Abstract
Using data on asset pricing anomalies, I test the idea that the act of arbitrage itself generates endogenous risk. Theoretically, I embed a set of mispriced anomaly assets in a model where arbitrageurs have limited capital. The act of arbitrage makes anomaly assets endogenously risky by causing their prices to comove with shocks to arbitrage capital. Crucially, this endogenous risk is larger for assets that were initially more mispriced since they attract correspondingly more arbitrage capital. Thus, arbitrage turns assets with high initial “alphas” into assets with high endogenous “betas.” Empirically, I study 34 anomaly assets from 1972 to 2015, splitting the sample into the period before 1993, when there was little arbitrage activity, and after. The data matches the model’s key cross-sectional predictions: (i) an anomaly’s initial profitability—its pre-93 return—predicts its subsequent endogenous risk—its post-93 beta with respect to arbitrage capital; (ii) this beta is explained by the amount of arbitrage capital devoted to the anomaly; (iii) this beta explains the anomaly’s expected return that survives in equilibrium.
1 Introduction

Asset pricing “anomalies” are investment strategies with high expected returns but low identifiable risks. These anomalies—such as value and momentum—first gained widespread recognition among finance academics and investment managers in the early 1990s.1 Since then, arbitrageurs such as hedge funds have allocated growing amounts of capital to these anomalies. As a result, the abnormal returns on these anomalies have fallen, but have not completely disappeared.2

What prevents arbitrageurs from completely eliminating anomaly returns? Are anomalies commonly exposed to hidden fundamental risks, so that the remaining anomaly returns represent fair compensation for these hard-to-measure risks? Or have anomalies become increasingly exposed to “endogenous risks” because of the very fact that many arbitrageurs are attempting to exploit them?

In this paper, I argue both theoretically and empirically that arbitrage activity exposes asset pricing anomalies to endogenous risks associated with the act of arbitrage. The emergence of these endogenous risks means that anomaly returns may survive in equilibrium even when the amount of arbitrage capital becomes large.

My key contribution is to draw out the implications of this endogenous risk view for the cross-section of “anomaly assets,” long-short portfolios that exploit asset pricing anomalies. Specifically, if demand curves for anomaly assets slope downward, the prices of anomaly assets comove with shocks to arbitrageur capital. This endogenous comovement is especially large for an anomaly with a large latent mispricing—abnormal return in the absence of arbitrageurs—since it attracts correspondingly more arbitrage capital. This way, arbitrage turns assets with high “alphas” into assets with high endogenous “betas.” This key insight allows me to develop cross-sectional tests that have more power than pure time-series tests.

To formalize my argument, I develop a model in which arbitrageurs can exploit many anomalies that differ solely in the degree of the latent mispricing that would prevail without arbitrageurs. In my three-period model, there is a continuum of anomaly assets with the same expected cash flow at the final date (time 2). A set of behavioral investors have a downward-sloping demand curve for each anomaly asset at times 0 and 1. Critically, behavioral investors undervalue the anomaly assets’ cash flow, and the degree of undervaluation—the latent mispricing—differs across anomalies.

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1Fama and French (1992, 1993, 1996) and Jegadeesh and Titman (1993) ignited this interest. However, anomalies such as size (Banz, 1981) and value (Rosenberg, Reid, and Lanstein, 1985) were documented earlier.

2McLean and Pontiff (2016) find an average 32% decline in the returns of 97 anomalies after their publication. Chordia, Subrahmanyam, and Tong (2014) also find that anomaly returns have not completely disappeared.
Arbitrageurs in my model are risk-neutral but face a stochastic funding constraint at time 1 that generates exogenous variation in the capital that they can deploy. As arbitrageur funding at time 1 improves, arbitrageurs devote greater capital to anomaly assets, raising their equilibrium prices. From the perspective of arbitrageurs at time 0, the existence of the stochastic funding constraint means that anomaly prices comove with their capital at time 1. And, crucially, this comovement is stronger for anomalies with higher degrees of latent mispricing. Since arbitrageurs want to hedge their time-1 capital shocks, this makes the more mispriced anomalies endogenously riskier for arbitrageurs to hold at time 0 (Merton, 1973). As a result, in equilibrium, anomalies with greater latent mispricing must offer higher endogenous risk compensation from time 0 to time 1.

In summary, in my model, arbitrage activity necessarily exposes anomaly assets to endogenous risks. These endogenous risks mean that anomaly returns persist in equilibrium. And these endogenous risks imply that an “intermediary asset pricing” model can explain the cross-section of anomaly expected returns: they line up with the anomaly’s exposure to arbitrageur funding shocks even though the anomaly asset has no fundamental link to those shocks.

The model makes three key predictions about the cross-section of anomaly assets. First, anomalies with greater latent mispricing become more exposed to endogenous risk—i.e., larger $\alpha$s turn into larger $\beta$s with respect to the funding conditions of arbitrageurs (Proposition 1). Second, this endogenous risk of an anomaly is explained by the amount of arbitrageur capital dedicated to the anomaly—i.e., arbitrageur funding $\beta$s line up with anomaly-level measures of arbitrage activity (Proposition 2). Third, an anomaly’s exposure to endogenous risk explains its expected return in equilibrium—i.e., arbitrageur funding $\beta$s can “price” the cross-section of anomalies (Proposition 3).

I test the model’s three main predictions using data on 34 equity anomaly assets from 1972 to 2015. Splitting the sample period in half, I proceed under the assumption that the pre-1993 period featured little arbitrage on anomalies whereas the post-1993 period features more arbitrage. I measure the funding conditions of arbitrageurs using the leverage of security broker-dealers, similar to the measure of financial intermediary funding conditions used in Adrian, Etula, and Muir (2014). For main empirical analyses, I use the generalized method of moments (GMM) to obtain conservative standard errors for the test parameters.

My empirical tests support the model’s predictions. In the pre-93 sample, anomalies generated large long-short returns but had little exposure to arbitrageur funding shocks (see Figure 1a). In the post-93 sample, however, as arbitrageur capital has flowed into anomaly assets, anomaly returns have fallen while their endogenous exposures to arbitrageur funding shocks have risen (see Figure 1b). And, consistent with the cross-sectional prediction of Proposi-
Notes: This is a preview of Figures 1 and 2 in Tables and Figures. Each circle is an anomaly asset. Pre-93 (1972-1993) and post-93 (1994-2015) periods respectively proxy for periods with little and more arbitrage on the anomalies. Long-short return is the difference between the value-weighted returns on the top and bottom deciles of stocks, where all stocks are sorted into deciles based on an anomaly signal (e.g., book-to-market ratio). Return is reported as an annualized percentage. Funding beta is the beta with respect to arbitrageur funding shocks measured by quarterly shocks to the leverage of broker-dealers.

An anomaly’s latent mispricing—its pre-93 return—predicts its subsequent endogenous risk—its post-93 beta with respect to arbitrageur funding (Figure 2). Furthermore, as predicted by Proposition 2, these post-93 funding betas are explained by anomaly-specific arbitrage capital inferred from short interests.

Consistent with the intermediary asset pricing logic of Proposition 3, the post-93 expected returns of different anomalies line up with their endogenous risks measured using post-93 betas with respect to arbitrageur funding (see the fitted line through the circles in Figure 1b). The intercept of the cross-sectional regression is not zero but positive, which is predicted by the model: anomaly assets generate risk-adjusted returns above the risk-free rate whenever arbitrage capital is insufficient to price all anomaly assets correctly. Interestingly, the price of risk estimated in the pooled period of 1972-2015 is larger than that estimated in the post-93 period, when arbitrageurs have become more important (the slope in Figure 3 is larger than the slope in Figure 1b). This is because anomalies with large funding betas and large equilibrium returns in the post-93 period had even larger returns in the pre-93 period before arbitrage began. The funding betas of all anomalies, however, were close to zero in the pre-93 period. As a result, pooling the two periods increases the spread of returns and decreases the spread of betas, generating an upward bias in the estimated price of risk.

Additional empirical tests support auxiliary implications of the model. First, treating the long and short sides of an anomaly as separate assets, I show that a unit of pre-93 abnormal return turns into a larger post-93 endogenous risk on the short side. This is consistent with the view that short sides of anomalies are primarily traded by leveraged arbitrageurs such as
hedge funds but long sides of anomalies are accessible to a wider set of investors. Second, I find that the covariation between anomaly returns and arbitrageur funding conditions occurs only when arbitrageurs are likely to be constrained, consistent with the model’s prediction that arbitrageurs exert price pressure on anomalies only when they are constrained (Proposition 4). Finally, I show that using the equity market-neutral index return from Hedge Fund Research (HFR) to measure shocks to arbitrageur capital delivers results similar to those obtained using my proxy for arbitrageur funding shocks.

What alternative explanations might account for my main findings? Suppose that anomalies with high average returns are exposed to some fundamental—as opposed to endogenous—risk factor that, for whatever reason, has become more correlated with arbitrageur funding shocks in recent years. In this case, an anomaly’s pre-93 mean return would appear to predict its post-93 arbitrageur funding beta, generating my key “αs into βs” result. If anomalies’ returns were driven by fundamental risks of this sort, one would expect the underlying firms’ cash flows to covary with arbitrageurs funding shocks. To examine this possibility, I examine whether anomaly assets’ cash flows covary with arbitrageur funding shocks, using the return on book equity to measure cash flows as in Campbell and Vuolteenaho (2004), and find no evidence that the anomaly assets have fundamental cash-flow exposures to arbitrageur funding shocks.

**Implications for the literature.** This paper tests cross-sectional predictions of the idea that the act of arbitrage makes mispriced assets endogenously risky. First formalized by Shleifer and Vishny (1997), this idea has been a central explanation for the occurrence of apparent arbitrage opportunities and has been extended to show that the act of arbitrage induces various forms of instability in financial markets. Empirical tests of the idea have relied on time-series variation in arbitrage capital and the ability of an arbitrageur-related risk factor to explain anomaly returns. For instance, Frazzini and Pedersen (2014) show that the “betting against market beta” portfolio realizes a low return when funding constraints tighten, highlighting the endogenous link between the amount of arbitrage capital and the prices of arbitraged assets. Drechsler and

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3 Although Shleifer and Vishny (1997) use noise traders to generate shocks to arbitrage capital, the specific source of the arbitrage capital shocks is unimportant. As pointed out in Shleifer (2000), the endogenous risk arises whenever arbitrageurs depend on external (debt or equity) capital, which prevents them from raising more capital when their capital level falls and the mispricing that they bet against widens.

4 Documented examples of apparent arbitrage opportunities include price divergence in Siamese-twin stocks (Rosenthal and Young, 1990; Froot and Dabora, 1999), negative stub values (Mitchell, Pulvino, and Stafford, 2002; Lamont and Thaler, 2003), and on-the-run vs. off-the-run bond spreads (Amihud and Mendelson, 1991; Warga, 1992; Krishnamurthy, 2002). The instabilities include contagion (Kyle and Xiong, 2001), fire sales (Gromb and Vayanos, 2002; Morris and Shin, 2004; Allen and Gale, 2005), liquidity spirals (Brunnermeier and Pedersen, 2009), and crash risks (Stein, 2009).

5 See Gärleanu and Pedersen (2011), Chordia, Subrahmanyam, and Tong (2014), Akbas, Armstrong, Sorescu,
Drechsler (2016) show that the cheap-minus-expensive-to-short (CME) portfolio, interpreted as the portfolio of arbitrageurs who focus on shorting, explains the returns on eight prominent equity anomalies.\textsuperscript{6} My paper complements these findings by testing a set of new cross-sectional implications of the endogenous-arbitrage-risk idea.

This paper’s findings suggest that, from arbitrageurs’ point of view, the equity anomalies represent mispricings turned into endogenous risks, contributing to the debate on the nature of asset pricing anomalies.\textsuperscript{7} This complements the time-series evidence that anomaly returns have decayed due to increased arbitrage activity following improved liquidity (Chordia, Subrahmanyam, and Tong, 2014) and academic publication (McLean and Pontiff, 2016), as well as the evidence that the return correlation between the top and bottom deciles of an anomaly falls after its academic publication (Liu, Lu, Sun, and Yan, 2015). Although my empirical tests focus on equity anomalies, my predictions apply to other asset classes. Interestingly, Brunnermeier, Nagel, and Pedersen (2009) observe that more profitable currency carry trades are subject to higher crash risks because they attract more arbitrage capital.

Finally, this paper proposes the origin of intermediary asset pricing betas. Intermediary asset pricing theories posit that, in the presence of financial frictions, shocks specific to financial intermediaries carry a risk premium (Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2012, 2013; and Brunnermeier and Sannikov, 2014). Adrian, Etula, and Muir (2014) test this empirically, finding that intermediary funding shocks inferred from the leverage of broker-dealers explain the returns on equity portfolios sorted by size, value, and momentum and bond portfolios sorted by maturity.\textsuperscript{8} However, existing work on intermediary asset pricing offers no explanation on the origin of betas—why some assets have larger exposures to financial sector shocks than others. I show that certain assets have high betas with respect to financial sector shocks because those assets have large latent mispricing and attract large arbitrage capital.

**Outline.** The paper proceeds as follows. Section 2 theoretically examines a model of arbitrageurs exploiting differently mispriced anomalies subject to a stochastic funding constraint. Section 3 empirically tests the model’s implications using the cross-section of anomaly assets. Section 4 presents additional empirical analyses. Section 5 concludes.

\textsuperscript{6}As I discuss in Section 2, they also solve a model of arbitrageurs that shares many similarities to mine.

\textsuperscript{7}See, e.g., Fama and French (1993); Lakonishok, Shleifer, and Vishny (1994); Daniel and Titman (1997); Davis, Fama, and French (2000); Campbell, Polk, and Vuolteenaho (2010); and Kozak, Nagel, and Santosh (2015). Some papers attribute anomaly returns to transaction costs (e.g., Korajczyk and Sadka, 2004; Novy-Marx and Velikov, 2016), although there is also an opposing view (e.g., Frazzini, Israel, and Moskowitz, 2015).

\textsuperscript{8}He, Kelly, and Manela (2016) and Kozak, Nagel, and Santosh (2015) also test intermediary asset pricing.
2 A model of arbitrageurs trading multiple assets

In this section, I develop a model in which arbitrageurs can exploit many different anomalies that differ solely in the degree of the latent mispricing that would prevail without arbitrageurs. The model shows that arbitrage activity exposes anomaly assets to endogenous risks and that this endogenous risk is higher for an anomaly with greater latent mispricing. The model generates additional testable predictions about the cross-section of anomaly assets.

2.1 Model setup

Time horizon, assets, and investors. Consider an economy with three time periods, \( t = 0, 1, 2 \). The economy has two types of assets: a risk-free asset and a continuum of risky assets which I call “anomaly assets.” The risk-free asset is in infinite supply with zero interest rate. An anomaly asset, indexed by \( j \in [0, 1] \), is a claim to an expected time 2 cash flow of

\[ v > 0 \quad (1) \]

and is in zero net supply. The assumption of no cash-flow news at \( t = 0, 1 \) and the zero risk-free rate normalization imply that the fundamental value of the anomaly assets to risk-neutral investors is always \( v \).

