Secular Stagnation, Land Prices, and Collateral Constraints

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Motivation

- The 2007-2008 Financial Crisis differs considerably from other postwar recessions
  - Larger declines of macro variables
  - Slower recovery
Motivation

Housing Price
GDP

The Great Recession
Previous Recessions

GDP
Investment
Labor
Housing Price

% deviation from trend vs. Quarters after recessions start
Overview

▶ Question:

1. Why slow recovery following the Great Recession?

2. What role did the real-estate(land) sector play?

▶ Proposes a standard neoclassical model with a land sector where land serves dual roles:

1. As consumption for the households

2. As collateral for the firms to finance borrowing and working capital

▶ Results:

1. Theory: existence of multiple steady states.

2. Quantitative: substantial persistence upon large recessions

   ▶ Large recessions trigger transitions across steady states
Mechanism

**Good Steady State**

- *High Capital Accumulation*
- *High Households Wealth*
- *High Land Price*
- *Relaxed Firm Working Capital Constraint*
- *High Employment*

**Bad Steady State**

- *Low Capital Accumulation*
- *Low Households wealth*
- *Low Land Price*
- *Tightened Firm Working Capital Constraint*
- *Low Employment*
Related Literature

- The paper relates to macro models with collateral constraints (Kiyotaki and Moore, 1997)

  - Typical model features a unique steady state

- Recent extensions still feature a unique steady state:

- The paper incorporates both the consumption role of land and working capital → Multiple steady states
Road Map

- Start with a stylized model
  - Isolate the key complementary forces
  - Characterize conditions steady state multiplicity arises
- Extended model
  - Sensitivity check
  - The mechanism generates substantial persistence
Model
A Stylized Model

- Discrete time. Infinite horizon
- A continuum identical households:
  - Consume consumption goods and land, supply labor, and accumulate capital
- Owns a single private firm
  - Constant-returns-to-scale production technology
  - Working capital subject to collateral constraint
- Land supply is fixed
Households problem

\[
\max_{c, l, n, n^d, k} \sum_{t=1}^{\infty} \beta^t U(c_t, n_t, l_{t-1})
\]

Subject to:

\[
c_t + p_t l_t + k_t \leq w_t n_t + \pi_t + p_t l_{t-1} + (1 - \delta) k_{t-1}
\]

\[
\pi_t = A(n_t^d)^{1-\alpha} k_{t-1}^{\alpha} - w_t n_t^d
\]

\[
w_t n_t^d \leq \xi p_t l_t + \kappa k_t \quad \text{Timeline}
\]

\[
0 \leq n_t \leq n_0, c_t, l_t, k_t \geq 0, l_0, k_0 \text{ given}
\]
Preference

\[ U(c, l, h) = \left( c - \chi \frac{n^{1+1/\nu}}{1 + 1/\nu} \right)^{1-1/\sigma} + \omega \frac{l^{1-1/\sigma}}{1 - 1/\sigma} \]

- No wealth effect on labor supply (GHH preference)
- \( \sigma \) is both:
  - Intratemporal elasticity of substitution (Matters)
  - (Inverse of) intertemporal elasticity of substitution (Not matter)
Competitive Equilibrium

Definition

A competitive equilibrium is \( \{c_t, k_{t+1}, l_{t+1}, n_t, n^d_t\}_{t=1}^\infty \) and \( \{p_t, w_t, q_t\}_{t=1}^\infty \) such that:

1. Given prices, allocations solve the households problem.

2. Land and labor market clears every period: \( l = l_0, n = n^d \)

- A steady state is a competitive equilibrium where capital stock \( k_t \) is time invariant.
Theorem

Suppose that

1. Consumption and land are complementary (Low $\sigma$)

2. Labor supply is elastic (High $\nu$)

Then there exists an open set $U \in R^2$ such that for any combinations of loan to value ratios $(\kappa, \xi) \in U$, there exists more than one locally-stable steady states.
Warm-Up: the Frictionless Case

- Suppose there is no credit constraint
- The steady state \((c, k, n, w, p)\) is fully characterized by:

\[
\begin{align*}
\text{(Resources Constraint)} \\
&c + \delta k = A k^\alpha n^{1-\alpha} \\
\text{(Labor supply)} \\
&n^{\frac{1}{\nu}} = w \\
\text{(Labor demand)} \\
&\left( \frac{(1 - \alpha)A}{w} \right)^{\frac{1}{\alpha}} k = n \\
\text{(Capital FOC)} \\
&\beta \left[ A\alpha(k/n)^{\alpha-1} + (1 - \delta) \right] = 1
\end{align*}
\]

