R}_+$ in the event of a successful attack on location i, and receives 0 in the event of an unsuccessful attack on either location. The defender experiences a loss of $d_i \in \mathbb{R}_+$ from a successful attack on location i, and experiences no loss from an unsuccessful attack.

The attacker knows the defender's valuations, d_1 and d_2. The attacker's preferences (a_1, a_2) are drawn from a commonly known cumulative distribution function $F : \mathbb{R}_+^2 \to [0, 1]$, and are known by the attacker but not the defender. For example, the value to the attacker of a success at location i may depend not only on the damage d_i, but also on the propaganda value of the target, the cost of mounting the attack, and other factors that the defender may not fully comprehend. We refer to a pair $(a_1, a_2) \in \mathbb{R}_+^2$ as a "type" of attacker.

ASSUMPTION 1: *The cumulative distribution function F is twice continuously differentiable, with density f that is positive on $\mathbb{R}_+^2$.*

An important element in the attacker's preferences may be the desire to inflict losses on the defender, and hence F may attach high probability to values close to d_1 and d_2. Our assumption allows this, requiring only that there is some possibility of any other value in $\mathbb{R}_+^2$ as well.

2.2. Technology

An attack on location i is a success with probability p_i. The defender can choose $p_i \in (0, 1]$ at cost $c_i(p_i)$, where $c_i : (0, 1] \to \mathbb{R}_+ (i = 1, 2)$. We think of $c_i(p_i)$ as identifying the cost of the defensive resources that must be allocated to location i in order to achieve a success probability p_i.

ASSUMPTION 2: *The function $c_i : (0, 1] \to \mathbb{R}_+0$ $(i = 1, 2)$ is twice continuously differentiable, with $c_i'(p_i) < 0$ and $c_i''(p_i) > 0$ for all $p_i \in (0, 1)$. In addition, $c_i(1) = 0$ and $\lim_{p \to 0} c_i(p) = \infty$ for $i = 1, 2$.*

Nature first draws a type for A (i.e., a pair (a_1, a_2)) from the distribution F. The attacker observes this draw, but the defender does not. The defender decides how to allocate resources between the two locations, selecting $(p_1, p_2) \in (0, 1]^2$. The attacker observes the defender's allocation (p_1, p_2), and then chooses a location to attack.

We say that the environment is symmetric if $d_1 = d_2$, $c_1(\cdot) = c_2(\cdot)$, and F is symmetric around the 45^o line (i.e., $F(a_1, a_2) = F(a_2, a_1)$ for all $(a_1, a_2) \in \mathbb{R}_+^2$).

2.3. Strategies and Payoffs

A pure strategy for the defender is a pair $(p_1, p_2) \in (0, 1]^2$. A pure strategy for the attacker is a function $s : \mathbb{R}_+^2 \times (0, 1]^2 \to \{1, 0\}$ that prescribes where to attack, given a type $(a_1, a_2) \in \mathbb{R}_+^2$ and an observed allocation (p_1, p_2) by D. We let $s(a_1, a_2, p_1, p_2) = 1$ denote an attack on location 1, and $s(a_1, a_2, p_1, p_2) = 0$ denote an attack on location 2.

Given a pair of strategies, (p_1, p_2) and s, the probability that location 1 will be attacked is given by

$$h(p_1, p_2, s) = \int_{a_1 \in \mathbb{R}_+} \int_{a_2 \in \mathbb{R}_+} s(a_1, a_2, p_1, p_2) f(a_1, a_2)\, da_1 da_2. \tag{1}$$

The attacker seeks to maximize the expected payoff from launching an attack. In *ex ante* terms, this expected payoff is given by

$$U(p_1, p_2, s) = \int_{a_1 \in \mathbb{R}_+} \int_{a_2 \in \mathbb{R}_+} [s(a_1, a_2, p_1, p_2) p_1 a_1 + (1 - s(a_1, a_2, p_1, p_2)) p_2 a_2] f(a_1, a_2)\, da_1 da_2. \tag{2}$$

The defender seeks to minimize the expected loss of an attack, given by

$$L(p_1, p_2, s) = h(p_1, p_2, s) p_1 d_1 + [1 - h(p_1, p_2, s)] p_2 d_2 + c_1(p_1) + c_2(p_2). \tag{3}$$

2.4. Equilibrium

We examine the analogue (for an infinite game) of a pure-strategy sequential equilibrium.

Definition 1: *An equilibrium is a pair of strategies,* (p_1^*, p_2^*) *and* $s^*(a_1, a_2, p_1, p_2)$, *such that:*

$$U(p_1, p_2, s^*) \geq U(p_1, p_2, s), \quad \forall p_1, p_2, s$$
$$L(p_1^*, p_2^*, s^*) \leq L(p_1, p_2, s^*) \quad \forall (p_1, p_2) \in (0, 1]^2.$$

The sequential-equilibrium consistency condition on beliefs is automatically satisfied, since the only player with private information (the attacker) moves last. Section 3 explains why mixed-strategy equilibria, defined in the obvious way, need not be considered.

2.5. An Example

Throughout this paper, we illustrate our results with an example in which a_1 and a_2 are distributed independently with exponential cumulative distributions given by

$$F_i(a_i) = 1 - e^{-4a_i},$$

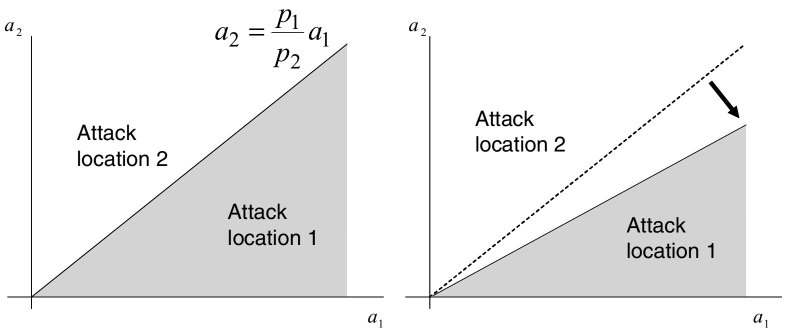

Figure 1: Location optimally attacked, as a function of the attacker's valuations a_1 and a_2 of locations 1 and 2. In the left-hand panel, $p_1 = p_2$. In the right-hand panel, we decrease p_1, causing the boundary line to rotate clockwise as shown.

and with the defensive technology (identical in the two locations) given by

$$c_i(p_i) = -\ln p_i. \tag{4}$$

3. Equilibrium Behavior

3.1. The Attacker

The attacker makes his choice with perfect information. His optimal behavior is straightforward and generically pure:[1]

LEMMA 1: *The strategy s^* is optimal for the attacker if and only if, for all* $(p_1, p_2) \in (0, 1]^2$,

$$\frac{a_2}{a_1} < \frac{p_1}{p_2} \Longrightarrow s^*(a_1, a_2, p_1, p_2) = 1, \tag{5}$$

$$\frac{a_2}{a_1} > \frac{p_1}{p_2} \Longrightarrow s^*(a_1, a_2, p_1, p_2) = 0. \tag{6}$$

If $p_1 = p_2$, for example, the attacker targets the location with the higher value. This optimal response is shown in the left panel of Figure 1. Location 2 becomes more likely to be attacked as the ratio p_1/p_2 falls, as shown in the right panel. Note that leaving a location undefended does not ensure it will be attacked. If location 1 is undefended $(p_1 = 1)$, an attack on location 1 will surely succeed but will occur only if $a_1 \geq p_2 a_2$.