There are two types of investors: “arbitrageurs” and “behavioral investors.” Behavioral investors generate mispricings in anomaly assets. They require, for an exogenous reason, positive expected returns for holding the anomaly assets, generating a downward price pressure. Risk-neutral arbitrageurs recognize that the anomaly assets have a fundamental value of \( v \) and trade against mispricings.

Mispricing. Anomaly assets differ only in the extent of behavioral investors’ mispricing. At each \( t \in \{0, 1\} \), behavioral investors’ demand for anomaly asset \( j \), in units of wealth, is

\[ B_{j,t} = \frac{E_t \left[ r_{j,t+1} \right]}{\bar{r}} - j, \quad (2) \]

where \( E_t \left[ r_{j,t+1} \right] \) denotes asset \( j \)’s conditional expected return and \( \bar{r} \) is a positive constant denoting the most-mispriced asset’s expected return in the absence of arbitrage.

This demand curve implies that (i) behavioral investors require a positive expected return for holding an anomaly asset and that (ii) an asset’s abnormal return, given any fixed amount of counteracting arbitrage position, increases with the index \( j \). To see (i), to clear the market with
just the behavioral investors \((B_{j,t} = 0)\), the expected return on any asset \(j\) must be positive:

\[
E_t [r_{j,t+1} | (B_{j,t} = 0)] = \tau j > 0
\]  

(3)

In the rest of the paper, I refer to this expected return in the absence of arbitrage capital as the anomaly’s “latent mispricing.” To see (ii), suppose now that arbitrageurs take the same wealth position \(x > 0\) on each anomaly asset. Then, to clear the market \((x + B_{j,t} = 0)\), asset \(j\)’s expected return is \(\tau (j - x)\):

\[
E_t [r_{j,t+1} | (x + B_{j,t} = 0)] = \tau (j - x)
\]  

(4)

The derivative of \(\tau (j - x)\) with respect to \(j\) is \(\tau > 0\), implying that an asset’s expected return, holding arbitrage position fixed, increases with \(j\).

Within each anomaly asset, the market-clearing expected return falls as arbitrage position increases. The slope of (4) with respect to \(x\) is \(-\tau\) for all assets, implying that the marginal effect of arbitrage position on the expected return is the same for all anomalies.

**Arbitrageurs.** The economy has a continuum of identical, risk-neutral arbitrageurs with aggregate mass \(\mu\). They live through all three periods and seek to maximize their expected wealth at time 2. Arbitrageurs have limited capital. At time \(t \in \{0, 1\}\), an arbitrageur’s deployable capital \(k_t\) is the sum of its own wealth \(w_t\) and a short-term funding \(f_t\):

\[
k_t = w_t + f_t
\]  

(5)

The wealth evolves according to

\[
w_t = w_{t-1} + \int_0^1 r_{j,t} x_{j,t-1} dj,
\]  

(6)

where \(r_{j,t}\) denotes the return on asset \(j\) at time \(t\) and \(x_{j,t}\) is the arbitrageur’s position on asset \(j\). I normalize the time-0 wealth of an individual arbitrageur to be \(w_0 = 1\) so that \(\mu\) is the aggregate arbitrageur wealth at time 0.

Short-term (uncollateralized) funding is available to each arbitrageur at the risk-free rate of zero but is capped at a stochastic funding constraint \(f_t\). Since part of the funding not used for an arbitrage activity can be invested at the zero interest rate, I assume for notational convenience that arbitrageurs always borrow to the limit \(f_t\). As we will see, the aggregate arbitrage capital...
\( \mu k_t \) (“arbitrage capital”) will be the only state variable in the model.\(^9\)

Arbitrageurs can take long or short positions on anomaly assets. However, they are required to put up a margin of one for each trade, which prevents them from levering up through a long-short trade.\(^{10}\) An arbitrageur’s capital constraint is, therefore,

\[
\int_0^1 |x_{j,t}| \, dj \leq k_t \tag{7}
\]

Arbitrageurs’ time-1 wealth may become negative. In this case, arbitrageurs are assumed to exit the economy, taking full responsibility for the liability incurred (unlimited liability) and paying any additional costs of default.

**Equilibrium.** Prices are determined in a competitive equilibrium. Arbitrageurs make optimal investment decisions taking current prices and their expectations of future prices as given, and those prices clear all asset markets. Formally, an equilibrium is defined as follows:

**Definition 1.** An *equilibrium* is the price functional \( p \), arbitrageur position \( x \), and behavioral investor demand \( B \) such that

(i) \( x \) is a solution to the arbitrageur’s optimization problem given price \( p \) and capital constraint (7).

(ii) Price \( p \) clears the market: \( \mu x + B = 0 \).

I solve for equilibrium prices iteratively, beginning with time 1 and moving to time 0. I look for a symmetric equilibrium in which all individual arbitrageurs make identical choices.

**Remarks on modeling choices.** This model is a simple way to deliver intuitions on how arbitrageurs trade multiple anomaly assets and what testable predictions this generates. Most of the modeling choices, however, are not crucial, and alternative specifications generate similar results.

The risk neutrality of arbitrageurs is one such assumption. I use risk neutrality for modeling purposes for two reasons: it most clearly highlights the emergence of endogenous risk and it is the framework used in Shleifer and Vishny (1997) and Brunnermeier and Pedersen

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\(^9\)To clarify, \( \mu \) captures the mass of arbitrageurs that changes over a long horizon; \( \mu = 0 \) indicates the period before extensive arbitrage and \( \mu > 0 \) indicates the period with extensive arbitrage. In contrast, variation in \( k_t \) captures the amount of arbitrageur capital that varies over a short horizon during which the mass of arbitrageurs \( \mu \) is fixed. Hence, for instance, I am assuming that \( k_t \) was low during the recent financial crisis although the mass of arbitrageurs \( \mu \) remained constant.

\(^{10}\)This ensures that the short-term funding is the only channel for levering up.
(2009), important precursors to my model. Under the risk neutrality assumption, the anomaly assets offer pure arbitrage opportunities in the absence of arbitrage capital since they generate expected returns above the risk-free rate. However, once arbitrageurs trade the anomaly assets with a nonnegligible amount of capital, they cause the prices of the assets to comove with the level of arbitrage capital, and this endogenous comovement becomes risk through arbitrageurs’ intertemporal hedging motive (Merton, 1973). Crucially, a more-mispriced anomaly becomes endogenously riskier since its greater exposure to arbitrage capital makes it a worse instrument for hedging (that is, it realizes a worse return than other assets when arbitrage capital falls and investment opportunities improve).  

With risk aversion, a more-mispriced anomaly still becomes endogenously riskier, but the mechanism is different. Under log utility, for instance, an asset’s risk is measured by its beta with respect to portfolio return. Since a more-mispriced anomaly offers a larger expected return, arbitrageurs assign a larger portfolio weight to the anomaly, which gives it a larger beta with respect to the portfolio return. In this way, different betas arise because of the different portfolios weights arbitrageurs assign to differently mispriced assets, and these betas explain the expected returns the anomaly assets earn in equilibrium. The model in the online appendix of Drechsler and Drechsler (2016) has this feature and shows a positive relationship between the degree of an asset’s underpricing and its beta with respect of arbitrageur portfolio return.

The source of variation in arbitrage capital ($\mu_k$) in this model is the stochastic funding constraint of arbitrageurs ($f_t$). However, any alternative source of shock that generates variation in arbitrageur capital—the key state variable in the model—generates analytically identical results. For instance, instead of a shock to the constraint on uncollateralized borrowing, one may use a shock to an arbitrageur’s margin requirement by making it stochastic (Gărbăleanu and Pedersen, 2011; Brunnermeier and Pedersen, 2009). Or one may shut down the borrowing channel altogether and use a shock to arbitrageur wealth ($w_t$) owing to interim cash-flow news, noise trades (that is, stochastic behavioral investor sentiment; the current model has constant sentiment), or stochastic investor flows to generate arbitrage capital shocks.

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11When the investor’s relative risk aversion $\gamma$ is below 1, as in the case of risk-neutrality, a hedge asset is the one whose return covaries positively with investment opportunities. This is because the speculative motive dominates.

12In the language of Shleifer and Vishny (1997), I assume that different anomaly assets are subject to different levels of pessimistic sentiments but not different volatilities of sentiment. This way, all anomaly assets would have the same fundamental risks (inherent volatilities), but they attain different endogenous risks (betas with respect to arbitrageur portfolio return).

13Shleifer and Vishny (1997) emphasize the wealth channel of arbitrageur capital. Although not emphasized, the same wealth channel exists in this model; a negative shock to funding $f_t$ also generates a negative wealth shock $w_t$ by lowering the values of anomaly assets in the arbitrageur’s portfolio. The difference is that the source of a negative wealth shock is the funding condition of arbitrageurs rather than the sentiment of noise traders.
To describe behavioral investors, I use demand curves rather than more primitive preferences. This allows me to abstract from the underlying cause of a mispricing, which is irrelevant for the rest of the analysis.\textsuperscript{14} The demand curves may be generalized to have different parameters govern the latent mispricing ($\bar{\tau}_j$) and the marginal effect of arbitrageur position on expected return ($\bar{\tau}$). Introducing a new parameter for this purpose does not affect the model’s analytical results.\textsuperscript{15}

2.2 Two benchmark scenarios: No arbitrage and a complete arbitrage

Before considering the more interesting case of limited arbitrage due to endogenous risks, I consider two benchmark scenarios.

The no-arbitrage case ($\mu_{k_t} \leq 0$)

The first is the “no-arbitrage” case in which arbitrageurs have zero or negative aggregate capital at all times ($\mu_{k0}, \mu_{k1} \leq 0$). As analyzed during the model setup, this induces the behavioral investors alone to price all assets, and the anomaly assets earn expected returns equivalent to their latent mispricings, $E_t[r_{j,t+1}] = \bar{\tau}_j$. The anomaly asset prices at time 0 and time 1 are $p_{j,0} = v / (1 + \bar{\tau}_j)^2$ and $p_{j,1} = v / (1 + \bar{\tau}_j)$, respectively. The prices are deterministic and do not depend on the specific realization of arbitrageur capital.

The complete-arbitrage case ($\mu_{k_t} \geq 1/2$)

At the other extreme is the “complete-arbitrage” case. Since arbitrageurs are risk-neutral, they competitively push all expected returns to zero when aggregate arbitrageur capital is large. If this is guaranteed to happen at times 0 and 1, the prices of anomaly assets equal their fundamental value $v$: $p_{j,0} = p_{j,1} = v$. The prices are deterministic and do not depend on the specific realization of arbitrage capital.

A complete arbitrage occurs if aggregate arbitrage capital $\mu_{k_t}$ is 1/2 or above almost surely both at time 0 and time 1. According to (4), the arbitrageur position required to push asset \(j\’s

\textsuperscript{14}Still, in Appendix A.2, I provide one way to endogenize the demand curves through heterogeneous beliefs.

\textsuperscript{15}Furthermore, although the demand curves are stated in terms of required expected returns, they can be restated in a more conventional form with prices on the left-hand side:

$$p_{j,t} = \frac{E_t[p_{j,t+1}]}{1 + \bar{\tau}(j + D_{j,t})}$$
expected return to zero is $j$. Integrating this over all assets, $\int_0^1 j \, d\,j$, gives $1/2$ as the aggregate arbitrage capital required to push all assets’ expected returns to zero.

The complete-arbitrage case seems to arise in the actual stock market. These are times when arbitrage capital is persistently sufficient to counteract all mispricings. In these times, anomaly assets have no endogenous risks and generate zero risk-adjusted returns. This point is reiterated theoretically in Proposition 4 and analyzed empirically in Section 3.3.

### 2.3 Limited arbitrage of multiple assets and the emergence of betas

Now I consider the more interesting case in which arbitrage capital may not be sufficient for a complete arbitrage at time 1. I first show that anomaly asset prices at time 1 comove with arbitrage capital due to arbitrageur trading. This makes the anomaly assets ex-ante risky from the perspective of arbitrageurs at time 0. This risk is larger for an anomaly asset with a larger latent mispricing, as it is expected to comove more strongly with arbitrage capital at time 1.

#### Equilibrium price at time 1 and endogenous risk

I first determine the prices of anomaly assets at time 1. Since arbitrageurs are risk-neutral, the expected return on any asset arbitrageurs hold will be the same, while the expected return on any asset arbitrageurs do not hold will be lower. This implies that there is a marginal asset. This marginal asset is determined as the point where the amount of capital needed to push down the expected return on all exploited anomalies up to the latent mispricing of the marginal asset is the amount of capital arbitrageurs have.

Let $j^*_1 \in [0, 1]$ be the marginal asset. Since latent mispricing increases with $j$, arbitrageurs hold assets $(j^*_1, 1]$ and earn expected returns $\tau j^*_1$ from them. This expected return implies that behavioral investors take a position $B_{j,1} = j^*_1 - j < 0$ on asset $j \in (j^*_1, 1]$, meaning arbitrageurs have a position $x_{j,1} = -B_{j,1} = j - j^*_1 > 0$ on $j \in (j^*_1, 1]$. Integrating this position of arbitrageurs over all exploited assets gives the amount of capital arbitrageurs must have to make $j^*_1$ the marginal asset: $\int_{j^*_1}^1 (j - j^*_1) \, d\,j = \frac{1}{2} (1 - j^*_1)^2$. Equating this with the actual capital of arbitrageurs, $\mu k_1$, gives the marginal asset when aggregate arbitrageur capital is in the intermediate region ($\mu k_1 \in [0, 1/2]$). Below this region, no arbitrage occurs, so $j^*_1 = 1$. Above this, arbitrageur capital has no further correcting role in anomaly assets, and $j^*_1 = 0$. The unexploited assets $[0, j^*_1)$ generate expected returns equal to their latent mispricings.\footnote{That variation in arbitrageur capital has a meaningful effect on asset prices only in the intermediate region of capital is an important feature of Gromb and Vayanos (2009).}
In summary, an anomaly asset’s equilibrium expected return at time 1 is

\[ E_1 [r_{j,2}] = \frac{v}{p_{j,1}} - 1 = \begin{cases} 
\tau j^*_1 & \text{if } j \geq j^*_1 \\
\tau j & \text{if } j \leq j^*_1 
\end{cases}, \tag{8} \]

where \( j^*_1 \) is the marginal asset given by

\[ j^*_1 = \begin{cases} 
1 & \text{if } \mu k_1 < 0 \\
1 - \sqrt{2\mu k_1} & \text{if } \mu k_1 \in [0, \frac{1}{2}] \\
0 & \text{if } \mu k_1 > \frac{1}{2} \tag{9} \]

Since \( E_1 [r_{j,2}] = \frac{v}{p_{j,1}} - 1 \), translating the expected returns into prices gives the following:

**Lemma 1.** (Equilibrium price at \( t = 1 \)). Equilibrium price of anomaly asset \( j \) at time \( t = 1 \) is

\[ p_{j,1} = \begin{cases} 
\frac{v}{1 + \tau j} & \text{if } j \leq j^*_1 \\
\frac{v}{1 + \tau j^*_1} & \text{if } j \geq j^*_1 \tag{10} \]

**Proof.** See Appendix A.1.

Since arbitrageurs equalize expected returns from all exploited assets, the prices of all exploited assets are the same. Figure 4 illustrates the equilibrium time 1 prices of anomaly asset \( j \) and anomaly asset \( j' > j \).