- The four equations in the box solve real allocations independent of land price \(p\).
Frictional Case

- The steady state \((c, k, n, w, p)\) is fully characterized by:

\[
\begin{align*}
  c + \delta k - Ak^\alpha n^{1-\alpha} &= 0 \\
  n^{\frac{1}{\nu}} &= w \\
  \min \left( \frac{\xi pl_0 + \kappa k}{w}, \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} k \right) &= n \\
  \beta \left[ A\alpha k^{\alpha-1} (n)^{1-\alpha} + (1 - \delta) + \left( \frac{1 - \alpha}{w} A(k/n)^\alpha - 1 \right) \kappa \right] &= 1
\end{align*}
\]

\[
F(p, c, k, w, n) := \omega l_0^{-\sigma} \left( c - \chi \frac{n^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} \right)^{1/\sigma} - (1 - \beta)p + \left( \frac{1 - \alpha}{w} A(k/n)^\alpha - 1 \right) \xi p = 0
\]

(Land FOC)

- Real allocations cannot be solved independent of land price \(p\)
Strategy

- Resources Constraint, labor demand, labor supply, capital FOC jointly define a mapping from land price \( p \) to \( (c, k, w, n) \)

  - This mapping is constant in the frictionless case.

- Write the land FOC as \( F(p, c, k, w, n) = 0 \)

- Plug the mapping into the land FOC \( \Rightarrow \) 1 equation 1 unknown:

\[
f(p) := F(p, c(p), k(p), w(p), n(p)) = 0
\]
Frictional Case

Willingness to buy function

- 'Willingness to buy' function:

\[
f(p) = \omega l_0^{-\sigma} \left( c(p) - \chi \frac{l(p)^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} \right)^{1/\sigma} + \left( \frac{(1 - \alpha) Ak(p)^{\alpha} l(p)^{-\alpha}}{w(p)} - 1 \right) \xi p - (1 - \beta)p
\]

- Benefit

- Net cost

- Land price \( p \) is part of steady state iff \( f(p) = 0 \).

- Crucial: \( f \) is nonmonotonic
$f(p)$ is Nonmonotonic
Why Nonmonotonic?

\[ f(p) := F(p, c(p), k(p), w(p), n(p)) = 0 \]

\[ \frac{\partial f(p)}{\partial p} = \frac{\partial F}{\partial p} + \frac{\partial F}{\partial c} \frac{\partial c}{\partial p} + \text{other terms} \quad (1) \]

Two opposing forces:

- **Direct Price Effect**: Land gets expensive, less willing to buy.
- **Indirect Collateral Effect**:

  Land Price \( \uparrow \implies \) Consumption \( \uparrow \implies \) More willing to buy land
Collateral Effect in Detail

1. Land Price \(\uparrow\)

2. Firm working capital constraint relaxed

3. Employment \(\uparrow\), Capital \(\uparrow\)

4. Households wealth \(\uparrow\), consumption \(\uparrow\)

5. Willingness to buy land \(\uparrow\) (Land-consumption complementarity)
Discussion of Assumptions

Land Price ↑ → Consumption ↑ → Willingness to buy land ↑

Labor Supply Elastic
(Need High $\nu$)

C and Land Complementary
(Need Low $\sigma$)

Otherwise

- If labor inelastically supplied ($\nu \to 0$)
  
  \[ \Rightarrow \text{Equilibrium labor not affected by working capital constraint} \]

  \[ \Rightarrow \text{Output and consumption not affected by land price} \]

- If consumption and land perfect substitutes ($\sigma \to \infty$)
  
  \[ \Rightarrow \text{Linearity structure implies level of consumption irrelevant} \]
When labor supply is perfectly elastic ($\nu \to \infty$), multiple steady states exist if and only if $\sigma < 1$. 
Quantitative Analysis
The Extended Model: Summary

- Two types of agents: households and firms. Households (HHs) are owners of the firms.