Proof: From (2), it is immediate that the attacker optimally attacks location 1 $(s(a_1, a_2, p_1, p_2) = 1)$ if $p_1 a_1 > p_2 a_2$, giving the result. ■

[1] We ignore the measure-zero cases in which $p_1 a_1 = p_2 a_2$, $a_1 = 0$, or $a_2 = 0$.

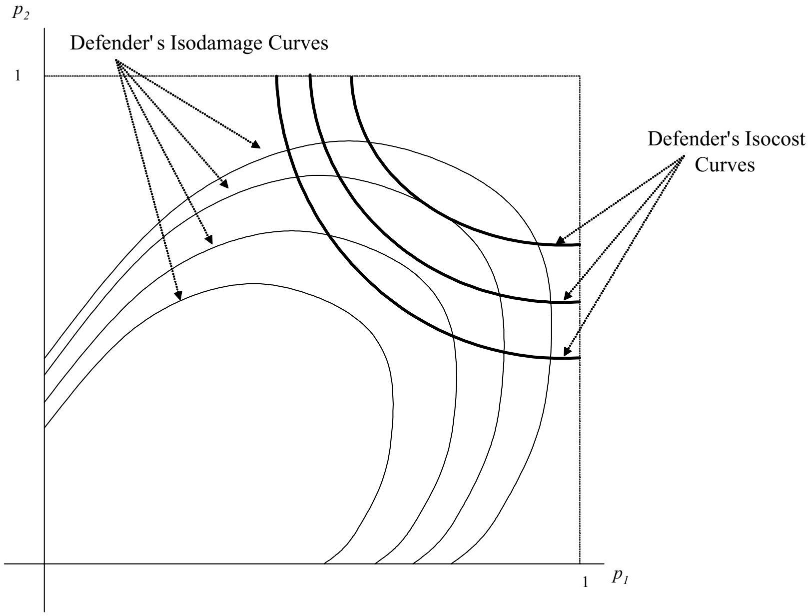

Figure 2: Partial iso-damage and iso-cost maps for the defender, for the example of Section 2.5 and the case of $d_1 = d_2$.

3.2. The Defender

The defender chooses (p_1, p_2) to minimize the loss function given in (3), given the attacker's optimal behavior:

$$\min_{(p_1,p_2)\in(0,1]^2} L\big(p_1, p_2, s^*\big). \tag{7}$$

To understand the defender's behavior, it will be helpful to draw her *iso-damage* and *iso-cost* curves. Let $D(\phi)$ be the "iso-damage" set $\{(p_1, p_2) \in [0, 1]^2 : h(p_1, p_2, s^*)p_1 d_1 + [1 - h(p_1, p_2, s^*)]p_2 a_2 = \phi\}$. Similarly, let $C(\psi)$ be the "iso-cost" set $\{(p_1, p_2) : c_1(p_1) + c_2(p_2) = \psi\}$.

Figure 2 plots part of the iso-damage and iso-cost map for the example of Section 2.5. Iso-damage curves closer to the origin describe lower levels of damage ϕ, and hence are preferred by the defender.

Figure 2 shows that the iso-damage curves can have upward-sloping segments. When p_1 is sufficiently small, for example, the defender prefers a higher success probability at location 1, even if accompanied by a higher success probability at location 2 (and similarly for small p_2). Thus, there are circumstances in which the defender would rather throw away resources than use them to further reduce the success probability of an attack at location 1.

These upward-sloping segments reflect the fact that the attacker's best-response behavior is incorporated in the defender's iso-damage calculations, and are a general phenomenon.

LEMMA 2: *Let $\phi/d_2 \in (0,1)$. Then the iso-damage condition $h(p_1, p_2, s^*)p_1 d_1 + [1 - h(p_1, p_2, s^*)]p_2 d_2 = \phi$ implicitly defines a function $p_2 = D(p_1, \phi)$* at $p_1 =$ 0, *with $dD(0,\phi)/dp_1 > 0$. A similar result holds for $p_2 = 0$.*

When p_1 is very small, the attacker is very likely to attack location 2, where he enjoys a relatively high success probability. Increasing p_1 diverts some of this attack probability to location 1 (where the probability of success is relatively low), to the benefit of the defender. If p_1 is sufficiently small, then the expected damage from diverting attacks to location 1 is small enough to ensure a reduction in the defender's total expected damage.

Proof: When $p_1 = 0$ (the case of $p_2 = 0$ is analogous), the attacker surely targets location 2, giving an expected damage of $p_2 d_2$, and hence a unique $p_2 \in (0, 1)$ with $p_2 d_2 = \phi$. We then implicitly differentiate $h(p_1, p_2, s^*)p_1 d_1 + [1 - h(p_1, p_2, s^*)]p_2 d_2 = \phi$ to obtain

$$\frac{dD(0,\phi)}{dp_1} = -\frac{\dfrac{dh}{dp_1}(p_1 d_1 - p_2 d_2) + h d_1}{\dfrac{dh}{dp_2}(p_1 d_1 - p_2 d_2) + (1-h) d_2}$$

$$= -\frac{\dfrac{dh}{dp_1} p_2 d_2}{\dfrac{dh}{dp_2} p_2 d_2 - d_2}$$

$$> 0,$$

where the second equality follows from $p_1 = 0$ (and $h(0, p_2, s^*) = 0$) and the inequality from $dh/dp_1 > 0$ and $dh/dp_2 < 0$. ■

3.3. Equilibrium

Equilibria exist, and mixing adds nothing to the payoff possibilities available to the defender:

PROPOSITION 1: *There exists a pure equilibrium (p_1^*, p_2^*, s^*). The attacker's strategy in any equilibrium is pure. If the defender's equilibrium strategy is mixed, then for any (p_1, p_2) in the support of the mixture, there is a pure equilibrium in which the defender plays (p_1, p_2) and receives the same payoff.*

Proof: Choose $\underline{p}_1$ and $\underline{p}_2$ such that $c_i(\underline{p}_i) > d_i$. Values of $p_i < \underline{p}_i$ are clearly suboptimal for the defender, and we can restrict attention to values

$(p_1, p_2) \in [\underline{p}_1, 1] \times [\underline{p}_2, 1]$. Since $L(p_1, p_2, s^*)$ is then a continuous function of (p_1, p_2) on a compact set, there exists an optimal strategy for the defender, giving (along with (5)–(6)) an equilibrium.

Lemma 1 has established that the attacker plays a pure strategy, while the characterization of the defender's strategy follows from the fact that the defender must be indifferent over any pairs (p_1, p_2) in the support of an equilibrium mixture. ■

Differentiating (3), the first-order conditions for an interior solution $(0 < p_i < 1)$ of the defender's optimization problem (7) are

$$\frac{dh}{dp_1}(p_1 d_1 - p_2 d_2) + h d_1 + \frac{dc_1}{dp_1} = 0 \tag{8}$$

$$\frac{dh}{dp_2}(p_1 d_1 - p_2 d_2) + (1 - h) d_2 + \frac{dc_2}{dp_2} = 0. \tag{9}$$

Sufficient second-order conditions are given in (16) below.

Will it ever be optimal to leave a location undefended, ensuring that an attack against that location would be successful? Even if $dc_i(1)/dp_i = 0$, so that marginal reductions of the success probability of an attack below unity are costless, a location will be left undefended if its relative value is sufficiently small.