How does arbitrage capital move the prices of different assets? The intensive margin is identical for all assets. When their capital changes, arbitrageurs rebalance their portfolios to ensure that the prices of all exploited assets equal. Hence, a change in arbitrage capital has the same effect on the prices of assets while they are being exploited. The extensive margin, however, applies differently. The larger the latent mispricing, the lower the level of capital from which arbitrageurs begin exploiting the asset. This makes the more-mispriced asset comove with arbitrage capital in a wider region of arbitrage capital. As the reader will see, this will make the more-mispriced asset ex-ante riskier since a larger price covariance with arbitrage capital means a more negative price covariance with the arbitrageur marginal value of wealth.

Since arbitrageurs maximize the expected wealth at time 2, their marginal value of wealth—the value of an additional unit of wealth at time 1—is the gross expected return the extra wealth will generate. This means that the gross expected return earned by exploited assets, \( 1 + \tau j^*_1 \), is the arbitrageurs’ marginal value of wealth at time 1. However, arbitrageurs’ marginal value of
wealth is not well-defined if they have negative realized wealth and exit the financial market (Brunnermeier and Pedersen, 2009). I assume that, in the event of a default, an arbitrageur incurs a marginal bankruptcy cost of $c$ for each additional dollar of default in addition to taking full responsibility for the negative realized wealth. I then impose a restriction on the value of $c$ to make an additional unit of wealth more valuable in the default region than in any part of the non-default region.\textsuperscript{17} Hence, the marginal value of wealth is as follows:

Remark 1. (Arbitrageur’s marginal value of wealth at $t = 1$). An arbitrageur’s marginal value of wealth at time 1, denoted $\Lambda_1$, is

$$\Lambda_1 = \begin{cases} 1 + c & \text{if } k_1 < 0 \\ 1 + \bar{r} j_1^* & \text{if } k_1 \geq 0 \end{cases},$$

where $j_1^*$ is the marginal asset specified in (9) and where I assume $c \geq \bar{r}$ so that the marginal value of wealth is higher in the default region.

Thus, marginal value of wealth decreases as arbitrage capital increases. This means that anomaly assets, which covary positively with arbitrage capital, covary negatively with the arbitrageur marginal value of wealth. This makes anomaly assets risky from the perspective of arbitrageurs at time 0. The risk is larger for a more-mispriced asset (higher $j$) with a larger covariance with arbitrage capital. This is summarized as Lemma 2.

**Lemma 2. (Anomaly asset’s endogenous risk).** An anomaly asset is risky as indicated by a negative price covariance with the arbitrageur’s marginal value of wealth:

$$\text{Cov}_0 (p_{j,1}, \Lambda_1) \leq 0 \forall j$$

(12)

Furthermore:

(i) This risk is endogenous, arising only if arbitrageurs have a positive mass in the market so as to generate price pressure:

$$\text{Cov} (p_{j,1}, \Lambda_1) \big|_{\mu=0} = 0 \forall j$$

(13)

\textsuperscript{17}Without this assumption of $c \geq \bar{r}$, the marginal value of wealth is lower in the default region than in some parts of the non-default region. This could make an asset that pays low in the state of default and pays high in the state of non-default (e.g., the most mispriced asset $j = 1$) safer than an asset that pays the same return in all states (the least-mispriced asset $j = 0$).
(ii) In the cross-section of assets \( j \in [0, 1] \), the riskiness increases with an asset’s latent mis-pricing:

\[
\frac{\partial \text{Cov}(p_{j,1}, \Lambda_1)}{\partial (\bar{r}_j)} = \frac{\partial \text{Cov}(p_{j,1}, \Lambda_1)}{\partial j} \leq 0
\]  

(14)

**Proof.** See Appendix A.1.

**Equilibrium price at time 0 and ex-ante pricing of endogenous risk**

To find anomaly asset prices at time 0, I first find the arbitrageur’s value function. Since each individual arbitrageur is small and risk-neutral, the arbitrageur’s value function at time 0 is simply wealth multiplied by the marginal value of wealth, \( \Lambda_0 w_0 \). Since there is no time-0 consumption or discount, this quantity has to equal the time-0 expectation of wealth multiplied by the marginal value of wealth at time 1:

\[
\Lambda_0 w_0 = E_0 \left[ \Lambda_1 \left( \int_0^1 \frac{p_{j,1}}{p_{j,0}} x_{j,0} d j + w_0 - \int_0^1 x_{j,0} d j \right) \right]
\]  

(15)

An arbitrageur then maximizes this value function subject to a capital constraint,

\[
\int_0^1 |x_{j,0}| d j \leq k_0
\]  

(16)

I analyze the equilibrium price in the unconstrained and constrained cases separately.

Suppose first that \( k_0 \) is large enough to make constraint (16) slack and arbitrageurs unconstrained. Then, taking the derivative of both sides of (15) with respect to \( x_{j,0} \) gives \( E_0 [\Lambda_1] = E_0 [\Lambda_1 p_{j,1} / p_{j,0}] \). Furthermore, taking the derivative with respect to \( w_0 \) gives \( \Lambda_0 = E_0 [\Lambda_1] \).

Hence,

\[
p_{j,0} = E_0 \left[ \frac{\Lambda_1}{E_0 [\Lambda_1]} p_{j,1} \right]
\]  

(17)

for all assets \( j \in [0, 1] \). Since arbitrageur trading at time 1 makes \( \text{Cov}_0(\Lambda_1 p_{j,1}) < 0 \) for \( j \in (0, 1] \), even if arbitrageurs are unconstrained at time 0, they do not push the price \( p_{j,0} \) all the way to \( E_0 [p_{j,1}] \).

Suppose now that \( k_0 \) is small, in which case the constraint (16) binds and arbitrageurs are constrained. Then, by (5) and (16), \( w_0 = k_0 - f_0 = \int_0^1 x_{j,0} d j - f_0 \), where I use the fact that

\[18\] This martingale property \( \Lambda_0 = E_0 [\Lambda_1] \) is a consequence of the zero risk-free rate assumption.
arbitrageurs have non-negative exposures to anomaly assets in equilibrium. Substituting \( w_0 \) with \( \int_0^1 x_{j,0} \, dj - f_0 \) and taking the derivative of the both sides of (15) with respect to \( x_{j,0} \) gives the optimality condition \( \Lambda_0 \geq E_0 \left[ \Lambda_1 p_{j,1} / p_{j,0} \right] \), which holds with equality if and only if the asset is exploited by arbitrageurs at time 0 and thus has an interior solution in the arbitrageur’s optimization problem. Thus, the price of an exploited asset satisfies the fundamental theorem of asset pricing:

\[
p_{j,0} = E_0 \left[ \frac{\Lambda_1}{\Lambda_0} p_{j,1} \right]
\] (18)

On the other hand, an unexploited asset is priced solely by behavioral investors, who require an expected return of \( \bar{r}_j \):

\[
p_{j,0} = \frac{E_0 \left[ p_{j,1} \right]}{1 + \bar{r}_j}
\] (19)

Conditions (18) and (19) imply that \( \Lambda_0 \) is pinned down by the expectation of returns on exploited assets multiplied by the marginal value of wealth at time 1:

\[
\Lambda_0 = \max_{j \in [0,1]} E_0 \left[ \Lambda_1 \left( 1 + r_{j,1} \right) \right]
\] (20)

Hence, arbitrageurs are constrained; they are not the marginal investor of all assets. Instead, arbitrageur’s stochastic discount factor \( m_1 = \Lambda_1 / \Lambda_0 \) prices assets only if they are held by arbitrageurs. Thus, unlike conventional pricing models, arbitrageur pricing is expected to work only on assets that are traded by arbitrageurs.

The equilibrium conditions in both the unconstrained and constrained cases imply that anomaly asset prices at time 0 decrease with \( j \). If \( j' < j'' \), anomaly \( j' \) is not only subject to a larger mispricing in the absence of arbitrageurs, but is also exposed to a larger endogenous risk. Thus, anomaly \( j'' \) must be valued less than anomaly \( j' \).

This monotonicity of prices at time 0 makes the endogenous risk results in Lemma 2 hold analogously with returns. Consider two assets \( j' < j'' \) so that \( j'' \) has a larger latent mispricing than \( j' \). Then, asset \( j'' \) not only has a more-negative price covariance with the marginal value of wealth at \( t = 1 \), but also has a lower price at time 0. Since gross return is \( 1 + r_{j,1} = p_{j,1} / p_{j,0} \), this necessarily means that asset \( j'' \) has a more negative return covariance with the marginal value of wealth at time 1. This gives Proposition 1, which states Lemma 2 in terms of returns:

**Proposition 1. (“Alphas” turn into “betas”).** In the cross-section of anomaly assets, an anomaly asset’s latent mispricing,

\[
\alpha_j \equiv \bar{r}_j
\] (21)
predicts its endogenous risk measured as the negative of the beta with the arbitrageur stochastic discount factor (SDF) \( m_1 \equiv \Lambda_1 / \Lambda_0 \):

\[
\beta_j \equiv - \frac{\text{Cov}_0(m_1, r_{j,1})}{\text{Var}_0(m_1)}
\]

(22)

That is,

\[
\frac{\partial \beta_j}{\partial \alpha_j} > 0
\]

(23)

Proof. See Appendix A.1.

Hence, anomaly assets’ risks—their betas with respect to SDF—arise endogenously in this model. That asset betas arise endogenously is not surprising, given that most nontrivial economies with multiple assets would imply different equilibrium risks of the assets.\(^{19}\) The difference in this model, however, is that the different risks are generated by arbitrage trading.\(^{20}\) To emphasize this point, I show that the amount of arbitrage capital devoted to an asset is expected to be larger for an asset with a larger \( \beta_j \). This is presented as Proposition 2.

**Proposition 2. (Beta is explained by anomaly-specific arbitrage capital).** \( \beta_j \) increases with the expected arbitrageur position in the asset:

\[
\frac{\partial \beta_j}{\partial E_0[x_{j,1}]} \geq 0
\]

(24)

Proof. See Appendix A.1.

This suggests that anomaly-specific measures of arbitrage activity should explain different amounts of endogenous risks in different anomaly assets.

Once arbitrageurs generate endogenous risks, they require a compensation for these risks. This implies that an “intermediary asset pricing” should work on anomaly assets if risk is measured by beta with respect to arbitrageur’s SDF. However, because limited capital can constrain arbitrageurs, the model makes a few nonconventional predictions about pricing assets with the arbitrageur SDF. These are summarized as Proposition 3.

\(^{19}\)For instance, Zhang (2005) shows that value firms can have returns covaring more with the SDF than growth firms if the adjustments in the investment-capital ratio are higher for value firms, especially in bad times. Brunnermeier and Pedersen (2009) show that fundamentally more volatile assets covary more with the speculator’s SDF since their market liquidity drops more quickly in times of low liquidity.

\(^{20}\)The emergence of the beta is perhaps most similar to high-margin securities attaining high funding-liquidity risks owing to their large sensitivities to funding liquidity events (Gârleanu and Pedersen, 2011).
Proposition 3. ("Intermediary asset pricing" with respect to arbitrageur’s SDF). Suppose asset j is exploited by arbitrageurs at $t = 0$. Then, the asset’s beta with arbitrageurs’ stochastic discount factor $\Lambda_1 / \Lambda_0$ explains its expected return:

$$E_0 [r_{j,1}] = \begin{cases} \bar{r}_j & \text{if } j \text{ is not exploited} \\ \frac{1}{E_0 [\Lambda_1 / \Lambda_0]} - 1 + \frac{\lambda \beta_j}{\text{risk premium}} & \text{if } j \text{ is exploited} \end{cases}$$

(25)

where

$$\lambda = \frac{\text{Var}_0 (\Lambda_1 / \Lambda_0)}{E_0 [\Lambda_1 / \Lambda_0]}$$

(26)

$$\beta_j = -\frac{\text{Cov}_0 (r_{j,1}, \Lambda_1 / \Lambda_0)}{\text{Var}_0 (\Lambda_1 / \Lambda_0)}$$

(27)

with $E_0 [\Lambda_1 / \Lambda_0] = 1$ if arbitrageurs are unconstrained ($k_0$ large) and $E_0 [\Lambda_1 / \Lambda_0] > 1$ if they are constrained ($k_0$ small).

Proof. Rearranging (18) implies that exploited assets are priced according to $1 = E_0 [\Lambda_1 \Lambda_0^{-1} (1 + r_{j,1})]$, so that $1 = E_0 [\Lambda_1 / \Lambda_0] E_0 [1 + r_{j,1}] + \text{Cov}_0 (\Lambda_1 / \Lambda_0, r_{j,1})$. This gives

$$E_0 [r_{j,1}] = \frac{1}{E_0 [\Lambda_1 / \Lambda_0]} - 1 + \lambda \beta_j$$

If arbitrageurs are unconstrained, $\Lambda_0 = E_0 [\Lambda_1]$ so that the zero-beta rate drops out. If arbitrageurs are constrained, $\Lambda_0 = \max_{j \in [0,1]} E_0 [\Lambda_1 (1 + r_{j,1})] > E_0 [\Lambda_1]$ since otherwise, arbitrageurs are not optimally choosing the exploited assets.

If arbitrageurs are constrained at time 0, some anomaly assets are exploited by arbitrageurs while others are not. Hence, arbitrageurs are not the marginal investor of all assets, and choosing the exploited assets is important when estimating the asset pricing model; this is in contrast to Adrian, Etula, and Muir (2014), who essentially assume that financial intermediaries are the marginal investor of all assets. The expected return on an unexploited asset is its latent mispricing $\bar{r}_j$, the expected return required by behavioral investors. The expected return on an exploited asset has two components: a zero-beta rate that is common across all exploited assets and a risk premium that is different for each exploited asset.

The zero-beta rate is above the risk-free rate of zero at time 0 if arbitrageur capital is insufficient to “price” all anomaly assets correctly. This is because, in the constrained case,
arbitrageurs earn expected returns higher than the levels that would just compensate for the risks that they face; that is, their risk-adjusted returns are positive. Hence, unlike a conventional cross-sectional asset pricing model that tests for a zero intercept, this model predicts that if arbitrageurs are sometimes constrained and expose the prices of anomaly assets to comove with their capital—if they form positive $\beta_j$—we should also expect to see a positive intercept in the cross-sectional regression.

What is the interpretation of the zero-beta rate? By definition, the difference between the zero-beta rate and the prevailing risk-free rate represents the arbitrage profit arbitrageurs are generating from their investments. Although not pursued in this paper, inferring the zero-beta rate from a cross-sectional regression of anomaly assets and relating it to the estimate of the price of risk may be an interesting route for an arbitrageur-based asset pricing model to take.