- There is a rental market for land

- HHs choose to own land (residential land) or rent it

- Firms accumulate land (commercial land) and can allocate it to rental or production use

- Land and capital can be used as collateral
The households problem

\[
\max_{c,l_n} \sum_{t=1}^{\infty} \beta^t U(c_t, n_t, l_{t-1})
\]

\[
c_t + p_t l_{ht} + r_t l_{ht-1}^r \leq p_t l_{ht-1} + d_t + w_t n_t
\]

\[
l_{t-1} = l_{1t-1} + l_{2t-1}
\]

\[
h_{10} \text{ given}, l_t \leq \bar{l}, l_{1t-1} + l_{2t-1} \geq 0
\]

- \(l_{ht}\): residential land; \(l_{ht}^r\): land rent by households;

- \(p_t\): land purchase price; \(r_t\): land rental rate;

- \(U(c, h, l) = \left[ \frac{\omega (c-\chi \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}})^{1-1/\sigma} + (1-\omega)l^{1-1/\sigma}}{1-\eta} \right]^{1-\eta} \)
The Firm’s Problem

\[
\max_{i,d,h_f,l} \sum_{t=1}^{\infty} M_t d_t
\]

\[
d_t + k_t + p_t l_{ft} \leq F(k_{t-1}, n_t, l^p_{ft-1}) + p_t l_{ft-1} - w_t n_t + r_t l^r_{ft} + (1 - \delta)k_{t-1}
\]

\[
w_t l_t \leq \xi(p_t l_{ft} + k_t)
\]

\[k_0, l_{f0} \text{ given}\]

- \(l_{ft}\): commercial land;
- \(l^r_{ft}\): land rent to households;
- \(l^p_{ft}\): land used for production;
- Cobb-Douglas Production Function: \(F(k, n, l) = k^{1-\gamma-\alpha}n^\gamma l^\alpha\)
Competitive Equilibrium

Definition

A competitive equilibrium is \( \{c_t, n_t, l_{ht}, l_{ht}^r, k_t, l_{ft}, l_{ft}^r, d_t, n_t^d\}_{t=1}^{\infty} \) and \( \{p_t, w_t, r_t\}_{t=1}^{\infty} \) such that:

1. Given prices, allocations solve the households problem.

2. Land, labor, and land rental market clears every period:

\[
l_h + l_f = h_0, n = n^d, l_h^r = l_f^r\]

An Equivalence Result

Suppose the land’s share in production $\alpha = 0$. Then the equilibrium allocations (consumption, labor, capital, and investment) and prices in the extended model are the same as those in the stylized model.
## Calibration

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<thead>
<tr>
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<td>Loan to value ratio</td>
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Table 1: Calibration
S-shaped Law of Motion for Capital

Transitional Dynamics

Capital
Accounting for the Slow Recovery

- Feed in unexpected shock to loan-to-value ratio $\xi$ that lasts seven quarters from 2007q4 to 2009q2...

- Such that land price drops by 25%
Accounting for the slow recovery

**Output**

- Data
- Model

**Investment**

- Data
- Model

**Labor**

- Data
- Model

**Land Price**

- Data
- Model
Conclusion

- I propose a theory where slow recoveries follow deep recessions.

- Crucial ingredient: the dual role of land as households consumption and firm collateral

- The model generates persistent recessions comparable to post-Great Recession data
Timeline

Firm Produces, Borrows to Finance Wage
Market Opens Firm Revenue Realizes Firm Repays Loan
If Default Lender seizes fraction of capital and land
t-1 t t+1
Cash Flow Mismatch
Define an operator $T$ mapping $[K'(K), P(K)]_n$ to $[K'(K), P(K)]_{n+1}$

- Existence (Schauder’s Theorem)
  - Problem is continuous and general fixed point theorem apply.

- Uniqueness
  - Peudo-Concavity
  - $x_0$ monotonicity
Little consensus in the literature

- Most micro estimates between 0.13 and 0.6
  - Flavin and Nakagawa (2008), Hanushek and Quigley (1980), Siegel (2008), Stokey (2009), Li, Liu, Yang, and Yao (2016)
  - Piazzesi, Schneider, and Tuzel (2007): > 1
  - Bajari, Chan, Krueger, and Miller (2012): > 6

- Set $\eta = 0.33$ as benchmark
- Constraint occasionally binding $\Rightarrow$ cannot estimate using steady state targets

- Here: Constraint binds with 7.5% drop in output

- $\xi = 0.04$
Accounting for the labor wedge
Accounting for the slow recovery

Credit shock ($\xi$)

Productivity shock (A)
Accounting for the slow recovery
Accounting for the slow recovery

**Output**
- Data
- Benchmark model
- Credit Shock only

**Investment**

**Labor**

**Land Price**
Computation

- Aggregate State $X = (K, W_-)$ where $W_-$ is previous period wage.