PROPOSITION 2:

(a) *In a symmetric environment, there exists a unique equilibrium, which is pure and in which $p_1 = p_2$.*

(b) *For any $d_1 > 0$, there exists $\underline{D}_2(d_1) > 0$ such that location 2 is undefended at any equilibrium if $d_2 < \underline{D}_2(d_1)$. Similarly, there exists $\underline{D}_1(d_2) > 0$ for $d_2 > 0$ such that location 1 is undefended at any equilibrium if $d_1 < \underline{D}_1(d_2)$.*

Proof:

(a) Let the environment be symmetric and suppose the defender attaches positive probability to a pair (p_1, p_2) with $p_1 \neq p_2$. Then replacing (p_1, p_2) with the pair $(\bar{p}, \bar{p})$ for $\bar{p} = \frac{1}{2}p_1 + \frac{1}{2}p_2$ reduces the defender's expected damage (since $d_1 = d_2$ and the attacker is now equally likely to target either location, by symmetry, whereas previously the attacker must have been more likely to target the location with the higher success probability) while reducing expected cost (because cost functions are strictly convex). Hence, the defender can attach positive probability only to pairs (p_1, p_2) in which $p_1 = p_2$. Any such pair must solve the defender's first-order conditions, which in a symmetric environment are (from (8)–(9))

$$h d_1 + \frac{dc_1}{dp_1} = 0 \quad \text{and} \quad (1 - h) d_2 + \frac{dc_2}{dp_2} = 0.$$

Using $d_1 = d_2$ and $c_1(\cdot) = c_2(\cdot)$, this system has a unique solution in which $dc_i/dp_i = -\frac{1}{2}d_i$.

(b) We provide the argument for $\underline{D}_2(d_1)$. Consider the first-order condition given by (9). Given d_1, the equilibrium probability p_1 is bounded below (*cf.* the proof of Proposition 1), while our assumptions on the distribution F ensure that dh/dp_2 is negative with finite (negative) upper and lower bounds. We have $h \in [0, 1]$ and $dc_2/dp_2 < 0$ for $p_2 < 1$. As a result, there is a value $\underline{D}_2$ such that if $d_2 < \underline{D}_2$, the left side of (9) must be negative for all $p_2 \in [0, 1]$. The equilibrium must then entail $p_2 = 1$. ■

Example: Consider the example of Section 2.5 with $d_1 = d_2 \equiv d$. Then the environment is symmetric and the equilibrium is given by

$$p_1 = p_2 = \frac{2}{d}$$

if $d \geq 2$ and $p_1 = p_2 = 1$ otherwise.

We now drop the requirement that $d_1 = d_2$. Figure 3 identifies which locations are defended (i.e., which have an equilibrium success probability less than 1) as a function of d_1 and d_2. Location 2 is undefended if and only if d_2 falls short of the value $\underline{D}_2(d_1)$ shown in Figure 3. Notice that as d_1

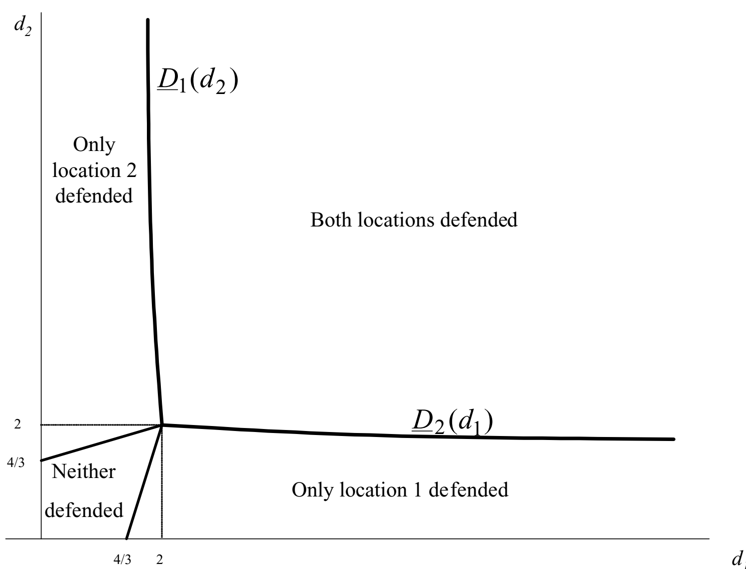

Figure 3: Specification of which locations are defended ($p_i < 1$) or undefended ($p_i = 1$) in equilibrium, as a function of the defender's valuations d_1 and d_2, for the example of Section 2.5.

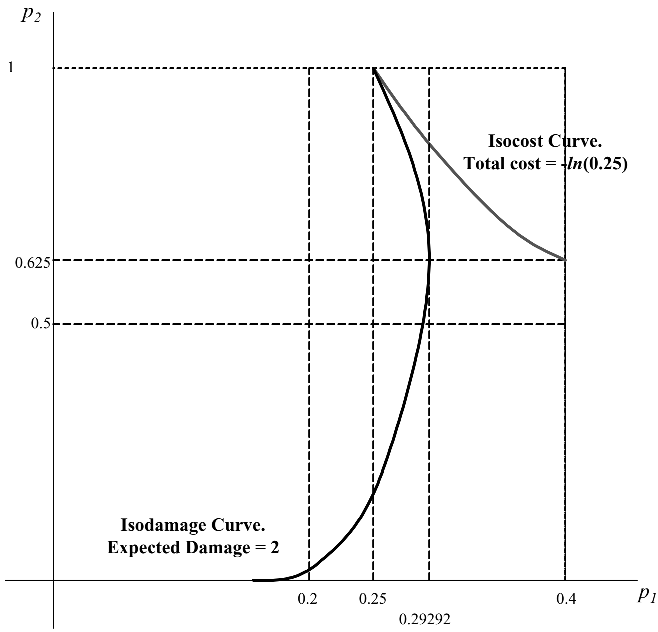

Figure 4: Iso-damage and iso-cost curves for which location 2 is optimally undefended.

increases, the boundary value $\underline{D}_2(d_1)$ for devoting resources to location 2 falls (with $\lim_{d_1 \to \infty} \underline{D}_2(d_1) = \frac{3}{2}$).[2] Increasing d_1 prompts the defender to reduce the success probability p_1, increasing the probability that location 2 is attacked and thus making it more valuable to allocate resources to location 2. This is not a general property, as there are other cost functions for which $\underline{D}_2(d_1)$ is positively sloped. Notice also that for values of d_1 and d_2 that both fall below 2, the defender expends resources on defense only if the locations are sufficiently asymmetric.

Figure 4 illustrates the iso-damage and iso-cost curves for an equilibrium in which location 2 is undefended. This is drawn for the case in which $d_1 = 14$ and $d_2 = 1.625$, with the cost function and type distributions from Section 2.5.

3.4. Comparative Statics

This section confirms that the equilibrium responds in the expected ways to variations in the underlying strategic environment. We typically state the

[2] Using (8)–(9), a calculation gives that for $d_2 \geq 2$,

$$\underline{D}_2(d_1) = \frac{3}{8d_1} + \frac{1}{2}\left(\frac{1}{2d_1} + \left(\frac{3}{4d_1}\right)\right)^{\frac{1}{2}} + \frac{3}{2}.$$

results for location 1, noting that an analogous statement obviously holds for location 2. We consider interior equilibria and assume that sufficient second-order conditions (derived in the course of proving Lemma 3) for the defender's minimization hold.

3.4.1. Defender's Valuations

Increasing location 1's value decreases the optimal success probability p_1:

LEMMA 3: *Let the equilibrium values* (p_1, p_2) *be contained in* $(0, 1)^2$. *Then* p_1 *is decreasing in* d_1.