Now I introduce Proposition 4, the last proposition of the model. The endogenous risks generated by arbitrageurs arise only if arbitrageurs are “constrained” at time 1. This is when aggregate arbitrageur capital at time 1 is in the intermediate region $\mu k_t \in [0, 1/2]$ so that the anomaly assets are mispriced and variation in arbitrage capital generates price pressure on the anomaly assets. In particular, if $\mu k_t > 1/2$, all anomaly assets are already correctly priced so that variation in arbitrageur no longer generates price pressure.

**Proposition 4. (Betas arise during constrained times).** Endogenous risk arises only during constrained times of $t = 1$. That is,

\[
\begin{align*}
\text{Cov}_0 (\Lambda_1, r_{j,1}|\mu k_1 > 1/2) &= 0 \\
\text{Cov}_0 (\Lambda_1, r_{j,1}|\mu k_1 < 1/2) &> 0
\end{align*}
\]

for all $j \in (0, 1]$. For this reason, if the funding condition follows a process $f_t^*$ such that $f_0^* > 1/2$ and $f_1^* > 1/2$ almost surely, then neither beta nor abnormal return arises:

\[
\beta_j = 0 \text{ and } E_0 [r_{j,1}] = 0 \text{ for all } j \in [0, 1]
\]

**Proof.** Follows from the price equation in Lemma 1 and the analysis in Section 2.2.

Empirically, I should observe that anomaly assets have zero endogenous risks and zero abnormal returns in times when arbitrageurs have persistently large capital.
3 Empirical test of the model

In this section, I test the model’s predictions about the cross-section of anomaly assets, using equity anomalies as the empirical counterparts of the differently mispriced anomaly assets in the model. Equity anomalies provide a convenient laboratory because they are easier to construct using publicly available data and straightforward to compare to one another.²¹

3.1 Test environment

Thirty-four equity anomalies as anomaly assets. The empirical counterpart of the anomaly assets in the model are 34 equity anomaly assets. For each one, I compute the time-series of quarterly value-weighted (VW) returns on a long-short self-financed portfolio over the period 1972 to 2015. The required data are downloaded from CRSP and Compustat.

I compute the long-short returns on each anomaly asset as follows. At the end of each month from 1972 to 2015, I allocate all domestic common shares trading on NYSE, AMEX, and NASDAQ into deciles based on an anomaly signal, such as the book-to-market ratio, with decile breakpoints determined by NYSE-listed stocks alone.²² Then, I calculate monthly value-weighted long-short return as the difference between the VW returns on the top and bottom deciles of stocks.²³ I aggregate the monthly returns to the quarterly frequency to match the frequency of the arbitrageur funding shock variable discussed below.

I use 34 distinct anomaly signals to construct the anomaly assets. These comprise 25 standard anomaly signals used in Novy-Marx and Velikov (2016), 6 industry-adjusted signals, and 3 “behavioral” signals meant to exploit investors’ behavioral biases.²⁴ The construction of the signals is similar to that of Novy-Marx and Velikov (2016) and of Green, Hand, and Zhang (2016); where I deviate, it is so that the signals resemble the actual signals arbitrageurs observe

²¹For instance, fixed-income arbitrage portfolios generated by Duarte, Longstaff, and Yu (2007) use proprietary data and require a separate, nontrivial valuation model for each anomaly asset. In contrast, equity anomaly portfolios can be readily constructed once the anomaly signals are calculated.

²²This ensures that the decile portfolios have comparable market capitalizations. For CRSP, domestic common shares on NYSE, AMEX, and NASDAQ are stocks with share code 10 or 11 and exchange code 1, 2, or 3.

²³Two exceptions are beta arbitrage and idiosyncratic volatility strategies, for which the plain-vanilla long-short portfolios have large negative exposures to the market portfolio. For these two strategies, I compute returns that go long \( \min\{5, \max\{0, \beta_{\text{Bottom Decile}} / \beta_{\text{Top Decile}}\}\} \) dollar of the top decile and short one dollar of the bottom decile, where \( \beta_{\text{Top Decile}} \) and \( \beta_{\text{Bottom Decile}} \) are the value-weighted market betas of the top and bottom deciles. The market beta used here is calculated at the end of each month using weekly returns in the previous one to three years, depending on data availability. The market factor is downloaded from Kenneth French’s website on June 25, 2016.

²⁴Out of 32 signals used in Novy-Marx and Velikov (2016), I exclude 7 for redundancy. For example, I exclude the “ValMomProf” signal since it is simply the sum of a stock’s decile numbers in the three univariate sorts based on value, momentum, and profitability.
at the end of each month. The online appendix provides more details on how I construct the anomaly signals.

Table 1 lists the 34 anomaly assets along with their mean returns, volatilities, CAPM betas, and arbitrageur funding betas (which I will come back to once I discuss my proxy for arbitrageur funding shocks) during the first-half of the sample period (1972Q1-1993Q4, “pre-93”) and the second-half (1994Q1-2015Q4, “post-93”). Twenty-nine of these have positive mean returns in the pre-93 sample, which I later use to measure an asset’s latent mispricing—the abnormal expected return that would prevail in the absence of arbitrageurs. For the five anomaly assets with negative mean returns, I will assume that arbitrageurs flip the direction of their trades to earn positive mean returns. There is some variation in the return volatility of the anomalies, and the ones with larger mean returns tend to have larger volatilities. Hence, it may be important to control for volatility in a regression that proxies for an anomaly’s latent mispricing using the pre-93 mean return. The anomalies’ CAPM betas are low, especially in the pre-93 sample, implying that their CAPM alphas are very similar to mean returns. I postpone the discussion of the funding betas.

**Broker-dealer leverage as arbitrageur funding condition.** The correct measure of risk of an anomaly asset is its beta with respect to arbitrageur’s stochastic discount factor (SDF). Since the SDF is unobserved, I look for an empirical proxy for the arbitrageur funding condition $f_t$, the variable underlying the variation in arbitrageur’s SDF in the model.\(^{25}\)

To measure arbitrageur funding shocks, I use shocks to the book leverage of broker-dealers,

$$f_t = \ln (\text{Leverage}_{BD}^t) - \ln (\text{Leverage}_{BD}^{t-1}),$$ \hspace{1cm} (30)

which Adrian, Etula, and Muir (2014) use to proxy for financial intermediary funding shocks. Here, a high $f_t$ or a high leverage shock indicates a favorable funding shock for arbitrageurs.\(^{26}\)

The book leverage of broker-dealers is defined as total financial assets net of repo assets divided by the difference between total financial assets and total liabilities.\(^{27}\)

Quarterly data are

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\(^{25}\)If arbitrageur’s SDF $m_1$ were approximately linear in the arbitrageur funding condition $f_1$, the model’s propositions could be identically stated with the beta with respect to the funding condition $f_1$. The approximation would be justified in a conditional model if the arbitrageur funding conditions were expected to vary over a small interval.

\(^{26}\)There is a slight abuse of notation since $f_t$ in the model is the level of arbitrageur funding, whereas $f_t$ here in the empirical analysis measures a shock to the arbitrageur funding condition.

\(^{27}\)Hence, the reverse repo (lending money through a repo) is not part of total assets. Instead, the difference between repo borrowing and repo lending ("net repo") enters into total liabilities. This amounts to assuming that only the relative increase in repo lending (equivalently, a fall in net repo) is taken as a positive funding shock to arbitrageurs.
} In actual analyses, I annualize the funding shock (30) by multiplying by four and winsorize the series at the 1\% and 99\% levels to mitigate the effects of outliers (this removes the smallest and largest values, both occurring around the recent financial crisis period).\footnote{Adrian, Etula, and Muir (2014) also make seasonal adjustments, which I do not do. Making seasonal adjustments does not affect my results. Asl and Etula (2012) and Adrian, Moench, and Shin (2013) also use this series to measure financial sector funding conditions.
} Figure 5 plots the original log leverage series and the leverage shock series $f_t$.

Adrian, Etula, and Muir (2014) show that the funding shock is procyclical and has expected signs of correlation with market volatility, Aaa-Baa spread, and financial stocks return. However, among different financial intermediaries, this measure is especially relevant for levered arbitrageurs such as hedge funds since a major part of security broker-dealers’ business is prime brokerage for hedge funds. As prime brokers, they provide their hedge fund clients with various types of financing, intermediate securities lending, and serve as custodians of the cash and stocks owned by hedge funds.\footnote{Aragon and Strahan (2012) empirically study the financial dependence of hedge funds on prime brokers, using the Lehman bankruptcy.
}

In the online appendix, I repeat my main analyses with stochastically detrended leverage series,

$$f_t = \ln(Leverage_{BD}^t) - \frac{1}{N} \sum_{s=1}^{N} \ln(Leverage_{BD}^{t-s}),$$

with $N = 4$, $N = 8$, and $N = 12$. I do these robustness checks because a leverage shock to prime brokers may affect their hedge fund clients with a lag. Hedge funds, especially larger ones, often arrange with their prime brokers to “lock in” the margin and collateral requirements for an agreed period. This margin lock-up is typically 90 days, but it can range from 30 to 120 days. Using the stochastically detrended leverage addresses this issue, since it assumes that an innovation to the arbitrageur funding condition at time $t$ is a weighted average of the innovations in broker-dealer leverage growth in the last four quarters. To see this, simply rewrite (30) as

$$f_t = \sum_{s=0}^{N-1} \frac{N-s}{N} [\Delta \ln(Leverage_{BD}^t)]_{t-s},$$

where $[\Delta \ln(Leverage_{BD}^t)]_t$ indicates the growth rate of leverage from $t - 1$ to $t$. Hence, the beta estimated using the factor resembles the Scholes-Williams beta (Scholes and Williams, 1977) and the Dimson beta (Dimson, 1979), which account for nonsynchronous data. Most test results are similar or stronger when I use $N = 4$, $N = 8$, or $N = 12$.\footnote{Aragon and Strahan (2012) empirically study the financial dependence of hedge funds on prime brokers, using the Lehman bankruptcy.
}
For the rest of the paper, I use $\beta_j$ to denote anomaly asset $j$’s beta with respect to the funding condition $f_t$, and I refer to it as the anomaly’s “funding beta.”31 I will sometimes refer to the funding beta as “endogenous risk” to highlight that the beta represents an endogenous risk exposure. When necessary, I will add a superscript to denote the sample period in which the beta is estimated.

**Identification through a sample split.** The model’s propositions rely on being able to observe the anomaly assets’ abnormal returns in the absence of arbitrage capital ($\mu = 0$) and endogenous risks in the presence of arbitrage capital ($\mu > 0$). Although no single year represents a clear jump in the mass of arbitrageurs in the anomaly assets, I argue that the first-half (pre-93) and the second-half (post-93) of my original sample of 1972Q1-2015Q4 are reasonable proxies for times when arbitrageurs have a negligible mass ($\mu = 0$) in the anomalies and for times when arbitrageurs have a positive mass ($\mu > 0$) in the anomalies, respectively.

There are three justifications for using 1993 as the cutoff year. First, arbitrage capital grew rapidly in the 1990s, with hedge fund assets under management expanding from $39$ billion in 1990 to $1.73$ trillion in 2008 (Stein, 2009). Second, 1993 is the year when some of the most influential papers in equity anomalies were published: Fama and French (1993) popularized the size and value anomalies by rationalizing them with a multifactor model, and Jegadeesh and Titman (1993) introduced the momentum anomaly. These papers spurred the search for new equity anomalies whose abnormal returns are not explained by exposures to size, value, and momentum factors. Third, Chordia, Subrahmanyam, and Tong (2014) also use years prior to 1993 as the period when the trading technology and liquidity had not sufficiently developed to allow for extensive arbitrage at reasonable costs. Main test results, however, are similar when I use different cutoff years within the early 1990s.32

An alternative to using a sample split is to use the anomalies’ publication years to study the effect of arbitrage trade. This approach is used by both McLean and Pontiff (2016) and Liu, Lu, Sun, and Yan (2015) (LLSY) in their studies of equity anomalies. However, LLSY find that strong arbitrage activities on size and value anomalies began around 1992, although some arbitrage activities occurred following their original discoveries by Banz (1981) and Rosenberg, Reid, and Lanstein (1985).33 This suggests that using the 1993 cutoff is a reasonable alternative to using publication years.

Given my sample split approach, I measure the latent mispricing of an anomaly by its mean

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31 Adrian, Etula, and Muir (2014) refer to this factor as a “leverage factor.” I refer to it as an “arbitrageur funding shock” to emphasize that it is the empirical counterpart of the arbitrageur funding condition in the model.
32 The online appendix repeats the main regressions using 1991, 1992, 1994, and 1995 as the cutoff year.
33 They attribute this to Fama and French (1992).
long-short return in the pre-93 sample ($\bar{r}_j^{pre93}$) and the endogenous risk of an anomaly by its beta with respect to the arbitrageur funding conditions in the post-93 sample ($\beta_j^{post93}$). Since the anomaly assets are not strongly exposed to market excess returns or arbitrageur funding shocks in the pre-93 sample, using pre-93 CAPM alphas, pre-93 Fama-French three-factor alpha, or pre-93 arbitrageur funding alphas to measure latent mispricings does not substantially change the paper’s main results (see the online appendix).

**Time-series evidence of endogenous risk exposures.** This paper’s main contribution is to use the cross-section of anomaly assets to test the idea that arbitrage generates endogenous risk by turning $\alpha$s into $\beta$s. However, I briefly highlight that time-series evidence is also consistent with the endogenous risk idea.

First, the returns on anomaly assets have fallen. Table 1 shows that the annualized long-short returns on anomaly assets have fallen from 5.24% in the pre-93 period to 3.57% in the post-93 period. This implies a 32% decline in expected returns as a result of increased arbitrage, similar to the 32% fall in expected returns that McLean and Pontiff (2016) find in anomaly assets after an academic publication. If I assume that arbitrageurs reverse the direction of the trades for the anomalies with negative pre-93 mean returns, the fall in expected returns is from 6.04% to 3.85%, which is a 37% decline.

Second, as the anomalies’ expected returns have fallen, their betas with respect to arbitrageur funding shocks have risen. Table 1 shows that the cross-sectional average of arbitrageur funding betas increases from -0.58 to 2.62 between the two sample periods. If I again assume that arbitrageurs reverse the direction of the trades for the anomalies with negative pre-93 mean returns, the change in beta is from -0.53 to 5.70. Although not reported in the table, in the post-93 period, 8.1% of the time-series variation in the return on an equal-weighted (EW) index of the 34 anomaly assets is explained by the variation in arbitrageur funding shock.\footnote{When constructing this index, I reverse the direction of the trade for anomalies with negative pre-93 long-short returns.}\footnote{34} In contrast, in the pre-93 period, the EW index return has an $R^2$ of only 0.3% in the same regression.

Next, I move on to cross-sectional tests. There, I show that cross-sectional evidence strongly points to anomaly assets becoming endogenously riskier due to arbitrage trade.

### 3.2 Mispricing turns into endogenous risk

I first test how well an anomaly’s latent mispricing (pre-93 mean return $\bar{r}_j^{pre}$) predicts its endogenous risk (post-93 beta with respect to arbitrageur funding shocks $\beta_j^{post}$) (Proposition 1). The simple intuition is that an anomaly with a larger latent mispricing attracts correspondingly
more arbitrage capital, which generates greater endogenous risk. The empirical test is to run the following regression in the cross-section of 34 anomaly assets

$$\beta_{j}^{\text{post}} = b_0 + b_1 r_{j}^{\text{pre}} + \eta_j$$

(33)

and test if $b_1 = 0$.