- Need to solve:

  1. Law of motion for capital: $K' = \Phi (K, W_-)$
  2. Labor function: $L = L (K, W_-)$
  3. Land price function: $P = P (K, W_-)$
  4. Law of motion for wage: $W = W (K, W_-)$
Policy Functions

\[
\begin{align*}
K' &= K - K \\
L &= 0.22, 0.24, 0.26, 0.28, 0.3, 0.32, 0.34 \\
w_{t-1} &= w_{ss} \\
P &= 3, 4, 5, 6, 7, 8 \\
W &= 1.98, 1.99, 2, 2.01, 2.02, 2.03
\end{align*}
\]
Typical Dynamics

\[ K_{t+1} \]

\[ \zeta^4 w_{ss} \]
\[ \zeta^3 w_{ss} \]
\[ \zeta^2 w_{ss} \]
\[ \zeta w_{ss} \]
Typical Dynamics

\[ K_{t+1} \]

\[ \zeta^4 w_{ss} \]
\[ \zeta^3 w_{ss} \]
\[ \zeta^2 w_{ss} \]
\[ \zeta w_{ss} \]

Small shock

Big shock

\[ C_2 \quad C_1 \quad C_3 \quad C \quad C_4 \quad B \quad B_1 \quad B_2 \]

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The Extended Model

- Representative households:
  - Consume consumption goods and land, supply labor, receive dividends from the representative firm.

- Representative firm
  - Hire labor and produce; accumulate capital and land; pay out dividend to households
  - Subject to working capital constraint

- Land supply is fixed
Real GDP and its linear trend 1970-2007
S&P/Case-Shiller U.S. National Home Price Index, deflated by GDP deflator
Lincoln Institute of Land Policy, Davis and Heathcote (2007)
Cross sectional evidence

Question: Is there a systematic relation between the extent of housing price drop and pace of recovery at the MSA level?
Labor Wedge

- Construct firm component of the labor wedge following Karabarbounis (2013)
- Large and persistent spike after Lehman’s bankruptcy

Data
Equilibrium Labor

- Frictionless
- Credit constraint only
- Sticky wage only
- Dual friction

Graph showing the relationship between $K_t$ and $L_t$.
Drop of labor in percentage terms

\[
\frac{(L_t^{\text{constrained}} - L_t^{\text{unconstrained}})}{L_t^{\text{unconstrained}}}
\]

Frictionless
Sticky wage

\[
\begin{align*}
&\text{Frictionless} \\
&\text{Sticky wage}
\end{align*}
\]
Drop of labor in percentage terms

\[
\frac{(L_t^{\text{constrained}} - L_t^{\text{unconstrained}})}{L_t^{\text{unconstrained}}} \text{ vs } K_t
\]
Recursive households problem with sticky wage

\[ V(k, h, b; K) = \max_{c, h', k', b', l, l^d} u(c_t, h_t) + \beta V(k', h', b'; K') \]

Subject to:

\[ c + p(K)h' + b + k' \leq w(K)l + \pi + p(K)h + q(K)b' + (1 - \delta)k \]

\[ 0 \leq l \leq \min\{l_0, l(K)\}, c, h', k' \geq 0 \]

\[ K' = \Phi(K) \]

Where

\[ \pi = \max_{l^d} A(l^d)k^\alpha - w(K)l^d \]

\[ w(K)l^d \leq \xi p(K)h' \]

\[ l^d \leq l(K) \]
Recursive Competitive Equilibrium

Definition

A \textit{sticky-wage} recursive competitive equilibrium is functions
\((h', k', b', l, l^d)(k, h, b; K), p(K), w(K), q(K), l(K), \Phi(K)\) such that:

1. Given the price functions, the decision rules solve the households problem

2. Housing and bond market clears every period: \(h = h_0, b = 0\)

3. \(w(K) = w_0\) for some \(w_0\).

\[l(K) = \min\{l(K, h_0, 0; K), l^d(K, h_0, 0; K)\}\]