Proof: The second derivatives of the defender's loss function are (letting h_i denote a derivative with respect to p_i and h_{ij} the second derivative with respect to p_i and p_j, and suppressing the arguments of L and h)

$$L_{11} = h_{11}(p_1 d_1 - p_2 d_2) + 2h_1 d_1 + \frac{dc_1^2(p_1)}{dp_1^2} \quad (10)$$

$$L_{12} = h_{12}(p_1 d_1 - p_2 d_2) + h_2 d_1 - h_1 d_2 \quad (11)$$

$$L_{22} = h_{22}(p_1 d_1 - p_2 d_2) - 2h_2 d_2 + \frac{dc_2^2(p_2)}{dp_2^2}. \quad (12)$$

Using (1) and Lemma 1, we can write

$$h(p_1, p_2, s^*) = \int_{a_1=0}^{\infty} \int_{a_2=0}^{\frac{a_1 p_1}{p_2}} f(a_1, a_2)\, da_2\, da_1.$$

Differentiating then gives

$$h_2 = -\frac{p_1}{p_2} h_1 \quad (13)$$

$$h_{21} = -\frac{1}{p_2} h_1 - \frac{p_1}{p_2} h_{11} \quad (14)$$

$$h_{22} = \frac{2p_1}{(p_2)^2} h_1 + \left(\frac{p_1}{p_2}\right)^2 h_{11}. \quad (15)$$

Combining these relationships with (10)–(12) and using the strict convexity of the cost function, we find that $L_{11}L_{22} - L_{12}L_{21} > 0$. Hence, the sufficient second-order (that the Hessian of L be positive definite) holds if

$$L_{11} = \frac{dh^2(p_1, p_2, s^*)}{dp_1^2}(p_1 d_1 - p_2 d_2) + 2\frac{dh(p_1, p_2, s^*)}{dp_1} d_1 + \frac{dc_1^2(p_1)}{dp_1^2} > 0. \quad (16)$$

It now follows from implicitly differentiating (8)–(9) and using (11)–(15) to simplify that the equilibrium value of p_1 is decreasing in d_1. ■

As d_1 increases, location 2 becomes relatively less valuable, creating a substitution effect tending to increase the optimal success probability at location 2. However, the decreased value of p_1 makes the attacker more likely to strike location 2, creating an offsetting effect that tends to decrease p_2. The following example illustrates a case in which p_2 declines as does p_1 (for interior equilibria), but the net effect is in general ambiguous.

Example: For the example of Section 2.5, the first-order conditions for interior solutions are

$$\frac{p_1 + 2p_2}{(p_1 + p_2)^2} p_1 d_1 - \frac{p_2}{(p_1 + p_2)^2} p_2 d_2 - \frac{1}{p_1} = 0$$

$$\frac{2p_1 + p_2}{(p_1 + p_2)^2} p_2 d_2 - \frac{p_1}{(p_1 + p_2)^2} p_1 d_1 - \frac{1}{p_2} = 0.$$

Using these, Figure 5 fixes $d_2 = 1.75$, and then identifies the optimal values of p_1 and p_2 as a function of d_1. When d_1 is very small, location 2 (only) is defended. As d_1 increases, defending location 2 becomes less valuable, as doing so pushes the attacks to an increasingly valuable location 1, and the defender allocates resources to neither target for

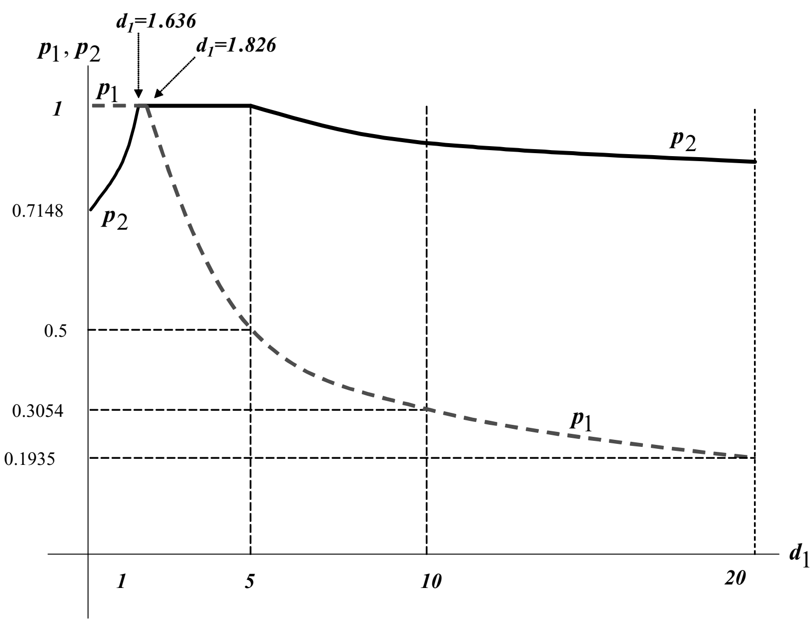

Figure 5: Defender's optimal values of p_1 and p_2, as a function of d_1 (with $d_2 = 1.75$), for the example of Section 2.5.

$d_1 \in [1.636, 1.826]$. As d_1 increases further, location 1 (only) is defended. As d_1 passes 5, expenditures on defending location 1 have made location 2 such an attractive target for the attacker that it now becomes optimal to defend both locations. As d_1 increases further, the optimal success probability at both locations falls.

As d_1 increases, p_2 cannot decline too much, in the sense that increasing location 1's value decreases the equilibrium probability of a successful attack on location 1.

LEMMA 4: *Fix d_2 and let $(\tilde{p}_1, \tilde{p}_2)$ and (p_1, p_2) be equilibrium values given $\tilde{d}_1$ and $d_1 < \tilde{d}_1$, respectively. Then $h(\tilde{p}_1, \tilde{p}_2, s^*)\tilde{p}_1 \leq h(p_1, p_2, s^*)p_1$.*

Proof: The optimality of the defender's choice, given value d_1, requires

$$h(p_1, p_2, s^*)p_1 d_1 + [1 - h(p_1, p_2, s^*)]p_2 d_2 + c_1(p_1) + c_2(p_2)$$
$$\leq h(\tilde{p}_1, \tilde{p}_2, s^*)\tilde{p}_1 d_1 + [1 - h(\tilde{p}_1, \tilde{p}_2, s^*)]\tilde{p}_2 d_2 + c_1(\tilde{p}_1) + c_2(\tilde{p}_2),$$

or

$$[h(p_1, p_2, s^*)p_1 - h(\tilde{p}_1, \tilde{p}_2, s^*)\tilde{p}_1]d_1$$
$$\leq -[1 - h(p_1, p_2, s^*)]p_2 d_2 - c_1(p_1) - c_2(p_2)$$
$$+ [1 - h(\tilde{p}_1, \tilde{p}_2, s^*)]\tilde{p}_2 d_2 + c_1(\tilde{p}_1) + c_2(\tilde{p}_2).$$

If $h(p_1, p_2, s^*)p_1 - h(\tilde{p}_1, \tilde{p}_2, s^*)\tilde{p}_1 < 0$, this precludes (recalling $\tilde{d}_1 > d_1$),

$$[h(p_1, p_2, s^*)p_1 - h(\tilde{p}_1, \tilde{p}_2, s^*)\tilde{p}_1]\tilde{d}_1$$
$$\geq -[1 - h(p_1, p_2, s^*)]p_2 d_2 - c_1(p_1) - c_2(p_2)$$
$$-[1 - h(\tilde{p}_1, \tilde{p}_2, s^*)]\tilde{p}_2 d_2 + c_1(\tilde{p}_1) + c_2(\tilde{p}_2),$$

which is necessary for the optimality of $(\tilde{p}_1, \tilde{p}_2)$ given $\tilde{d}_1$, a contradiction. ■

3.4.2. Defender's Costs

Let the cost function for location i be given by $\theta_i c_i(p_i)$, so that reductions in θ_i reduce total and marginal costs. A location's success probability is reduced as the cost of doing so declines.