A complication arises because pre-93 mean return is estimated. If the estimated mean $r_{j}^{\text{pre}}$ is a noisy signal of the actual latent mispricing, and if arbitrageurs observe the true latent mispricing whereas the econometrician does not, then the standard errors for the cross-sectional regression (33) need to be adjusted for the fact that the regressor is generated. Hence, I jointly estimate the coefficients in the cross-sectional regression (33), pre-93 mean long-short returns, and post-93 funding betas using the generalized method of moments (GMM). Since arbitrageurs may actually have the identical information set that the econometrician has, using realized returns in the past to gauge an anomaly asset’s latent mispricing, I also report ordinary least squares (OLS) t-statistics.

To use GMM, consider the following data-generating process. In the pre-93 period, a mean return is a noisy realization of the latent mispricing:

$$r_{j,t} = r_{j,t}^{\text{pre}} + \epsilon_{j,t}$$

(34)

This latent mispricing determines an anomaly’s endogenous exposure to funding shocks:

$$\beta_{j}^{\text{post}} = b_0 + b_1 r_{j}^{\text{pre}} + \eta_j$$

(35)

Here, $\eta_j$ has a cross-sectional mean of zero. This funding beta then determines the equilibrium expected return in the post-93 period:

$$r_{j,t} = a_{j}^{\text{post}} + \beta_{j}^{\text{post}} f_t + \epsilon_{j,t}$$

(36)

These conditions imply the following 4J moment conditions:

$$g_{4J \times 1}(b) = \begin{bmatrix} E \left[ (r_{j,t} - r_{j,t}^{\text{pre}}) 1(t \in \text{Pre}) \right] \\ E \left[ (r_{j,t} - a_{j}^{\text{post}} - \beta_{j}^{\text{post}} f_t) 1(t \in \text{Post}) \right] \\ E \left[ (r_{j,t} - a_{j}^{\text{post}} - \beta_{j}^{\text{post}} f_t) f_t 1(t \in \text{Post}) \right] \\ E \left[ (\beta_{j}^{\text{post}} - b_0 - b_1 r_{j,t}) 1(t \in \text{Pre}) \right] \end{bmatrix},$$

(37)
I then use a selection matrix to ensure that a cross-sectional expectation is taken over the last set of $J$ moments,

$$A_{(3J+2)\times 4J} = \begin{bmatrix} I_{3J\times 3J} & 0_{3J\times J} \\ 0_{1\times 3J} & 1_{1\times J} \\ 0_{1\times 3J} & \tilde{r}_1'_{1\times J} \end{bmatrix}$$  \quad (38)

where $\tilde{r}$ is a $J \times 1$ vector of pre-93 mean returns. Then, I find parameter estimates $\hat{b}$ such that

$$A_g(\hat{b}) = 0_{(3J+2)\times 1}$$  \quad (39)

The chosen selection matrix generates the identical set of moment conditions as in sequential OLS estimations of (34), (36), and then (35).

The first column of Table 2 reports the parameter estimates, OLS t-statistics, and GMM t-statistics from estimating the effect of latent mispricing on funding beta specified in (33). Each percentage of pre-93 mean return turns into a post-93 funding beta of 1.24. To interpret this number, since returns are in percentages whereas the funding shocks are not, a beta of 1.24 means that a 100% increase in the leverage of broker-dealers leads to a 1.24% increase in the anomaly asset return. This “turning alphas into betas” effect is statistically significant based on both GMM and OLS standard errors. The large $R^2$ implies that the pre-93 return is a strong predictor of an anomaly’s endogenous exposure to arbitrageur funding. The intercept is statistically insignificant, implying that there is no strong trend in the anomalies’ funding betas apart from the endogenous effect coming from arbitrage activity.

Additional control variables have little influence on the results. In the second column, I add pre-93 funding beta as an additional regressor to show that it is not the persistence or magnification of pre-93 beta that drives the large post-93 funding betas. The coefficient of 0.75 on the pre-93 funding beta implies some persistence in the beta, although the effect is significant only based on t-OLS and not based on t-GMM. Despite this persistence, since pre-93 funding beta was small in magnitude, including it in the regression has little effect on the coefficient on pre-93 mean return.

In the third column, I add pre-93 return volatility as an additional regressor, thus addressing the concern that the anomalies with larger pre-93 mean returns also tend to have large pre-93 volatilities; that is, pre-93 mean return may proxy not only for latent mispricing, but also for volatility. We see, however, that controlling for pre-93 volatility has little effect on the slope of pre-93 mean return and that pre-93 volatility is a highly insignificant predictor of post-93 funding beta. This shows that it is the latent mispricing rather than volatility that predicts an anomaly asset’s endogenous risk.
Thus far I have treated the year 1993 as a “jump” in the mass of arbitrageurs in the anomaly assets. In reality, the increase in the mass of arbitrageurs in the anomalies (µ in the model) and hence the anomalies’ endogenous exposures to arbitrageur funding shocks would have been gradual even within the post-93 period. Therefore, I allow the arbitrageur funding beta to be an increasing and concave function of time \( t \) within the post-93 period. In particular, I assume the following data-generating process in which an exposure to arbitrageur funding grows at the rate \( \beta_{j,1}^\text{post} t^{-1} \), where \( t \) here is the number of quarters into the post-93 sample (1994Q1 being \( t = 1 \)).

\[
\begin{align*}
\text{Pre-93 return:} & \quad r_{j,t} = \bar{r}_{j}^\text{pre} + \epsilon_{j,t} \\
\text{Post-93 return:} & \quad r_{j,t} = a_{j}^\text{post} + \left( \beta_{j,0}^\text{post} + \beta_{j,1}^\text{post} \ln (t) \right) f_t + \epsilon_{j,t} \\
\text{Beta determination:} & \quad \beta_{j,1}^\text{post} = b_0 + b_1 \bar{r}_{j}^\text{pre} + \eta_j
\end{align*}
\]

In this case, the latent mispricing of an anomaly measured by the pre-93 mean return should predict the rate of increase in beta \( \beta_{j,1}^\text{post} \).

The fourth column of Table 2 reports the “turning alphas into betas” effect estimated under the assumption of a gradual endogenous risk exposure. Again, the latent mispricing measured by the pre-93 mean return predicts an anomaly’s endogenous risk measured by \( \beta_{j,1}^\text{post} \). Although the coefficient is not statistically significant based on t-GMM because of large standard errors, the estimated coefficient is similar in magnitude to the one from the first column. Since there are 88 quarters in the post-93 period (1994Q1-2015Q4), it follows that each percentage of pre-93 mean return leads to a funding beta increase of \( 0.34 \times \ln(88) = 1.52 \) by the end of the post-93 period. The intercept is small, implying that there is no strong trend in the funding beta other than through arbitrage activity.

Although I have used funding beta to measure an anomaly asset’s exposure to arbitrageur funding, beta can change spuriously because of volatility rather than correlation. For instance, to elaborate on the point made earlier, suppose that anomalies with large pre-93 mean returns are precisely the ones with large return volatilities. If such anomalies’ large volatilities persist through the post-93 period, then even if anomalies have equal post-93 correlations with

\[\text{When implementing the GMM, I set } \ln(t) = 0 \text{ for all } t \in \text{Pre}. \text{ The values of } \ln(t) \text{ in the pre-93 sample do not affect the results, as all moment conditions involving } \ln(t) \text{ are multiplied by the post-93 dummy. Admittedly, the specific function through which I introduce concavity in } t \text{ is arbitrary. However, I find that the specific way of introducing concavity in } t \text{ does not lead to large changes in the parameter estimate, unless the function is too “linear” over } t \in \text{Post.}\]

\[\text{The GMM implementation of this model is explained further in Appendix B.1.}\]
arbitrageur funding, pre-93 return would appear to predict post-93 funding beta:

\[ \beta_{j}^{\text{post}} = \rho_{j}^{\text{post}} \frac{\sigma_{j}^{\text{post}}}{\sigma_{j}^{\text{pre}}} \approx \rho_{j}^{\text{pre}} \]  \hspace{1cm} (41)

To rule out this possibility, I repeat the baseline regression using funding correlation \( \rho_{j}^{\text{post}} \) as the dependent variable.\(^{37}\)

The last column of Table 2 reports the results from predicting an anomaly’s post-93 funding correlation using its pre-93 mean return. The results are strong both based on the t-statistics and on \( R^2 \). This suggests that anomalies’ large post-93 funding betas are driven by correlations, not volatilities. In terms of magnitude, each percentage of pre-93 mean return raises the anomaly’s funding correlation by 1.42 percentage points (%p). Since the anomaly assets’ pre-93 mean mean returns range from \(-5\%\) to 21%, the predicted post-93 correlation with arbitrageur funding shock ranges from \(-7\%\) to 30%.

The results presented here are robust to alternative measures of latent mispricing. The online appendix shows that using pre-93 CAPM alpha, pre-93 Fama-French three-factor alpha, or pre-93 arbitrageur funding alpha generates similar results. The same appendix also shows that using a volatility-neutral measure of latent mispricing generates similar results.

The results here suggest that anomaly assets’ exposures to the arbitrageur funding conditions are an endogenous outcome of arbitrage trading. In Section 3.5, I show that these exposures help explain the anomaly assets’ equilibrium returns in the post-93 period. Hence, the endogenous risk theory explains the origin of betas before using the betas to explain equilibrium expected returns—a response to the question, “Why are betas exogenous?” (Cochrane, 2011: p.1063). Next, I provide further evidence that the betas are generated by arbitrage activity.

### 3.3 Endogenous risk is explained by anomaly-specific arbitrage capital

Here, I test the prediction that, if anomaly assets’ funding betas are indeed a byproduct of arbitrage activity, then the funding betas must be explained by anomaly-specific measures of arbitrage activity (Proposition 2). To test this, I find measures of arbitrage activity specific to an anomaly. Then, I run a cross-sectional regression to test if an anomaly with greater arbitrage activity has a larger post-93 funding beta during the same period.

I use three measures of anomaly-specific arbitrage capital based on how much shorting there is in the bottom decile of the anomaly relative to the top decile: the difference in the short

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\(^{37}\)I leave more detailed discussions of the GMM implementation to Appendix B.1.
interest ratio in the bottom and top deciles of an anomaly (“short interest ratio difference”); percentage (log) difference in the short interest ratio in the bottom and top deciles (“log short interest ratio difference”); and difference in the days to cover in the bottom and top deciles (“days to cover difference”). Net shorting is a relatively clean way to measure arbitrage activity since most shorting is done by hedge funds.\textsuperscript{38}

Specifically, short interest ratio, previously used by Hanson and Sunderam (2014), is defined as the number of shares being shorted (short interest) divided by the number of shares outstanding at a given time. I compute the short interest ratio of each stock in each month by dividing its mid-month short interest amount (from Compustat) by shares outstanding on the same day (from CRSP). Then, to obtain the short interest ratio difference measure, I take the post-93 time-series average of the monthly difference between the VW average short interest ratios in the bottom decile and the top decile:

\[
DSIR_{\text{post}}^{j} = 100 \times T^{-1} \sum_{t=1}^{T} \left( SIR_{j,t}^{\text{bottom decile}} - SIR_{j,t}^{\text{top decile}} \right)
\]

(42)

where the value-weighted average short interest ratio of a decile is computed as

\[
SIR_{j,t}^{\text{decile}} = \sum_{i \in \text{decile}_j} \omega_{i,t} \frac{\text{Short Interest}_{i,t}}{\text{Shares Outstanding}_{i,t}}
\]

(43)

with $\omega_{i,t}$ denoting the weight of stock $i$ in the relevant extreme decile and $t$ denoting month.

The second measure of arbitrage activity, log short interest ratio difference, is a variant of the short interest ratio difference measure. Here, I take a percentage difference rather than a level difference in the short interest ratio in the bottom and top deciles because the level difference has a strong positive trend over time; that is, taking a time-series average of the level difference puts disproportionate weights on recent years. Hence, the log short interest ratio difference measure is the following:

\[
DLSIR_{\text{post}}^{j} = T^{-1} \sum_{t=1}^{T} \left( \ln \left( SIR_{j,t}^{\text{bottom decile}} \right) - \ln \left( SIR_{j,t}^{\text{top decile}} \right) \right)
\]

(44)

where $t$ is month.

As for the third measure, days to cover (DTC) of a stock is defined as its short interest

\textsuperscript{38}A report by Goldman Sachs (2014) estimates that 85% of the short interest is held by hedge funds. Similarly, Boehmer, Jones, and Zhang (2013) and Ben-David, Frazoni, and Moussawi (2012) both argue that hedge funds are responsible for most of the short interest. Dechow et al. (2001), Hirshleifer, Teoh, and Yu (2011), and Cao et al. (2012) also interpret short activity on an anomaly as arbitrage activity.
divided by average daily trade volume. Thus, it measures the expected number of days required to recover all shorted stocks from the market, also interpreted as the liquidity cost of exiting the short positions. Since DTC normalizes short interest by the stock’s liquidity, Hong et al. (2015) argues that DTC is a more accurate measure of arbitrage intensity than short interest ratio. To measure an anomaly’s average DTC during the post-93 period, I first compute a stock’s DTC each month by dividing its monthly short interest ratio by the average share turnover (trade volume divided by shares outstanding) during the same month. I subtract the VW average DTC in the top decile of an anomaly from that in the bottom decile to obtain the DTC difference between the two deciles. I then compute the time-series average of this monthly DTC measure during the post-93 sample period to obtain the average DTC difference between the two deciles of an anomaly:

\[
DDTC_{post}^{j} = T^{-1} \sum_{t=1}^{T} \left( DTC_{bottomdecile, j}^{t} - DTC_{topdecile, j}^{t} \right)
\]  

where the value-weighted average days to cover of a decile is computed as

\[
DTC_{decile, j}^{t} = \sum_{i \in \text{decile}_{j, t}} \frac{\omega_i}{\text{Short Interest}_{i, t}} \frac{\text{AvgDailyTradeVolume}_{i, t}}{\text{AvgDailyTradeVolume}_{i, t}}
\]

with \(\omega_i\) denoting the weight of stock \(i\) in the relevant extreme decile. For all three measures, I compute the same measure for the pre-93 period, although the short interest data starts from 1973 rather than 1972, when my pre-93 period begins.