4. Consistency: \(\Phi(K) = k'(K, h_0, 0; K);\)
Functional equations of RCE

The system of equations characterizing \((C(K), P(K), \Phi(K), l(K))\) are:

\[
\theta h_0^{-\eta} + \beta p(\Phi(K))(C(\Phi(K)))^{-\sigma} = \]

\[
(C(K))^{-\sigma} \left( 1 - \left( \gamma AK^\alpha (l(K))^{\gamma-1} - w_{ss} \right) \frac{\xi}{w_{ss}} \right) \frac{1}{w_{ss}} \left\{ \frac{w_{ss}}{\gamma A} \frac{1}{\gamma-1} K^{\frac{\alpha}{1-\gamma}} > \frac{\xi p(K) h_0}{w_{ss}} \right\} p(K)
\]

\[
(C(K))^{-\sigma} = \beta (C(\Phi(K)))^{-\sigma} \left( \alpha A (\Phi(K))^{\alpha-1} (l(\Phi(K)))^{\gamma} + 1 - \delta \right)
\]

\[
l(K) = \min \left\{ \left( \frac{w_{ss}}{\gamma A} \right)^{\frac{1}{\gamma-1}} K^{\frac{\alpha}{1-\gamma}}, \frac{\xi p(K) h_0}{w_{ss}}, l_0 \right\}
\]

\[
C(K) = AK^\alpha [l(K)]^\gamma + (1 - \delta) K - \Phi(K) + y_0
\]
## Numerical Example

<table>
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Dynamics

Sticky wage model + credit constraint
Dynamics

\[ K_{t+1} - K_t \]

- Frictionless model
- + credit constraint

Graph showing dynamics with two lines representing different models.
Recursive households problem

\[ V(k, h, b; K) = \max_{c, h', k', b', l, l^d} \frac{c^{1-\sigma}}{1 - \sigma} + \theta \frac{h^{1-\eta}}{1 - \eta} + \beta V(k', h', b'; K') \]

Subject to:

\[ c + p(K)h' + b + k' \leq w(K)l + \pi + p(K)h + q(K)b' + (1 - \delta)k \]

\[ 0 \leq l \leq l_0, c, h', k' \geq 0 \]

\[ K' = \Phi(K) \]

Where

\[ \pi = \max_{l^d} A(l^d)^\gamma k^\alpha - w(K)l^d \]

\[ w(K)l^d \leq \xi p(K)h' \]

⇒ Decision rules: \((h', k', b', l, l^d)(k, h, b; K)\)
Recursive Competitive Equilibrium

Definition

A *flexible-wage* recursive competitive equilibrium is a collection of functions \((h', k', b', l, l^d)(k, h, b; K), p(K), w(K), q(K), \Phi(K)\) such that:

1. Given price functions, decision rules solve the hh’s problem

2. Housing, labor, bond market clears: \(h = h_0, l^d = l, b = 0\)

3. Consistency: \(k'(K, h_0, 0; K) = \Phi(K)\)
Local Stability

Theorem
Suppose $\xi \in (\xi_{ss}, \xi_0]$ for some $\xi_0$, $\alpha + \gamma < 1$ and $\sigma$ sufficiently big. There exists multiple locally stable steady states.

Proof: Still working in progress
Step 1: the continuity of $f$

1. Identify the ’kink’ $\bar{p}$:

$$\xi \bar{p} h_0 = w_{ss} l_0$$

2. $f(p)$ is continuous for $p$ less than $\bar{p}$ and for $p$ greater than $\bar{p}$:

$$f(p) = \begin{cases} 
\theta h_0^{-\eta} c_s^\sigma - (1 - \beta) p, & \text{for } p \geq \bar{p} \\
 a_0 p^{1-\alpha \sigma} + a_1 p^{1-\alpha} - a_2 p, & \text{for } p < \bar{p}
\end{cases}$$

3. $f(p)$ is continuous at $\bar{p}$. 
Step 2: existence of multiple s.s. at $\xi_{ss}$

- when $\xi = \xi_{ss} = \frac{w_{ss}l_0}{p_{ss}h_0}$, $\bar{p} = p_{ss}$, thus $f(\bar{p}) = f(p_{ss}) = 0$.

- Assumptions required: $\sigma$ sufficiently big, $\alpha + \gamma < 1$.