LEMMA 5: *Let the equilibrium values (p_1, p_2) be contained in $(0, 1)^2$. Then p_i is increasing in θ_i.*

Proof: A straightforward implicit differentiation of(8)–(9) shows that p_i declines as does θ_i. ■

In general, a decrease in θ_i may induce the defender to either increase or decrease p_j.

3.4.3. Attacker's Valuations

The attacker's valuations are described by the distribution F over $(a_1, a_2) \in \mathbb{R}^2_+$. The effects of changes in the distribution F can be complicated. We restrict attention to the cost function given in the example of Section 2.5, noting as we proceed where this structure is used.

For distributions F and $\tilde{F}$ over the attacker's valuations, let us say that $F \succ_1 \tilde{F}$ if, for all $\phi \in (0, \infty)$ (where f and $\tilde{f}$ are the densities associated with F and $\tilde{F}$),

$$\int_{a_1 \in \mathbb{R}_+} \int_{a_2 : \frac{a_1}{a_2} > \phi} f(a_1, a_2)\, da_1\, da_2 > \int_{a_1 \in \mathbb{R}_+} \int_{a_2 : \frac{a_1}{a_2} > \phi} \tilde{f}(a_1, a_2)\, da_1\, da_2.$$

The relation $\succ_1$ thus induces a partial order over distributions of valuations based on first-order stochastic dominance. Intuitively, we think of ϕ as a candidate success-probability ratio p_1/p_2. If $F \succ_1 \tilde{F}$, then for any ratio ϕ, the attacker is more likely to target location 1 under distribution F than under $\tilde{F}$. The defender's response to a shift from $\tilde{F}$ to F is to decrease the relative success probability at location 1.

LEMMA 6: *If the the cost function is given by (4) and optimal values* $(\tilde{p}_1, \tilde{p}_2)$ *corresponding to distribution* $\tilde{F}$ *satisfy* $(\tilde{p}_1, \tilde{p}_2) \in (0, 1)^2$, *then* $F \succ_1 \tilde{F} \Rightarrow p_1/p_2 < \tilde{p}_1/\tilde{p}_2$.

Proof: The defender's first-order conditions (8)–(9) can be combined to give

$$\frac{p_2}{p_1} = -\frac{\frac{dh}{dp_1}}{\frac{dh}{dp_2}} = \frac{\left| hd_1 + \frac{dc_1}{dp_1} \right|}{\left| (1-h)\, d_2 + \frac{dc_2}{dp_2} \right|}, \tag{17}$$

for the case in which $p_1 d_2 - p_2 d_2 \neq 0$,[3] where the first equality follows from (13). Fix the attacker's valuation at $\tilde{F}$. Suppose now that we restrict attention to values (p_1, p_2) which incur a fixed expenditure $C = c_1(p_1) + c_2(p_2)$. Then the leftmost term in (17) is decreasing in p_1/p_2 and the rightmost increasing, ensuring that there is a unique equality. Replacing $\tilde{F}$ with F has no effect on the left side while increasing the right side (by increasing h and decreasing $1 - h$), ensuring that the optimal value p_1/p_2 decreases, conditional on C. The argument is now competed by noting that, from (4), the set of values (p_1, p_2) satisfying (17) is a ray. ■

No assumptions on the cost functions are required to show that, conditional on an expenditure level, $F \succ_1 \tilde{F}$ implies $p_1/p_2 < \tilde{p}_1/\tilde{p}_2$. This is a substitution effect. The cost function given by (4) suffices to ensure that this

[3]If $p_1 d_2 - p_2 d_2 = 0$, we substitute $hd_1/(1-h)\, d_2 = (dc_1/dp_1)/(dc_2/dp_2)$.

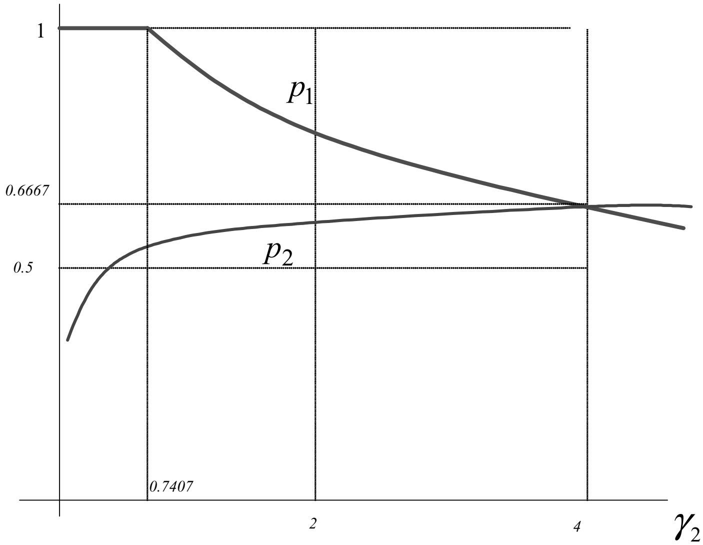

Figure 6: Defender's optimal values p_1 and p_2, for the example of Section 2.5, as a function of γ_2 (with $\gamma_1 = 4$ and $d_1 = 2$, $d_2 = 2$.

substitution effect is not overwhelmed by "income effects" caused by a change in the optimal expenditure level.

Example: Let $\tilde{F}$ denote the distribution in our example, under which a_1 and a_2 are distributed independently and exponentially, with parameters denoted γ_1 and γ_2, where $\gamma_1 = \gamma_2 = 4$. Now, hold γ_1 fixed while increasing γ_2, giving rise to a distribution F with $F \succ_1 \tilde{F}$. As a result, the defender's optimal response is to decrease p_1 and increase p_2, shifting resources to location 1 in response to the increased likelihood of its being attacked. Figure 6 illustrates.

4. Strategic Policy

We now put our model to work at addressing questions concerning optimal policies of strategic deterrence.

4.1. Decentralization

Should strategic defensive decisions be centralized or decentralized? We focus on the symmetric case in answering this question.

Let the decentralized defender be represented by two agents, where agent 1 chooses p_1 and agent 2 chooses p_2, to minimize, respectively,

$$h(p_1, p_2, s)p_1 d_1 + c_1(p_1)$$
$$(1 - h(p_1, p_2, s))p_2 d_2 + c_2(p_2).$$

The attacker's equilibrium behavior when facing decentralized defenders will be given by s^*, defined in Lemma 1. Fixing this behavior, we can view defenders 1 and 2 as playing a two-player game, choosing p_1 and p_2, with each seeking to minimize her own payoff. The first-order conditions for a Nash equilibrium of this game are

$$\frac{dh}{dp_1} p_1 d_1 + h d_1 + \frac{dc_1}{dp_1} = 0 \tag{18}$$

$$-\frac{dh}{dp_2} p_2 d_2 + (1 - h) d_2 + \frac{dc_2}{dp_2} = 0. \tag{19}$$

PROPOSITION 3: *Let the environment be symmetric and let the centralized equilibrium be an interior solution. Then equilibrium success probabilities for the centralized defender are larger than those in any equilibrium of the decentralized game.*

Proof: Proposition 2 has established that, in the symmetric case, the centralized defender's optimal strategy must be pure and symmetric (i.e., $p_1 = p_2$).

The decentralized equilibrium must similarly be pure. Given any mixed strategy for player i, there exists a convex combination of the prices in the support of this mixed strategy that yields an identical expected damage at a lower expected cost. Using the first equality from (17) in (18)–(19) then allows us to conclude that

$$h d_1 + \frac{dc_1}{dp_1} = (1 - h) d_2 + \frac{dc_2}{dp_2},$$

which, given the symmetry of the environment, implies that the decentralized equilibrium must be symmetric.