For each of the three measures of anomaly-specific arbitrage capital, I ask if an anomaly with more arbitrage activity has a larger funding beta. This amounts to running the following regression, where \(ArbCapital_{post}^{j}\) is an anomaly-specific arbitrage capital measure:

\[
\beta_{post}^{j} = b_0 + b_1 ArbCapital_{post}^{j} + \eta_j
\]

I estimate the parameters using OLS and report t-statistics based on heteroskedasticity-consistent standard errors. Although the explanatory variables in this regression are estimated, I do not need to correct for their variances because it is the realized arbitrage activity, not the unobserved “true mean,” that matters for generating the observed betas.\(^{39}\)

Table 3 reports the test results. During the post-93 period, funding betas are explained by anomaly-specific arbitrage capital. The highly significant slope coefficients, large \(R^2\), and small

\(^{39}\)However, GMM t-statistics would be useful to control for the cross-asset correlations.
intercepts imply that once arbitrage activity has been controlled for, there is little idiosyncratic or common variation in the anomalies’ funding betas. Interpretations of the slope coefficients are as follow. First, the coefficient on the short interest ratio difference implies that, in the cross-section of anomalies, the funding beta increases by 4.94 for an increase of 1/100 in the difference in the short interest ratio between the bottom and the top deciles. Next, the coefficient on the log short interest ratio difference implies that, again in the cross-section of anomalies, the funding beta increases by 0.14 for each 1% point increase in the short-interest-ratio difference between the bottom and the top deciles. Finally, the coefficient on DTC implies that, in the cross-section of anomalies, the funding beta increases by 4.11 for each increase in the number of days it takes to recover the short interest in the bottom decile relative to the top decile.

Interestingly, the ability of anomaly-specific arbitrage capital to explain the cross-section of funding betas is strong in the pre-93 period as well, implying that the pre-93 funding betas were also an outcome of arbitrage activity. However, as noted in the table, the magnitudes of the arbitrage activities were substantially smaller in the pre-93 period, causing the pre-93 funding betas to be smaller in magnitudes than post-93 funding betas.

3.4 Mispricing turns into a larger endogenous risk on the short side

Before testing Proposition 3, I carry out a test that is outside the scope of the model but provides further evidence for the endogenous arbitrage risk view. This test separates out the long-short returns on anomaly assets into long (top decile) and short (bottom decile) portfolios and asks whether the same amount of abnormal return turns into a larger endogenous risk on the short side than on the long side of the anomaly. Intuitively, the long side of an anomaly can be exploited by a large class of investors including mutual funds, pension funds, and individual investors who are not exposed to arbitrageur funding shocks, whereas the short side of an anomaly is primarily exploited by arbitrageurs, such as hedge funds, as argued in Section 3.3. Hence, if an anomaly’s endogenous risk is a byproduct of arbitrage activity, we should expect a larger “turning alphas into betas” effect on the short side.

To test this, I repeat the test of Proposition 1 in Section 3.2 using 68 portfolios representing long and short sides of the 34 anomaly assets. Since the test portfolios now have significant market exposures, I take the robust approach of measuring the latent mispricing using pre-93 CAPM alpha (instead of simple mean return) and endogenous risk using post-93 funding beta net of market exposure. Hence, the baseline regression is

$$\beta_{j, \text{post}} = b_0 + b_1 \text{Short}_j + b_2 \alpha_{j, \text{pre, CAPM}} + b_3 \alpha_{j, \text{pre, CAPM}} \times \text{Short}_j + \eta_j,$$  

(48)
where $j$ now indexes one of the 68 long- and short-side anomaly portfolios, and $\text{Short}_j$ is a dummy variable for short-side portfolios. I test if $b_3$ is positive and statistically different from zero.

Figure 6 reports the test result. Consistent with the hypothesis, the turning-alphas-into-betas effect is much stronger on the short side than on the long side. When the long and short sides of the anomalies are separately analyzed (the first two columns), the slope of the coefficients are significant based on OLS t-statistics. However, the magnitude of the slope and the $R^2$ of the regression are much larger on the short side. This difference in the slope is statistically significant, as reported in the last column. This result is consistent with arbitrageurs generating larger endogenous risks on the short sides of the anomalies where they have a larger relative presence.

The result here is difficult to reconcile with the idea that a hidden fundamental risk explains the large returns earned by anomaly assets. To see this, suppose that the anomaly assets have always been commonly exposed to single latent risk factor $L_t$: $r_{jt} = \delta_j L_t + \epsilon_{jt}$. Suppose also that, for whatever reason, this latent risk factor has become more correlated with the arbitrageur funding conditions in recent years: $\rho(L_t, f_t) = 0$ in pre-93 but $f_t = \gamma_{\text{post}} L_t + \eta_t$ in post-93 for some noise $\eta_t$. This would at least explain why pre-93 mean returns appear to explain anomalies’ post-93 betas with respect to the arbitrageur funding conditions and why post-93 funding betas explain post-93 expected returns, consistent with the empirical results in Section 3.2 and Section 3.5. In this case, however, the short and long sides of the anomaly assets are expected to have the same coefficient in the “turning alphas into betas” regression. This is because, in this scenario, the coefficient would simply measure how well the arbitrageur funding shock proxies for the latent risk factor during the post-93 period. Analytically, since pre-93 expected return is $\bar{r}^{\text{pre}}_j = \delta_j \bar{L}^{\text{pre}}$ and post-93 beta is $\beta^{\text{post}}_j = \delta_j \gamma^{\text{post}}$, the ratio between the two is a constant:

$$\frac{\beta^{\text{post}}_j}{\bar{r}^{\text{pre}}_j} = \frac{\gamma^{\text{post}}}{\bar{L}^{\text{pre}}}$$

(49)

Hence, the result here cannot be rationalized by the hidden fundamental risk view, at least under the assumption that there is one latent risk factor to which the anomalies are commonly exposed. Rationalizing it with multiple latent risk factors would require a much more elaborate story in which the long and short sides of the anomalies are exposed to different latent risk factors and those latent risk factors have come to attain different post-93 correlations with the arbitrageur funding conditions.
3.5 “Intermediary asset pricing” of anomaly assets based on endogenous risks

The empirical tests up to this point have focused on showing that funding betas arise as an endogenous outcome of different arbitrage activities on differently mispriced anomaly assets. An empirical question, however, is whether arbitrageurs take these funding betas into consideration when assessing risks of the anomaly assets. Here, I present some evidence that arbitrageurs are mindful of being exposed to funding betas, the endogenous risks they themselves have generated. However, I also find that jointly explaining the expected returns of 34 different equity anomalies with single factor is a challenging task.

The objective is to test Proposition 3, which predicts that the anomaly assets’ endogenous exposures to arbitrage risk explain the expected returns that survive in equilibrium. To do this, in the post-93 period, I run a cross-sectional asset pricing regression using beta with respect to arbitrageur funding. I then test if the price of risk is positive and if the variation in expected returns is explained by the variation in funding betas. In addition, I test the model’s nonconventional asset pricing predictions.

Before I delve into the test, I highlight this exercise’s close connection to the intermediary asset pricing test of Adrian, Etula, and Muir (2014) (AEM). Proxying for the financial intermediary funding condition using the broker-dealer leverage shocks, they show that the single factor explains the cross-section of returns on 25 equity portfolios sorted by size and value, 10 equity portfolios sorted by momentum, and 6 bond portfolios sorted by maturity. They do this over the years 1968 to 2009, and they test the usual restriction that the intercept of the cross-sectional regression must be zero.

At the most basic level, the exercise here can be viewed as extending the AEM result on size, value, and momentum anomalies to a much wider set of equity anomalies. More importantly, however, I use this exercise to show that the endogenous risks generated by arbitrageurs prevent the initial anomaly returns from disappearing completely. Furthermore, I use this exercise to highlight the endogenous arbitrage risk model’s nonconventional asset pricing predictions.

The endogenous-arbitrage-risk model makes two nonconventional predictions in asset pricing. First, if anomaly assets tend to have positive funding betas, we must also see a positive intercept in a cross-sectional regression. For arbitrageurs to generate positive $\beta$s in anomalies through price pressure, the anomaly assets must sometimes be mispriced and generate abnormal returns. This means that the zero-beta rate—the risk-adjusted return that arbitrageurs are earning from anomaly assets—must be higher than the risk-free rate, on average, during the sample period. Second, the price of risk is biased upward if the cross-sectional regression
is run in a sample that includes both the “pre-arbitrage” and “post-arbitrage” periods. This is because such a regression would try to explain the large returns during the “pre-arbitrage” period driven by mispricing using large betas during the “post-arbitrage” period, when the actual causality flows in the opposite direction from the “pre-arbitrage” return to “post-arbitrage” beta. Although this second point is not formalized as part of Proposition 3, I do examine the issue empirically.

I now use a cross-sectional asset pricing regression to explain anomaly assets’ returns through their exposures to arbitrageur funding shocks. To do this, I jointly estimate the anomaly assets’ betas in the time-series of returns and the price of risk in the cross-section of returns in a GMM framework:

\[
\text{Time-series regressions:} \quad r_{j,t} = a_j + \beta_j f_t + \varepsilon_{j,t} \quad \text{for } j = 1, \ldots, J
\]

\[
\text{Cross-sectional regression:} \quad \bar{r}_j = \lambda_0 + \lambda_1 \beta_j + e_j \quad (50)
\]

where the cross-sectional regression is an OLS regression that puts an equal weight on all anomaly assets. As articulated by Cochrane (2005), mapping these regressions into GMM allows me to obtain standard errors that account for both the fact that \(\beta\)s are estimated and the fact that returns can be correlated across anomaly assets. However, the baseline regression uses only 88 quarters in the post-93 sample to test if exposure to arbitrageur funding has a positive price of risk within this period of large arbitrageur presence. Hence, the estimates of the full variance-covariance matrix of errors \(\varepsilon\) may be subject to noise. Given this consideration, I also report t-statistics that adjust the standard errors for the fact that betas are estimated but use the conventional heteroskedasticity-consistent matrix of residuals \(\varepsilon\) that restrict the cross-anomaly correlations to be zero (“t-GenReg”). Furthermore, I compare the arbitrageur funding shock’s ability to explain the cross-section of anomaly mean returns to that of conventional multifactor models.

Table 5 presents the results. In terms of \(R^2\), during the post-93 sample, the single arbitrageur funding shock explains 37% of the cross-sectional variation in the long-short returns on 34 anomaly assets. This \(R^2\) is somewhat lower than the \(R^2\)s I obtain from the Fama-French three-factor model (Fama and French, 1993), the Carhart four-factor model (Carhart, 1997), and the Fama-French five-factor model (Fama and French, 2008). However, that the market and size factors exhibit negative prices of risk in most of these multifactor models speaks of the difficulty of jointly explaining the cross-section of anomaly expected returns.

The estimated price of risk is 0.18 so that each additional unit of funding beta is compensated by an annualized return of 0.18%. This estimated slope, however, is not unequivocally
significant. When cross-anomaly correlations are restricted to be zero, the standard error implies a t-statistic of 2.30. However, based on the most conservative GMM standard errors where cross-anomaly correlations are freely estimated, the t-statistic is 1.30. This implies that, in the data, some of the anomaly assets are highly correlated with other anomaly assets, which increases the standard errors.

To see if this is the case, I try restricting my attention to a smaller set of anomaly assets that “span” my 34 anomalies. Starting with the 34 anomalies, I iteratively eliminate an anomaly asset that has the largest $R^2$ when projected onto the other anomalies until the remaining anomalies have $R^2$s of less than 50% in such an exercise. Using these more “independent” anomalies shows a similar price of risk estimate (0.20 rather than 0.18) but a substantially smaller GMM standard error, making the price of risk statistically significant at the 10% level. There are other avenues for dealing with cross-anomaly correlations. One possibility is to use the generalized least squares (GLS) to penalize the anomalies whose residuals are volatile or highly correlated with the residuals of other anomalies, but this still relies on correctly estimating the cross-anomaly correlations of 34 anomaly assets using 88 time-series observations. Another possibility would be to estimate cross-anomaly correlations of the residuals in the entire sample rather than in the post-93 sample, an approach taken by Greenwood (2005) to deal with the short time-series. I do not explore this approach in this paper.

To summarize the discussion on the price of risk, there is some evidence that arbitrageurs demand compensation for the funding risks that arbitrageurs themselves have created in the anomalies, causing the anomaly returns to survive in equilibrium. Although the GMM t-statistics for the price of risk is not large in the baseline regression, both the large $R^2$ and the improved performance using the more independent anomaly assets suggests that focusing on the risks that arbitrageurs face is a fruitful path to take in order to jointly explain the cross-section of expected returns of a large set of asset pricing anomalies.

The intercept of the regression is positive and significant, consistent with arbitrageurs being constrained and anomaly assets being mispriced in some parts of the sample period. The intercept of 3.10% measures the average investment opportunity faced by arbitrageurs: arbitrageurs generate a risk-adjusted return that is on average 3.10% above the risk-free rate.

How well does the pricing work in other sample periods? In the pre-93 sample, the estimated price of risk is actually similar in magnitude to that of the post-93 sample, but the low t-statistics suggest that the estimate is unreliable. This suggests that, during this period, arbitrageurs were still small ($\mu \approx 0$) and did not have large presence in the 34 anomaly assets.

Combining the two sample periods, however, generates a much larger estimated price of risk than that in either of the two sample periods. The reason is that large returns come from
the pre-arbitrage period and large betas come from post-arbitrage period. Hence, pooling them together inflates the price of risk. This point can be better illustrated using graphs. Figure 1b shows that, during the post-93 period, the anomalies with larger funding betas command larger expected returns. However, from the “alphas to betas” regression (Figure 2), we know that the anomalies with larger post-93 funding betas had even larger expected returns in the pre-93 period because the anomaly’s pre-93 return was what turned into post-93 endogenous risk. Furthermore, Figure 1a shows that pre-93 funding betas were centered around zero, which means that pooling the two samples leads to an attenuation of funding betas. Putting all this together, pooling the pre-arbitrage and post-arbitrage periods means that the betas are attenuated while the expected returns are inflated, which results in an inflated price of risk in the entire sample (Table 5 and Figure 3). The $R^2$ is slightly below that of the post-93 sample regression since funding betas in the entire sample are essentially white noise added to post-93 funding betas.

In Adrian, Etula, and Muir (2014) too, the estimated price of risk in the entire sample is larger than that in the subsamples. There, the estimated price of risk is (with appropriate scaling to match the price of risk in this paper) is 0.62 in the pooled sample period of 1968-2009 but 0.21 and 0.18, respectively, in the subsamples 1968-1988 and 1989-2009.\footnote{I take the last two numbers (0.21 and 0.18) from the online appendix to Adrian, Etula, and Muir (2014) available on Tyler Muir’s website. Their leverage factor is expressed as a percentage whereas my funding shock is expressed as the original number, so I apply a scaling factor of 100.} This is still true even when the price of risk of the leverage factor is estimated in a two-factor setting with the market excess return as the additional factor.\footnote{This criticism nonetheless applies to this study’s post-93 cross-sectional asset pricing as well. Because the mass of arbitrageurs $\mu$ grew gradually over time even within the post-93 period, by the same logic as above, the prices of risk estimated from splitting the post-93 period into two are lower than that estimated from the entire post-93 period.}

### 3.6 Endogenous risk is generated during constrained times

I now test Proposition 4, which predicts that the endogenous covariation between anomaly asset returns and arbitrageur funding occurs only in times when arbitrageurs are constrained. At such times, a favorable funding shock leads them to dedicate more capital to anomaly assets, increasing their valuations and realized returns. When they are unconstrained, all anomaly assets are already being fully exploited, so a variation in arbitrage capital does not lead to variation in the prices of anomaly assets. Furthermore, during a period in which arbitrageurs are persistently unconstrained, neither endogenous risks nor abnormal returns should arise in anomaly assets.