- Steps:
  1. $f(0; \xi_{ss}) = 0$ and $f''^+(0; \xi_{ss}) > 0$
     $\Rightarrow \exists p_1 > 0 \text{s.t.} f(p_1, \xi_{ss}) > 0$
  2. $f(p_{ss}; \xi_{ss}) = 0$ and $f'^-(p_{ss}; \xi_{ss}) > 0$
     $\Rightarrow \exists p_2 > p_1 \text{s.t.} f(p_2, \xi_{ss}) < 0$
  3. By continuity of $f$, we are done.
Step 3: extend the result to $\xi > \xi_{ss}$

- Pick $\varepsilon > 0$ sufficiently small such that for any
  $$\xi \in (\xi_{ss}, \xi_{ss} + \varepsilon]:$$
  - $f(\bar{p}, \xi) > 0$ where $\bar{p} > p_2 > p_1$
  - $f(p_1, \xi) > 0$
  - $f(p_2, \xi) < 0$

- Slight complication: need to show $f$ is continuous w.r.t $\xi$
  for $\xi$ sufficiently close to $\xi_{ss}$, at $p = p_1$ and $p = p_2$. 
Introduction

▶ "Secular stagnation" episodes:

▶ Follow large crises: Great Recession, Japanese 1990 crisis, and Great Depression.

▶ Feature slow recovery of output, employment, and investment.

▶ Existing theories: focus on insufficient demand arising from zero lower bound.

Endogenous Land Allocation

- Land price is volatile whereas residential and commercial land growth is largely uniform (Davis and Jonathan 2007, Davis 2009)

- Introduce adjustment cost for land

\[ \phi(h_t^f, h_{t-1}^f) = \tau\left(\frac{h_t^f - h_{t-1}^f}{h_{t-1}^f}\right)^2 h_{t-1}^f \]

- Set \( \tau \) to \( \infty \).
Conclusion

- This paper proposes a theory of how land price dynamics contributes to secular stagnation.

- **Slower** recovery speed upon **large** negative shocks.

- Crucial elements: wage rigidities and working capital constraint on firm.

- Future work: more serious quantitative experiment.
Fruitionless Case: GHH

- Consider an environment with no working capital constraint.

- There exists a unique steady state \( \{l_{ss}, k_{ss}, w_{ss}, c_{ss}, p_{ss}\} \) where:

\[
\begin{align*}
l_{ss} &= \left[ (1 - \alpha) A \left( \frac{\frac{1}{\beta} - 1 + \delta}{A\alpha} \right)^{\frac{\alpha}{\alpha-1}} \right]^\nu \\
k_{ss} &= ((1 - \alpha) A)^\nu \left( \frac{\frac{1}{\beta} - 1 + \delta}{A\alpha} \right)^{\frac{1 + \alpha \nu}{\alpha-1}} \\
w_{ss} &= \gamma A \left( \frac{\frac{1}{\beta} - 1 + \delta}{A\alpha} \right)^{\frac{\alpha}{\alpha-1}}
\end{align*}
\]
The model with perfectly sticky wage

- Discrete time. Infinite horizon
- A continuum identical households/entrepreneur:
  - Consume consumption goods and housing, supply labor inelastically, and accumulate capital
- Owns a single private firm
  - CD production technology
  - Working capital subject to collateral constraint
- Land supply is fixed
Households problem

\[
\max_{c,h,l,l^d,k} \sum_{t=1}^{\infty} \left[ \left( \omega c_t^{1-1/\eta} + (1 - \omega) h_t^{1-1/\eta} \right)^{1/(1-1/\eta)} \right]^{1-\sigma} - \chi \frac{l_t^{1+1/\nu}}{1 + \frac{1}{\nu}}
\]

subject to

\[
c_t + p_t h_t + b_t + k_t \leq w_t l_t + \pi_t + p_t h_{t-1} + q_t b_{t+1} + (1 - \delta) k_{t-1}
\]

\[
\pi_t = \max_{l^d_t} A \left( l^d_t \right)^{\gamma} k_t^{\alpha} - w_t l^d_t
\]

\[
q_t b_{t+1} + \theta w_t l^d_t \leq \xi p_t h_t + \kappa k_t
\]

\[
0 \leq l_t \leq l_0, c_t, h_t, k_t \geq 0, h_0, k_0 \text{ given}
\]
Competitive Equilibrium

Definition

A competitive equilibrium given sticky wage $w_0$ is

$$\{c_t, k_{t+1}, h_{t+1}, l_t, l^d_t, b_t\}_{t=1}^\infty$$ and $$\{p_t, w_t, q_t\}_{t=1}^\infty$$ such that:

1. Given prices, allocations solve the households problem.

2. Housing and bond market clears every period: $h = h_0, b = 0$

3. $w(K) = w_0$ for some $w_0$. Equilibrium labor is determined by the minimum of labor demand and labor supply.