From Proposition 2 again, the centralized defender gives a unique equilibrium in which $dc_i/dp_i = -\frac{1}{2} d_i$. Comparing this with the first-order conditions for the decentralized defenders in (18)–(19), we find that the conditions in (18)–(19) each contain an additional, positive term, pushing the decentralized decision maker toward lower attack success probabilities. These terms appear because a decentralized defender does not internalize the externality that their allocation imposes on the other location. As a result, any symmetric solution for the case with decentralized defenders must give lower success probabilities than does the centralized defender. ■

Example: Returning to our example, let $d_1 = d_2 = d \geq 2$. Then a centralized defender will choose

$$p_1 = p_2 = \frac{2}{d},$$

while decentralized decision making gives

$$p_1 = p_2 = \frac{4}{3d}.$$

It is tempting to view the lower success probabilities provided by the decentralized solution as an advantage. However, these levels are inefficiently low. The centralized defender achieves a higher level of utility (or, equivalently, a lower overall loss) than the sum of the utilities achieved by the decentralized defenders.

4.2. Costly Attacks

We have assumed so far that the attacker invariably attacks, with the only uncertainty being the target of the attack. In practice, however, one objective of a defensive policy is to deter an attack.

To capture this, we assume that the attacker has an outside option or opportunity cost, K. An attack will then be launched on location i only if the expected value $a_i p_i$ exceeds K. The cost K may reflect the value of attacking an alternative defender, the direct cost of mounting an attack, the cost of retaliatory military action, or the cost of withdrawn goodwill or foreign aid as a result of an attack.

Some valuation profiles (a_1, a_2) are now associated with no attack. Moreover, the probability of an attack on any location depends not just on the ratio $\frac{p_1}{p_2}$, but also on the absolute levels of p_1 and p_2. Let us now define $h_i(p_1, p_2, s^*)$ to be the probability that location i is attached when $K > 0$, where

$$h_1\big(p_1, p_2, s^*\big) = \int_{a_1 \geq \frac{K}{p_1}} \int_{\frac{p_1}{p_2} a_1 \geq a_2} f(a_1, a_2)\, da_1\, da_2$$

$$h_2\big(p_1, p_2, s^*\big) = 1 - h_1\big(p_1, p_2, s^*\big) - \int_{a_1 \leq \frac{K}{p_1}} \int_{a_2 \leq \frac{K}{p_2}} f(a_1, a_2)\, da_1\, da_2.$$

The last term in the specification of $h_2(p_1, p_2)$ is the probability of A launching no attack at all. Figure 7 shows the attacker's optimal strategy as a function of his valuations.

Decreasing p_1 now has two effects—shifting the attacker to target location 2 in some cases, but also shifting the attacker to not attack in others. In Figure 7, decreasing p_1 rotates the diagonal boundary of the region in which location 1 is attacked downward, and shifts the vertical boundary to the right.

This suggests that the presence of an outside option $K > 0$ may make it more valuable to reduce the success probability of an attack, leading to smaller equilibrium values of p_1 and p_2. However, a countervailing effect arises out of the fact that an outside option reduces the probability of an attack, making smaller values of p_1 and p_2 less valuable. The net effect is ambiguous.

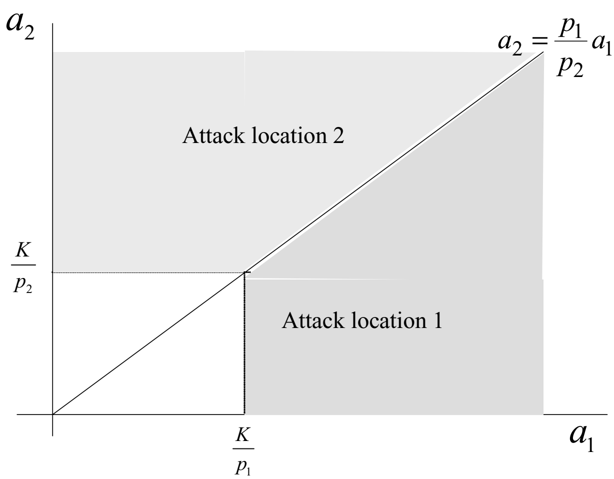

Figure 7: Attacker's optimal response with an outside option. In this illustration, we assume that $p_1 = p_2$. The open square in the lower left identifies the region in which $p_1 a_1 < K$ and $p_2 a_2 < K$, and hence no attack is optimal.

PROPOSITION 4: *As $K \to \infty$, the optimal values of p_1 and p_2 approach unity. The optimal values may be decreasing in K for small values of K.*

Proof: The first statement is straightforward. As $K \to \infty$, the probability of an attack converges to zero, even if $p_1 = p_2 = 1$. As a result, the optimal expenditures on defensive measures converges to zero.

We establish the second result with a variation on our running example. Let $d_1 = d_2 = 10$ and $a_i \sim \exp(\theta_i), i = 1, 2$, where $\theta_1 = \theta_2 = 1$. The equilibrium solution in this case is symmetric, with $p_1 = p_2 \equiv p$ given by the first-order condition

$$\left(1 + \frac{K}{p}\right) e^{-\frac{K}{p}} - \left(\frac{1}{2} + \frac{K}{p}\right) e^{-2\frac{K}{p}} + \frac{c'(p)}{d} = 0.$$

Figure 8 illustrates the optimal value of p as a function of K.[4] When $K < 0.12642$, increases in K raise the optimal level of protection (i.e., reduce the optimal value of p), while for $K > 0.12642$, increases in K lead to reductions in the level of defensive resources (increases in p). For $K \geq 3.8659$, no resources are allocated to defense ($p = 1$). ■

[4] The comparative statics behind this figure are given by $\frac{dp}{dK} = \frac{K}{p}\frac{1}{p}(1 - 2e^{-\frac{K}{p}})e^{-\frac{K}{p}} \frac{\frac{K}{(p)^2}(\frac{K}{p})e^{-\frac{K}{p}} + (1+\frac{K}{p})\frac{1}{p}e^{-2\frac{K}{p}}}{H}$, where H is the determinant of the Hessian of the defender's objective function.

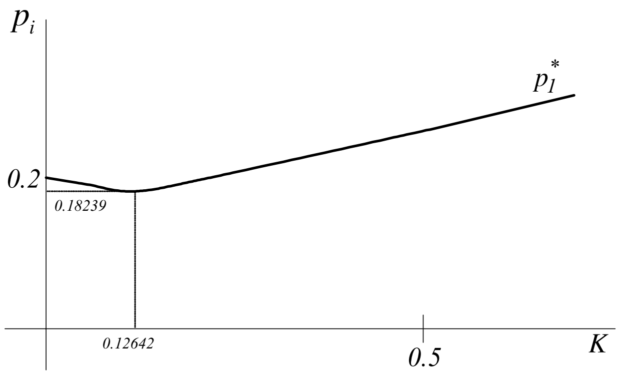

Figure 8: Optimal value of the success probability $p_1 = p_2 = p^*(K)$, as a function of the attacker's outside option K.

4.3 A Simultaneous Game

The attacker may be unable to observe the defender's choice of p_1 and p_2. What difference does this make, and which arrangement should the defender prefer?

4.3.1. Simultaneous Moves

A game in which the attacker cannot observe the defender's choice of (p_1, p_2) is a simultaneous-move game. A pure strategy for the defender is, as before, a pair (p_1, p_2). A pure strategy for the attacker is now a function from the space of valuations $\mathbb{R}^2_+$ to the selected location for an attack, rather than a function from the space of valuations and pairs (p_1, p_2) to a selected location. We let $s_1(a_1, a_2) = 1$ if location 1 is attacked, and $s_2(a_1, a_2) = 1$ if location 2 is attached. Note that only one of $s_1(a_1, a_2) = 1$ and $s_2(a_1, a_2) = 1$ can equal one, while both may be zero (in the event of no attack).