To test these predictions, I first need to identify times when arbitrageurs are more likely or...
less likely to be constrained. To proxy for the level (as opposed to the growth) of the arbitrageur funding condition, I take the four-year moving average of the arbitrageur funding shock \( f_t \),

\[
f_t^{MA} \equiv \frac{1}{17} \sum_{s=-8}^{8} f_{t+s},
\]

where \( s \) indexes quarter. This proxy for the level of arbitrageur funding is preferred to a more intuitive measure that simply removes a constant time trend from the original log leverage series, since such a measure would be exposed to medium-term changes in the leverage of broker-dealer leverage unrelated to the funding conditions of arbitrageurs. To mitigate concerns about data mining, I also try measuring the level of arbitrageur funding condition based on the quarterly series of average month-end VIX obtained from the Chicago Board Options Exchange (CBOE).\(^{42}\) This generates similar results.

Given the measure of arbitrageur funding level, I define “unconstrained” ("constrained") times within the post-93 period as the quarters in which the level of arbitrageur funding is above (below) the post-93 median. \( \text{Figure 7} \) plots the constrained and unconstrained quarters based on the baseline moving-average measure \( f_t^{MA} \). For comparison, I also plot the log leverage series with constant detrending and quarterly VIX. Based on either the moving-average of funding condition or VIX (top and bottom figures), the constrained quarters approximately fall into (i) the late 1990s to early 2000s and (ii) the financial crisis period. The late-1990s to early-2000s period includes the fall of long-term capital management (LTCM), the dot-com bubble and crash, and the 2003 mutual fund scandal, which also affected hedge funds. These three events would have contributed to persistently low arbitrageur funding conditions during the period. In contrast, the log leverage series with constant detrending (middle figure) treats the mid-1990s and early-to-mid-2010s as constrained quarters. Including these periods as constrained quarters weakens the results.

I proceed with the moving-average arbitrageur funding \( f_t^{MA} \) as the measure of arbitrageur funding level. With this, I first test whether funding betas arise only during constrained quarters. Defining a dummy variable \( 1 (t \in \text{Constrained}) \) to indicate the above-median \( f_t^{MA} \) quarters within the post-93 sample, I estimate

\[
r_{j,t} = \left( a_{j}\text{uncon} + \beta_{j}\text{uncon} f_t \right) 1 (t \in \text{Unconstrained}) + \left( a_{j}\text{const} + \beta_{j}\text{const} f_t \right) 1 (t \in \text{Constrained}) + \epsilon_{j,t},
\]

in the time series for individual anomaly assets as seemingly unrelated regressions. I then

\(^{42}\)The monthly series was downloaded from the CBOE website on August 10, 2016.
test the following joint hypotheses: funding betas are jointly zero during the unconstrained
time period ($\beta^\text{uncon}_j = 0$); funding betas are jointly zero during the constrained time period
($\beta^\text{const}_j = 0$); and change in returns from unconstrained to constrained times net of funding
exposure are jointly zero ($a^\text{const}_j - a^\text{uncon}_j$). Theory predicts the first hypothesis but rejects the
latter two.

Panel A of Table 6 presents test results. The first two columns show that funding betas
arise only during the constrained period, consistent with the prediction that arbitrageurs do
not generate endogenous risks when they are unconstrained and all anomalies are fully ex-
ploded. The average funding betas are 0.83 and 4.65, respectively, during the unconstrained
and constrained time periods. The last column tests if anomaly asset returns increase from
unconstrained to constrained times. If constrained times are indeed when arbitrage capital is
insufficient to eliminate anomaly returns, I would expect to see an increase in the zero-beta rate
and hence positive $a^\text{const}_j - a^\text{uncon}_j$. Indeed, the changes $a^\text{const}_j - a^\text{uncon}_j$ are jointly different from
zero with a cross-sectional average of 4.26, which loosely implies that the zero-beta rate rises
by 4.26% from unconstrained to constrained periods.

Panel B of Table 6 asks how well an anomaly’s latent mispricing—its pre-93 long-short
return $\bar{r}_j^{\text{pre}}$—predicts the cross-section of unconstrained time betas, of constrained time betas,
and of the change in $a^\text{const}_j - a^\text{uncon}_j$. For instance, the second column of Panel B is based on the
regression

$$
\beta^\text{const}_j = b_0 + b_1 \bar{r}_j^{\text{pre93}} + \eta_j,
$$

estimated through GMM, as explained in Appendix B.1. Theory implies that pre-93 return only
predicts the constrained time betas. It should not strongly predict $a^\text{const}_j - a^\text{uncon}_j$ for the follow-
ning reason. In the model, an anomaly’s abnormal return is zero if arbitrageurs are unconstrained
and $\min \{\bar{r}_j, 1/E_0 [\Lambda_1/\Lambda_0] - 1\}$ if arbitrageurs are constrained at time 0. That is, in the con-
strained case, all anomalies should have the same abnormal returns unless the anomaly’s latent
mispricing $\bar{r}_j$ is smaller than the abnormal returns arbitrageurs are earning from exploited as-
sets. This means that, in the cross-section of anomalies, pre-93 return should predict the change
in abnormal return from unconstrained to constrained times during the post-93 period only for
the small subset of anomaly assets that are not exploited by arbitrageurs during constrained
times.

The results in Panel B of Table 6 are consistent with the prediction. Although the slope
coefficients are positive in all columns, pre-93 mean return strongly predicts only the post-
93 constrained time funding betas. The regressions in the first two columns are illustrated in
Figure 8. Anomalies’ funding betas during unconstrained time are scattered around zero and
show no strong relationship with their latent mispricings measured by pre-93 mean returns. In contrast, funding betas during constrained times tend to be positive and show a strong positive relationship with pre-93 mean returns.

4 Auxiliary evidence for endogenous arbitrage risk

In this section, I carry out two additional tests of the endogenous arbitrage risk model. In Section 4.1, I show that the anomaly assets’ post-93 exposures to the arbitrageur funding conditions are not explained by fundamental cash-flow exposures. In Section 4.2, I show that two main empirical tests show similar results if I measure an anomaly’s endogenous risk as the beta with respect to arbitrageur’s portfolio returns, which I proxy using the portfolio returns of equity market-neutral hedge funds.

4.1 Funding beta is not a fundamental cash-flow risk

In Section 3, I argued that anomaly assets have become endogenously risky due to arbitrage activity. This implies that the anomalies’ risks are driven by arbitrageurs generating discount-rate news in the anomalies. Here, I test an alternative explanation, which is that the anomalies’ betas with respect to arbitrageur funding are driven by their cash-flow exposure to the arbitrageur funding conditions.

To do this, I first obtain cash-flow news of an anomaly asset. Building on the Campbell-Shiller decomposition (Campbell and Shiller, 1988), Vuolteenaho (2002) show that a firm’s price-to-book ratio can be decomposed into a discounted sum of future return-on-equity (ROE) and that of future returns:

\[
\ln \left( \frac{ME_{t-1}}{BE_{t-1}} \right) = \sum_{j=0}^{\infty} \rho^j \ln \left( 1 + ROE_{t+j} \right) - \sum_{j=0}^{\infty} \rho^j \ln \left( 1 + R_{t+j} \right),
\]

(54)

where \( ROE \) is defined as the ratio of clean surplus earnings \( X_t = BE_t - BE_{t-1} + D_t^{\text{gross}} \) to the firm’s beginning-of-the-period book equity \( (BE_{t-1}) \), and where \( R \) denotes the net return on the firm’s stock. Rearranging (54) gives an expression that decomposes the firm’s return into

---

43 An analogous decomposition of market betas appears in Campbell and Vuolteenaho (2004), Cohen, Polk, and Vuolteenaho (2009), and Campbell, Polk, and Vuolteenaho (2010).
cash-flow news and discount-rate news components:

\[
\begin{align*}
    r_{i,t} - E_{t-1}r_{i,t} &= \sum_{j=0}^{\infty} \rho^j \left( \text{roe}_{i,t+j} - E_{t-1}\text{roe}_{i,t+j} \right) - \sum_{j=1}^{\infty} \rho^j \left( r_{i,t+j} - E_{t-1}r_{i,t+j} \right) \\
    &\equiv CF_{i,t} + DR_{i,t},
\end{align*}
\]

where \( \text{roe} \) and \( r \) respectively denote \( \ln(1 + \text{ROE}) \) and \( \ln(1 + R) \). This means that anomaly \( j \)'s conditional beta with respect to the arbitrageur funding condition \( f_t \), denoted \( \beta_{j,t} \), can be decomposed into cash-flow and discount-rate betas:

\[
\begin{align*}
    \beta_{j,t} &= \sum_{i \in I_j} w_{i,t}^{j} \frac{\text{Cov}(CF_{i,t+1}, f_{t+1})}{\text{Var}(f_{t+1})} + \sum_{i \in I_j} w_{i,t}^{j} \frac{\text{Cov}(DR_{i,t+1}, f_{t+1})}{\text{Var}(f_{t+1})} \\
    &\equiv \beta_{j,t}^{CF} + \beta_{j,t}^{DR},
\end{align*}
\]

where \( I_j \) represents the constituents of anomaly asset \( j \), \( w_{i,t}^{j} \) is the portfolio weight of \( i \) (which can be negative), and \( N_{j,t}^{CF} \) and \( N_{j,t}^{DR} \) respectively denote cash-flow and discount-rate news about anomaly \( j \). In summary, an anomaly’s funding beta can be decomposed into cash-flow beta \( \beta_{j,t}^{CF} \) and discount-rate beta \( \beta_{j,t}^{DR} \), and the cash-flow beta of an anomaly can be computed using the underlying firms’ ROEs.

In implementing the test, I follow Campbell, Polk, and Vuolteenaho (2010) in proxying for a stock’s cash-flow news with its \( \text{ROE} \) adjusted for inflation: \( \text{roe}_{i,t} = \ln(1 + \text{ROE}_t) - 0.4\ln(1 + r_f) \), where \( r_f \) is the risk-free rate. To prevent shorter-term trends in profitability from driving cash-flow betas, I follow Campbell and Vuolteenaho (2004), Cohen, Polk, and Vuolteenaho (2009), and Campbell, Polk, and Vuolteenaho (2010) in proxying for a stock’s cash-flow news using a discounted sum of realized return on equities:

\[
CF_t \approx \sum_{j=0}^{K-1} \rho^j \ln(1 + \text{ROE}_{t+j}),
\]

where \( \rho = 0.975^{1/4} \) is the quarterly discount rate and \( K \) is the number of quarters. I use \( K = 1, 4, 8, \) and 12 quarters. When using a discounted sum of \( \text{ROE} \)s, I use the analogously discounted sum of arbitrageur funding shocks \( f_t \) as the factor.

I run two kinds of test on the cash-flow betas of 34 anomaly assets in the post-93 period.
First, I test if the anomalies’ cash-flow funding betas are positive and jointly different from zero. Second, I test how much of anomalies’ funding betas are explained by cash-flow betas.

Table 7 reports the test results. In Panel A, I find that the anomalies’ cash-flow funding betas are on average slightly negative. This suggests that anomaly assets do not have positive fundamental exposure to the variation in the arbitrageur funding conditions. In Panel B, I find that the cash-flow betas are only a small part of funding betas. Although the anomalies with higher cash-flow funding betas do tend to have higher total betas, the relationship is not statistically significant, and both the large intercept and the small $R^2$s imply that most of the total betas remain unexplained.

A related question is whether the anomalies are linked through the discount-rate components of their returns. Both the theoretical and empirical results of this paper suggest that this must occur in the post-93 period with more arbitrage activities. Lochstoer and Tetlock (2016) find that, between 1964 and 2015, anomalies have little commonality in the discount-rate or cash-flow components of their returns. It would be interesting to see whether this continues to be true in the more recent years with heightened arbitrage activities and with more extreme deciles of the anomalies. If so, it would be strong evidence against my results.

4.2 Equity market-neutral hedge fund return as an alternative proxy for shocks to arbitrage capital

Thus far, the empirical tests have used funding beta to measure an anomaly’s endogenous risk. This is consistent with a model in which shocks to arbitrageur capital $k_t = w_t + f_t$ come entirely from funding shocks $f_t$, but in reality, shocks to arbitrageur wealth $w_t$ orthogonal to funding shocks may generate large variation in arbitrage capital. For example, some of the anomaly assets held by arbitrageurs may underperform for reasons other than the arbitrageur funding conditions, and such a shock to the portfolio may lead the arbitrageurs to de-lever their other positions (e.g., Shleifer and Vishny, 1997). This channel of arbitrageur capital shock would be better measured by the portfolio return of the arbitrageurs.

In Table 8, I report results from testing Proposition 1 and Proposition 3 using beta with respect to equity market-neutral hedge fund return (“HF return beta”), from Hedge Fund Research, as the measure of endogenous risk.\footnote{The monthly series was downloaded on June 27, 2016 and converted to quarterly frequency.} Consistent with Proposition 1, an anomaly asset with a larger pre-93 long-short return attains a larger post-93 hedge fund return beta, although the result is not statistically significant at the 5% level based on GMM t-statistics. The second column shows that the result is stronger when I use post-93 hedge fund return correlation as
the dependent variable, since it controls for changes in anomaly assets’ volatilities unrelated to arbitrageur trading, which would add noise to the estimated post-93 betas. The last column examines the cross-section of anomaly asset returns with their hedge fund return betas. The estimated price of risk is not large enough to completely overcome large GMM standard errors, but the $R^2$ of 21% suggests that hedge fund return betas still help explain the cross-sectional variation in anomaly asset returns.

Figure 9 visualizes the regression results in the first and last columns of Table 8. Compared to Figure 2 and Figure 1b, the figures show slightly larger residuals, suggesting that arbitrageur funding shock is a better measure of shocks to arbitrageur capital $k_t$ than hedge fund portfolio returns.

5 Conclusion

This paper shows that the act of arbitrage generates endogenous risk by turning assets’ alphas into betas. The act of arbitrage causes the prices of anomaly assets to comove with shocks to arbitrage capital, and the strength of this comovement—the beta—depends on the amount of arbitrage capital devoted to each anomaly asset. Once these betas arise, they explain the anomaly returns that survive in equilibrium, since arbitrageurs require a compensation for the risk they themselves have created.

This paper links two seemingly disparate points of view: limits to arbitrage and intermediary-based asset pricing. From the limits-to-arbitrage point of view, this paper represents an extension of the idea that arbitrage activity generates endogenous risk to a cross-section of differently mispriced anomaly assets. From the intermediary-based asset pricing point of view, measuring this endogenous risk as the beta with respect to arbitrage capital shocks allows for a cross-sectional pricing of anomaly assets.

There are at least three avenues for future research. First, although I use equity anomalies in the U.S. market as a convenient test laboratory, the model’s implications apply to any asset pricing anomalies. It would be interesting to empirically examine if arbitrage has turned mispricings into endogenous risks in other asset classes or other markets. Second, by a mechanism similar to the one explored in this paper, arbitrage activity may cause differently mispriced assets to be differently exposed to crash risks or tail risks.\textsuperscript{45} One may formalize this conjecture in a model and take its cross-sectional predictions to data, thereby endogenizing the cross-sectional exposures to higher-order risks that may be priced in equilibrium (e.g., Bali, Cakici, Stein (2009) provides such a framework in a single-asset environment. An extension would require embedding differently mispriced assets into such a model.