- A steady state a competitive equilibrium where capital stock $k_t$ is time invariant.
Consider an environment with no working capital constraint, and wage is flexible.

there exists a unique steady state \( \{l_{ss}, k_{ss}, w_{ss}, c_{ss}, p_{ss}\} \) given by:

\[
\begin{align*}
\omega h_0^{-1/\eta} c^{1/\eta} + \beta p &= p \\
\beta \left( \alpha A \gamma k^{\alpha-1} + 1 - \delta \right) &= 1 \\
\omega c^{-1/\eta} \left[ \omega c^{1-1/\eta} + (1 - \omega) h_0^{1-1/\eta} \right]^{(1-\sigma)/(1-1/\eta)-1} &= w \chi l^{1/\nu} \\
c &= A \gamma k^{\alpha} - \delta k \\
w &= \gamma A \gamma^{-1} k^{\alpha}
\end{align*}
\]

focus on equilibrium with \( w_t = w_{ss} \) (Shimer, 2012)
Suppose $\eta < 1$. Then there exists an $\bar{\eta}(\eta) < 1$ such that for any $\alpha + \gamma > \bar{\eta}$ and $\kappa$ sufficiently small, there exists an interval $U^\xi$ such that, if $\xi \in U^\xi$, then there exists more than 1 locally stable sticky-wage steady states given $w_{ss}$. 
Steady state characterization

- The steady state \((c, k, p)\) is fully characterized by:

\[
\frac{1 - \omega}{\omega} h_0^{-\frac{1}{\eta}} + \beta pc^{-\frac{1}{\eta}} + \left( \frac{\gamma Ak^{\alpha \ell(k, p)} \gamma^{-1}}{w_{ss}} - 1 \right) \xi pc^{-\frac{1}{\eta}} = pc^{-\frac{1}{\eta}}
\]

\[
\beta \left[ A\alpha k^{\alpha - 1} (l(k, p))^\gamma + (1 - \delta) + \left( \frac{\gamma Ak^{\alpha \ell(k, p)} \gamma^{-1}}{w_{ss}} - 1 \right) \xi \right] = 1
\]

\[
c + \delta k - Ak^{\alpha \ell(k, p)}^\gamma = 0
\]

where

\[
l(k, p) = \min \left\{ \frac{\xi (ph_0 + k)}{w_{ss}}, \left( \frac{\gamma A}{w_{ss}} \right)^{\frac{1}{\gamma - 1}} k^{\frac{\alpha}{1 - \gamma}}, l_0 \right\}
\]

- The last two equations define a mapping from \(p\) to \(k, c\).
Willingness to pay function

- Define a ’willingness to pay’ function in consumption units:

\[
f(p) = \frac{1 - \omega}{\omega} h_0^{\frac{-1}{\eta}} c(p)^{\frac{1}{\eta}} + \left( \frac{\gamma A k(p)^{\alpha} l(k(p), p)^{\gamma-1}}{w_{ss}} - 1 \right) \xi p - (1 - \beta)p
\]

- \((c, k, p)\) is a steady state if and only if the associated \(p\) satisfies \(f(p) = 0\)
Graphic Illustration of $f(p)$
Outline of the proof:

- $f(p)$ is continuous w.r.t. $p$ for any $\xi$

- Define $\xi_{ss} = \frac{w_{ss}l_0}{p_{ss}h_0 + k_{ss}}$. Show that given $\xi_{ss}$, there exists multiple nontrivial steady states:
  
  1. $f(0; \xi_{ss}) > 0$
  2. $f(p_{ss}; \xi_{ss}) = 0$ and $f'(p_{ss});$

- Show that the case extends to $\xi \in (\xi_{ss}, \xi_0]$ for some $\xi_0$. 