Given a pair of pure strategies (p_1, p_2) and s, the attacker's expected utility is given by

$$U(p_1, p_2, s) = \int_{a_1 \in \mathcal{A}_1} \int_{a_2 \in \mathcal{A}_2} [(a_1 p_1 - K) s_1(a_1, a_2) + (a_2 p_2 - K)(s_2(a_1, a_2))] f(a_1, a_2)\, da_1\, da_2. \tag{20}$$

The defender's expected loss is

$$L(p_1, p_2, s) = d_1 p_1 \Pr(1) + d_2 p_2 \Pr(2) + c_1(p_1) + c_2(p_2), \tag{21}$$

where $\Pr(i) = \int_{a_1 \in \mathcal{A}_1} \int_{a_2 \in \mathcal{A}_2} s_i(a_1, a_2) f(a_1, a_2)\, da_1\, da_2$ is the probability of the attacker striking location i. We are interested in Nash equilibria of this simultaneous-move game.

DEFINITION 2: *A pure-strategy equilibrium of the attacker–defender simultaneous game is a function* $s^* = (s_1^*, s_2^*) : \mathcal{A}^2 \to \{(1, 0), (0, 1), (0, 0)\}$ *and a pair* (p_1^*, p_2^*) *such that:*

$$U(p_1^*, p_2^*, s^*) \geq U(p_1^*, p_2^*, s) \quad \forall s$$
$$L(p_1^*, p_2^*, s^*) \leq L(p_1, p_2, s^*) \quad \forall (p_1, p_2) \in (0, 1]^2.$$

PROPOSITION 5: *There is a Nash equilibrium in the simultaneous attacker–defender game. Any such equilibrium must be pure. The attacker uses a cut-off strategy, in the sense that there are positive constants* k_1^*, k_2^*, α_1^* *and* α_2^* *such that location i is attacked if* $a_i > k_i^*$ *and* $a_i/a_j > \alpha_i^*$.

Proof: Glicksberg's (1952) fixed point theorem ensures the existence of an equilibrium, though leaving open the possibility that the equilibrium might be mixed. It is immediate from (20) that the attacker uses a cutoff strategy (with $k_i^* = K/p_i^*$ and $\alpha_i^* = p_j^*/p_i^*$ for a pure equilibrium (p_1^*, p_2^*, s^*)). Suppose the defender's strategy is not pure. Let $\bar{p}_1$ and $\bar{p}_2$ be the expected values of p_1 and p_2, under the defender's mixed strategy. Then it follows from the linearity of the attacker's objective that the pure strategy $\bar{p}_1$ and $\bar{p}_2$ elicits the same attacker response, and hence the same expected damage for the defender (even if $d_1 \neq d_2$), at smaller expected costs (given the convexity of $c_i(p_i)$), a contradiction to the optimality of the mixed strategy. ■

In the case of simultaneous moves, with or without an outside option for the attacker, centralized and decentralized defenders give identical solutions. When defenders are decentralized, defender i potentially has an incentive to decrease p_i in order to shift some of the attack risk to location j. However, this requires the attacker to observe p_i, in order to respond. Simultaneous moves precludes such a possibility, causing the centralized and decentralized equilibria to coincide.

4.3.2. Simultaneous versus Sequential Choices

How do the (centralized) defender's outcomes compare when moving first versus when moving simultaneously? The following is immediate from a comparison of first-order conditions:

PROPOSITION 6: *If the game is symmetric and the attacker has no outside option, then the sequential and simultaneous games give identical outcomes. If the attacker has a positive outside option, then the equilibrium success probability of an attack in the sequential game is lower than the corresponding probability in the simultaneous game.*

The intuition is straightforward. The attacker's best-response functions in the sequential and simultaneous games are identical. Any differences in equilibrium thus arise out of differences in the defender's incentives. In the sequential game, the defender could simply choose the success probabilities that are optimal for the simultaneous game, eliciting a like response from the attacker. Alternatively, the defender could use her first-mover status to choose something different, shifting the attacker away from the simultaneous-game optimum (and toward the outside option). In the symmetric case, however, the simultaneous game prompts the attacker to target either location with equal probability. In the absence of an outside option (and hence the ability to affect the overall probability of an attack), the defender in a sequential-move game can do no better than to mimic the optimal behavior of the simultaneous-move game.

Things are different in the presence of a positive outside option. Here, starting from the equilibrium behavior of the simultaneous game, the defender in the sequential game can improve her payoff by decreasing both success probabilities. In the absence of any response from the attacker, a marginal decrease would have only a second-order effect on the defender's payoff (because the point of departure is an equilibrium of the simultaneous game). In the presence of a nontrivial outside option, this second-order effect is joined by a first-order gain resulting from the attacker's attendant shift from attacking to taking the outside option in some cases. As a result, there will be an equilibrium with lower success probabilities and a lower probability of an attack.[5]

Would the defender prefer to move in advance of the attacker, or simultaneously? The key consideration here is that the defender in the sequential game can always choose the actions that would be optimal in the simultaneous game, ensuring at least as favorable an outcome as in an equilibrium of the latter. Anything else she finds optimal can only improve her situation. Hence, we have (for asymmetric as well as symmetric games):

PROPOSITION 7: *An equilibrium in the sequential game must give the defender as least as high a payoff as any equilibrium of the simultaneous game. If the outcome of the sequential game is not an equilibrium of the simultaneous game, then the defender must strictly prefer the former. A sufficient condition for the latter is that the simultaneous game give an interior solution and the attacker have a nontrivial outside option.*

Even when the attacker has no outside option, the defender will (save in exceptional cases) strictly prefer the sequential game when the environment is asymmetric and the simultaneous game gives an interior solution.

5. Related Literature

For surveys of work on the optimal defensive strategy in response to a terrorist threat, see Bier (2004) and Sandler and Enders (2004). A useful point of

[5]A corresponding result for asymmetric games is more complicated. Here, we can still be assured that at least one location receives a higher allocation in the sequential than in the simultaneous game, but one location may actually receive a lower allocation.

entry into the literature is provided by Arce and Sandler (2005), Heal and Kunreuther (2004), Keohane and Zeckhauser (2003), Kunreuther and Heal (2004) and Sandler and Lapan (1988), who focus on the externalities that arise in formulating defensive policy when there are multiple locations to be defended, and especially when there are multiple defenders allocating resources. For example, increased efforts on the part of one defender may make other defenders more likely targets, giving rise to a "security race" that culminates in inefficiently excessive defensive measures. This effect plays a key role in our model, shaping the decisions of a centralized defender and leading to the inefficiency of decentralized defenses.[6]

Arce and Sandler (2001), Heal and Kunreuther (2004), Kunreuther and Heal (2004), and Lakdawalla and Zanjani (2002) examine possible policy responses to these externalities.[7] In much of this literature, the attacker's behavior and the externalities to which it gives rise are generally exogenously specified. Our model is fully strategic, deriving the attacker's behavior (and the externalities that arise in the defender's allocation problem) directly from the agents' underlying optimization problems.[8] This provides a natural setting for studying a first-mover advantage that the attacker can use to exploit the attacker's strategic behavior.[9]

Major (2004) considers a strategic model with complete information, in which the attacker's gain is precisely equal to the defender's loss, perhaps because the attacker cares only about inflicting harm on the defender. The attacker–defender interaction is then treated as a simultaneous-move zero-sum game, whose solution is an application of the minmax theorem.[10] Because of the zero-sum payoff structure, locations may optimally be left undefended in Major's model, but only if they are of sufficiently small value that they will never be attacked. This contrasts with the "weakest link" hypothesis of Woo (2002, 2003) (in which the least heavily

---

[6]Alternatively, an increased allocation on the part of one defender could make everyone more secure, creating incentives for other defenders to free ride and leading to inefficiently little investment in defense. This would be the case in our model if decentralized defenders could increase the attacker's outside option K.