Graphic Illustration of $f(p)$
Dynamics with perfectly sticky wage
Dynamics with perfectly sticky wage
Wage adjustment rule II

- wage adjustment rule:

\[ w_t \geq \zeta w_{t-1} \]

- set \( \zeta = 0.995 \)

- Nondecreasing nominal wage in an economy with 2% inflation.
Dynamics with wage adjustment rule II
Wage adjustment rule

- wage adjustment rule:

\[ \log(w) - \log(w_{ss}) = \varepsilon^w (\log(y) - \log(y_{ss})) \]

- set \( \varepsilon^w = 0.45 \)


- Steady state multiplicity arises when \( \eta \leq 0.3 \)
Equilibrium Functions

- **Capital**: The graph shows the capital function as a function of $k$. The curve peaks at a certain value of $k$.

- **Labor**: The labor function increases as the $k$ value increases.

- **Land price**: The land price increases with increasing $k$.

- **Wage**: The wage increases with increasing $k$.
Dynamics with different $\eta$

Li etc. (2016): $\eta = 0.487$
Benchmark: $\eta = 0.3$
FN (2008): $\eta = 0.13$
Accounting for the slow recovery

![Graph showing data and model for investment, land price, labor, and output.](image)
Accounting for the labor wedge

- Labor wedge spikes during the recession and remains high after the recession:

\[
\text{labor wedge} = \log(MPN) - \log(MRS)
\]
Outline of the proof:

- $f(p)$ is continuous w.r.t. $p$ for any $\xi$

- Solve the unique unconstrained steady state $(c_{ss}, k_{ss}, p_{ss}, w_{ss}, l_{ss})$.

- If $\kappa$ is sufficiently small, define $\xi_{ss} = \frac{w_{ss}l_{ss} - \kappa k_{ss}}{p_{ss}h_0}$. Show that given $\xi_{ss}$, there exists multiple nontrivial steady states:
  
  1. $f(0; \xi_{ss}) > 0$

  2. $f(p_{ss}; \xi_{ss}) = 0$ and $f'(\cdot)(p_{ss})$;

- Show that the case extends to $\xi \in (\xi_{ss}, \xi_0]$ for some $\xi_0$. 
Graphic Illustration of $f(p)$
Frictional Case

- The steady state \((c, k, p, w, n)\) is fully characterized by:

\[
\begin{align*}
    c + \delta k &= Ak^\alpha n^{1-\alpha} \\
    n^{1/\nu} &= w \\
    \left( \frac{(1 - \alpha)A}{w} \right)^{\frac{1}{\alpha}} k &= n \\
    \beta \left[ A\alpha k^{\alpha-1} (n)^{1-\alpha} + (1 - \delta) \right] &= 1 \\
    \omega h_0^{-\sigma} \left( c - \chi \frac{n^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} \right)^{1/\sigma} + \beta p &= p
\end{align*}
\]
Frictional Case

- The steady state \((c, k, p, w, l)\) is fully characterized by:

\[
c + \delta k = Ak^\alpha n^{1-\alpha}
\]

\[
n^{\frac{1}{\nu}} = w
\]

\[
\min \left( \frac{\xi pl_0 + \kappa k}{w}, \left( \frac{(1 - \alpha)A}{w} \right)^{\frac{1}{\alpha}} k \right) = n
\]

\[
\beta \left[ A\alpha k^{\alpha-1} (n)^{1-\alpha} + (1 - \delta) + \left( \frac{(1 - \alpha)Ak^\alpha (n)^{-\alpha}}{w} - 1 \right) \kappa \right] = 1
\]

\[
\omega l_0^{-\sigma} \left( c - \chi \frac{n^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} \right)^{1/\sigma} + \beta p + \left( \frac{(1 - \alpha)Ak^\alpha (n)^{-\alpha}}{w} - 1 \right) \xi p = p
\]
View of This Paper

- Economy exhibits **multiple ”regimes”** (locally stable steady states)
  - Good regime: high capital accumulation $\iff$ high land price
  - Bad regime: low capital accumulation $\iff$ low land price
- Thus **asymmetric** response to small and large shocks:
  - Mild recession $\Rightarrow$ Quick recovery
  - **Severe** recession $\Rightarrow$ **Regime switch** $\Rightarrow$ **Permanent** impact
Model Overview

Standard Neoclassical model with a land sector

- Land serves dual roles:
  - Consumption for the households
  - Collateral for the firm to finance its working capital

- Result:

  The equilibrium law of motion for capital is S-shaped with multiple locally stable steady states.