[7]Arce and Sandler (2001) show that treaties (taking on the role of a moderator in a correlated equilibrium) may solve the coordination issue when the free-riding problem is present. Heal and Kunreuther (2004) and Kunreuther and Heal (2004) argue that the interdependent risks facing defenders may give rise to tipping behavior: seemingly small shifts in the environment can lead to significant changes in the overall defensive configuration. Lakdawalla and Zanjani (2002) show that insurance can be used to coordinate the behavior of defenders facing incentives to free ride.

[8]Examples of studies including fully strategic models include Arce and Sandler (2005) and Sandler (2003).

[9]Woo (2002, 2003) directs attention to the fact that any adjustments in a defender's resource allocation can be expected to prompt adjustments in an attacker's behavior.

[10]Major's model belongs to a class traditionally called *Colonel Blotto games*: zero-sum games involving two players with fixed resources and n battlefields (see Shubik and Weber 1981 for a classic reference, and Coughlin 1992 for a generalization).

defended locations are the most likely to be attacked), and with our finding that leaving a location undefended virtually always carries some level of risk.

6. Discussion

The theme of our work is that, when facing the threat of an intentional attack, it is important to model the strategic behavior of the attacker. Concentrating on the last target struck by the the attacker, perhaps heightening airline security, may be effective if the attacker continues to concentrate on airlines, but may be of no avail if the attacker simply switches to targeting the food supply. Inspecting containers from ports shipping 95% of the containers entering the country would provide significantly improved security if attackers made no effort to alter their pattern of access to incoming containers, but may be of no avail if attackers can shift their activities to those ports shipping the remaining 5%.

As Frey and Luechinger (2002, 2003) observe, strategic models of the attacker–defender interaction tend to focus attention on a relatively small set of choices. In our case, the defender can choose only how to allocate defensive resources. There may be other useful policy options, with Frey and Luechinger calling attention to the possibility of altering the attackers' preferences. This could be introduced into our model by allowing the defender to affect the value K of the attacker's outside option, suggesting a promising direction for further work.

Perhaps more importantly, there is much that is not captured by our model. For example, we allow the attacker's preferences to be linked to the defender's valuations d_1 and d_2 (although not exclusively so), but we exclude a link between the attacker's preferences and the defender's allocation of defensive resources. Our model thus does not capture the behavior of computer hackers who delight in attacking the most secure systems for the sheer joy of the challenge. Similarly, we do not capture terrorists who value the destruction of a target *and* the destruction of the defensive resources allocated to that target. The latter may enter the attacker's preferences either because destroying resources now means that fewer are available for defending against future attacks, or simply because the attacker cares about total destruction, whether of the target itself or the resources assigned to defend it. Incorporating such considerations would lead to a (more complicated) model exhibiting the basic features of our analysis, including the externalities that arise across targets, the attendant superiority of a centralized defender, the ability to exploit an attacker's strategic responses to the defender's advantage, and the accompanying superiority of a public defense, though the details could change considerably.

Our model allows the attacher only a single attack. In some cases, involving opportunistic criminal behavior or an individual plotting a suicide attack, this may be a useful approximation. However, terrorist activity often occurs as part of a centrally directed and long-term campaign.[11] Expanding the model to capture such intertemporal considerations must begin with a more careful consideration of what makes a target valuable to the attacker.

We have excluded psychological considerations from our model. We suspect that this is an especially important omission in the case of terrorism. The losses imposed by terrorist attacks are magnified by psychological considerations, just as the gains from defending against such attacks may often be primarily psychological. We accept thousands of traffic fatalities each year as routine, without abandoning our cars, while terrorist attacks causing hundreds of airline fatalities could cripple our air transportation system. Victory over terrorism may ultimately require us to view terrorist acts as routine rather than extraordinary. This again is an important direction for further research.

References

ARCE D. G., and T. SANDLER (2001) Transnational public goods: Strategies and institutions, *European Journal of Political Economy* **17**, 493–516.

ARCE, D. G., and T. SANDLER (2005) Terrorism and game theory, *Simulation and Gaming* **34**.

ARCE, D. G., and T. SANDLER (2005) Counterterrorism: A game theoretic analysis, *Journal of Conflict Resolution* **49**, 183–200.

BIER, V. (2004) Game-theoretic and reliability methods in counter-terrorism and security, Center for Human Performance and Risk Analysis, University of Wisconsin-Madison.

COUGHLIN, P. J. (1992) Pure strategy equilibria in a class of systems defense games, *International Journal of Game Theory* **20**, 195–210.

GLICKSBERG, I. L. (1952) A further generalization of the Kakutani fixed point theorem, with application to Nash equilibrium points, *Proceedings of the American Mathematical Society* **3**, 170–174.

FREY, B. S., and S. LUECHINGER (2002) Terrorism: Deterrence may backfire, Institute for Empirical Research in Economics Working Paper 136, University of Zurich.

FREY, B. S., and S. LUECHINGER (2003) How to fight terrorism: Alternatives to deterrence, *Defence and Peace Economics* **14**, 237–249.

HEAL, G., and H. KUNREUTHER (2004) Interdependent security: A general model, NBER Working Paper 10706.

KEOHANE, N. O., and R. J. ZECKHAUSER (2003) The ecology of terror defense, *The Journal of Risk and Uncertainty* **26**, 201–229.

KUNREUTHER, H., and G. HEAL (2004) Interdependent security, *The Journal of Risk and Uncertainty* **26**, 231–249.

LAKDAWALLA, D., and G. ZANJANI (2002) Insurance, self protection, and the economics of terrorism, NBER Working Paper 9215.

LAPAN, H. E., and T. SANDLER (1993) Terrorism and signalling, *European Economic Reivew* **9**, 383–397.

[11]See Lapan and Sandler (1993) and Overggaard (1994) for models with intertemporal considerations.

MAJOR, J. A. (2002) Advanced techniques for modelling terrorism risk, *Journal of Risk Finance* **4**, 15–24.

OVERGAARD, P. B. (1994) Terrorist attacks as a signal of resources, *Journal of Conflict Resolution* **38**, 452–478.

SANDLER, T. (2003) Collective action and transnational terrorism, School of International Relations Research Paper 2003/13, University of Southern California.

SANDLER, T., and W. ENDERS (2004) An economic perspective on transnational terrorism, *European Journal of Political Economy* **20**, 301–316.

SANDLER, T., and H. E. LAPAN (1988) The calculus of dissent: An analysis of terrorists' choice of targets, *Synthese* **76**, 245–261.

SHUBIK, M., and R. J. WEBER (1981) Systems defense games: Colonel Blotto, command and control, *Naval Research Logistics Quarterly* **28**, 281–287.

WEAVER, R., B. G., SILVERMAN, H., SHIN, and R. DUBOIS (2001) Modelling and simulating terrorist decision-making: A 'Performance Moderator Function' approach to generating virtual opponents, University of Pennsylvania (http://hms.upenn.edu/2001/terrist.pdf).

WOO, G. (2002) Quantitative terrorist risk assessment, *Journal of Risk Finance* **4**, 7–14.

WOO, G. (2003) Insuring against Al Qaeda, NBER Insurance Project Workshop presentation (http://www.nber.org/~confer/2003/insurance03/woo.pdf).