\textsuperscript{45}
and Whitelaw, 2014; Kelly and Jiang, 2014; Amaya, Christoffersen, Jacobs, and Vasquez, 2015; Bollerslev, Todorov, and Xu, 2015). Third, this paper implies that arbitrageurs such as hedge funds have strong preferences over the characteristics of securities that are known to generate abnormal returns. Hence, in the spirit of Berry, Levinsohn, and Pakes (1995) and Koijen and Yogo (2016), one can estimate arbitrageurs’ preferences over those characteristics using institutional holdings data and short interest data. From this, one can infer how arbitrage trading—through both long and short positions—shapes the liquidity of the securities with anomaly characteristics.
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### Tables and Figures

#### Table 1: Summary Statistics of Anomaly Assets by Sample Period

This table summarizes the 34 equity anomaly assets used in the empirical section. The abbreviations for the categories are: NMV = Novy-Marx and Velikov (2016); Ind.Adj. = industry-adjusted signals; Behavi. = meant to exploit behavioral biases. Sign indicates whether a higher level of the signal indicates a higher (1) or lower (-1) expected return. Return $\sigma$ indicates the standard deviation of the time-series of returns. All returns except for those of beta arbitrage and idiosyncratic volatility are value-weighted (VW) long-short returns calculated by subtracting bottom-decile VW return from top-decile VW return. Beta arbitrage and idiosyncratic volatility strategies are hedged for their market exposures based on the CAPM betas of the top and bottom deciles estimated from weekly returns in the previous three years.

<table>
<thead>
<tr>
<th>No</th>
<th>Anomaly</th>
<th>Category</th>
<th>Sign</th>
<th>Mean Return</th>
<th>$\sigma$</th>
<th>CAPM $\beta$</th>
<th>Funding $\beta$</th>
<th>1972-1993 (&quot;pre-93&quot;)</th>
<th>1994-2015 (&quot;post-93&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Beta arbitrage</td>
<td>NMV</td>
<td>-1</td>
<td>4.15</td>
<td>24.10</td>
<td>-0.05</td>
<td>11.70</td>
<td>5.17</td>
<td>28.79</td>
</tr>
<tr>
<td>2</td>
<td>Ohlson’s O-score</td>
<td>NMV</td>
<td>-1</td>
<td>5.31</td>
<td>37.00</td>
<td>-0.44</td>
<td>3.94</td>
<td>1.69</td>
<td>34.53</td>
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<tr>
<td>3</td>
<td>Size</td>
<td>NMV</td>
<td>-1</td>
<td>1.90</td>
<td>43.04</td>
<td>0.43</td>
<td>-12.28</td>
<td>2.17</td>
<td>35.51</td>
</tr>
<tr>
<td>4</td>
<td>PEAD(SUE)</td>
<td>NMV</td>
<td>1</td>
<td>12.60</td>
<td>25.53</td>
<td>-0.01</td>
<td>-0.88</td>
<td>5.57</td>
<td>25.12</td>
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<tr>
<td>5</td>
<td>Value</td>
<td>NMV</td>
<td>1</td>
<td>11.53</td>
<td>39.42</td>
<td>-0.17</td>
<td>0.88</td>
<td>3.92</td>
<td>37.41</td>
</tr>
<tr>
<td>6</td>
<td>36-month momentum</td>
<td>Behavi.</td>
<td>-1</td>
<td>4.45</td>
<td>45.55</td>
<td>0.18</td>
<td>-9.56</td>
<td>2.96</td>
<td>37.80</td>
</tr>
<tr>
<td>7</td>
<td>Long-run reversals</td>
<td>NMV</td>
<td>-1</td>
<td>5.38</td>
<td>44.51</td>
<td>0.04</td>
<td>-9.84</td>
<td>2.74</td>
<td>36.49</td>
</tr>
<tr>
<td>8</td>
<td>Short-term reversals</td>
<td>NMV</td>
<td>-1</td>
<td>5.13</td>
<td>23.88</td>
<td>0.05</td>
<td>-1.39</td>
<td>-3.72</td>
<td>40.20</td>
</tr>
<tr>
<td>9</td>
<td>Momentum</td>
<td>NMV</td>
<td>1</td>
<td>20.93</td>
<td>45.69</td>
<td>-0.22</td>
<td>6.31</td>
<td>8.76</td>
<td>52.91</td>
</tr>
<tr>
<td>10</td>
<td>Annual sales growth</td>
<td>Ind.Adj.</td>
<td>-1</td>
<td>3.35</td>
<td>27.12</td>
<td>-0.21</td>
<td>-4.88</td>
<td>1.33</td>
<td>24.48</td>
</tr>
<tr>
<td>11</td>
<td>employees</td>
<td>Ind.Adj.</td>
<td>-1</td>
<td>-1.49</td>
<td>21.74</td>
<td>-0.29</td>
<td>2.38</td>
<td>4.23</td>
<td>20.49</td>
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<tr>
<td>12</td>
<td>Accruals</td>
<td>NMV</td>
<td>-1</td>
<td>6.16</td>
<td>19.68</td>
<td>-0.06</td>
<td>0.07</td>
<td>4.16</td>
<td>24.28</td>
</tr>
<tr>
<td>13</td>
<td>Ind-adj book-to-market</td>
<td>Ind.Adj.</td>
<td>1</td>
<td>7.69</td>
<td>31.79</td>
<td>0.01</td>
<td>-0.95</td>
<td>1.99</td>
<td>26.91</td>
</tr>
<tr>
<td>14</td>
<td>Industry momentum</td>
<td>NMV</td>
<td>1</td>
<td>2.98</td>
<td>37.07</td>
<td>-0.22</td>
<td>4.97</td>
<td>1.03</td>
<td>38.02</td>
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<tr>
<td>15</td>
<td>Ind-adj firm size</td>
<td>Ind.Adj.</td>
<td>-1</td>
<td>4.49</td>
<td>21.83</td>
<td>0.23</td>
<td>-1.18</td>
<td>1.73</td>
<td>20.91</td>
</tr>
<tr>
<td>16</td>
<td>Ind-adj cash-flow-to-</td>
<td>Ind.Adj.</td>
<td>1</td>
<td>2.17</td>
<td>23.24</td>
<td>0.03</td>
<td>-8.13</td>
<td>-2.93</td>
<td>31.99</td>
</tr>
<tr>
<td>17</td>
<td>price ratio</td>
<td>NMV</td>
<td>1</td>
<td>2.05</td>
<td>16.19</td>
<td>-0.14</td>
<td>2.58</td>
<td>3.62</td>
<td>19.67</td>
</tr>
<tr>
<td>18</td>
<td>Idiosyncratic volatility</td>
<td>NMV</td>
<td>-1</td>
<td>3.97</td>
<td>26.96</td>
<td>-0.17</td>
<td>9.37</td>
<td>3.66</td>
<td>27.65</td>
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<td>19</td>
<td>Price delay</td>
<td>Behavi.</td>
<td>-1</td>
<td>-1.71</td>
<td>21.88</td>
<td>-0.22</td>
<td>3.56</td>
<td>-3.48</td>
<td>17.48</td>
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<tr>
<td>20</td>
<td>Failure probability</td>
<td>NMV</td>
<td>-1</td>
<td>9.70</td>
<td>48.19</td>
<td>-0.45</td>
<td>7.49</td>
<td>7.66</td>
<td>56.63</td>
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<td>21</td>
<td>Asset growth</td>
<td>NMV</td>
<td>-1</td>
<td>6.27</td>
<td>24.27</td>
<td>-0.17</td>
<td>-3.06</td>
<td>5.83</td>
<td>25.47</td>
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<tr>
<td>22</td>
<td>Net issuance</td>
<td>NMV</td>
<td>-1</td>
<td>5.46</td>
<td>18.74</td>
<td>-0.03</td>
<td>-3.52</td>
<td>8.13</td>
<td>29.96</td>
</tr>
<tr>
<td>23</td>
<td>Seasonality</td>
<td>NMV</td>
<td>1</td>
<td>12.43</td>
<td>28.82</td>
<td>0.10</td>
<td>-0.16</td>
<td>6.59</td>
<td>27.84</td>
</tr>
<tr>
<td>24</td>
<td>Ind-adj change in profit margin</td>
<td>Ind.Adj.</td>
<td>1</td>
<td>0.21</td>
<td>21.99</td>
<td>-0.09</td>
<td>-5.58</td>
<td>-3.00</td>
<td>23.64</td>
</tr>
<tr>
<td>25</td>
<td>Ind-adj change in asset turnover</td>
<td>Ind.Adj.</td>
<td>1</td>
<td>1.57</td>
<td>18.53</td>
<td>-0.06</td>
<td>-3.48</td>
<td>5.35</td>
<td>23.10</td>
</tr>
<tr>
<td>26</td>
<td>PEAD(CAR3)</td>
<td>NMV</td>
<td>1</td>
<td>12.09</td>
<td>17.16</td>
<td>0.03</td>
<td>4.49</td>
<td>8.14</td>
<td>22.72</td>
</tr>
<tr>
<td>27</td>
<td>Investment</td>
<td>NMV</td>
<td>-1</td>
<td>7.76</td>
<td>22.37</td>
<td>-0.17</td>
<td>2.03</td>
<td>4.81</td>
<td>22.20</td>
</tr>
<tr>
<td>28</td>
<td>Return on market equity</td>
<td>NMV</td>
<td>1</td>
<td>16.25</td>
<td>30.84</td>
<td>-0.11</td>
<td>-4.84</td>
<td>11.12</td>
<td>46.46</td>
</tr>
<tr>
<td>29</td>
<td>Return on book equity</td>
<td>NMV</td>
<td>1</td>
<td>9.30</td>
<td>34.29</td>
<td>-0.09</td>
<td>-4.44</td>
<td>8.25</td>
<td>42.22</td>
</tr>
<tr>
<td>30</td>
<td>Return on assets</td>
<td>NMV</td>
<td>1</td>
<td>7.47</td>
<td>31.69</td>
<td>-0.12</td>
<td>2.14</td>
<td>6.24</td>
<td>40.84</td>
</tr>
<tr>
<td>31</td>
<td>Asset turnover</td>
<td>NMV</td>
<td>1</td>
<td>4.04</td>
<td>26.93</td>
<td>0.16</td>
<td>-1.42</td>
<td>4.32</td>
<td>32.00</td>
</tr>
<tr>
<td>32</td>
<td>Gross margins</td>
<td>NMV</td>
<td>1</td>
<td>-2.40</td>
<td>21.71</td>
<td>-0.11</td>
<td>-3.77</td>
<td>2.39</td>
<td>20.88</td>
</tr>
<tr>
<td>33</td>
<td>Gross profitability</td>
<td>NMV</td>
<td>1</td>
<td>0.45</td>
<td>26.34</td>
<td>-0.03</td>
<td>-0.58</td>
<td>5.08</td>
<td>30.10</td>
</tr>
<tr>
<td>34</td>
<td>Ind-adj reversals</td>
<td>NMV</td>
<td>-1</td>
<td>-3.36</td>
<td>19.81</td>
<td>0.05</td>
<td>-1.66</td>
<td>-4.21</td>
<td>33.92</td>
</tr>
</tbody>
</table>

Average across anomalies: 5.24 28.47 -0.07 -0.58 3.57 31.14 -0.17 2.62
Standard deviation across anomalies: 5.66 9.15 0.18 5.41 3.79 9.50 0.34 13.20
Table 2: Mispricing Turns into Funding Beta

Baseline: $\beta_{\text{post}} = b_0 + b_1 \bar{r}_{\text{pre}} + \eta_j$

This table reports the results from the cross-sectional regression predicting an anomaly asset’s post-1993 funding beta using pre-1993 mean long-short return. The dependent variable in the first three columns is post-1993 funding beta $\beta_{\text{post}}$, calculated under the assumption that the beta is constant during the sample period: $r_{j,t} = a_0 + \beta_{\text{post}} f_t + \epsilon_t$. The dependent variable in the fourth column is the post-1993 rate of increase in beta $\beta_{\text{post}}$, calculated under the assumption that anomaly return attains an increasingly large exposure to arbitrageur funding during the post-1993 sample: $r_{j,t} = a_0 + \left( \beta_{\text{pre}}^{\text{post}} + \beta_{\text{post}}^{\text{in}} \ln(t) \right) f_t + \epsilon_t$, where $t$ is the number of quarters into the sample ($t = 1$ for 1994Q1). The dependent variable in the last column is the post-1993 funding correlation $\rho_{\text{post}} = \beta_{\text{post}} \sigma_{\text{f}} \left( \sigma_{\text{post}}^{\text{f}} \right)^{-1}$ in percentage (%), calculated under the assumption that the correlation is constant during the post-1993 sample. t-OLS is the t-statistic calculated using only the residuals from the cross-sectional regression and accounts for a possible heteroskedasticity of residuals across anomaly assets. t-GMM refers to a t-statistic obtained from the GMM estimation procedure and accounts for the effects of generated regressors and cross-anomaly correlations. Arbitrageur funding shocks are measured by quarterly shocks to the leverage of broker-dealers.

<table>
<thead>
<tr>
<th></th>
<th>Post-93 Funding Beta</th>
<th>Post-93 Funding Corr</th>
<th>( \beta_{\text{post}}^{\text{f}} )</th>
<th>( \beta_{\text{post}}^{\text{inc}} )</th>
<th>( \rho_{\text{post}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-93 Mean Long-Short Return</strong> ( \bar{r}_{\text{pre}} )</td>
<td></td>
<td></td>
<td>1.24</td>
<td>1.10</td>
<td>1.19</td>
</tr>
<tr>
<td>(t-OLS)</td>
<td>(3.51)</td>
<td>(2.82)</td>
<td>(3.19)</td>
<td>(2.49)</td>
<td>(2.80)</td>
</tr>
<tr>
<td>(t-GMM)</td>
<td>(2.03)</td>
<td>(1.98)</td>
<td>(2.15)</td>
<td>(0.81)</td>
<td>(2.41)</td>
</tr>
<tr>
<td><strong>Pre-93 Funding Beta</strong> ( \beta_{\text{pre}} )</td>
<td></td>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-OLS)</td>
<td>(2.50)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-GMM)</td>
<td>(1.17)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pre-93 Return Volatility</strong> ( \sigma_{\text{pre}} )</td>
<td></td>
<td>0.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-OLS)</td>
<td></td>
<td>(0.26)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-GMM)</td>
<td></td>
<td>(0.10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Intercept</strong></td>
<td>-3.84</td>
<td>-2.69</td>
<td>-5.49</td>
<td>-1.04</td>
<td>-3.89</td>
</tr>
<tr>
<td>(t-OLS)</td>
<td>(-1.44)</td>
<td>(-1.00)</td>
<td>(-0.80)</td>
<td>(-1.00)</td>
<td>(-0.89)</td>
</tr>
<tr>
<td>(t-GMM)</td>
<td>(-1.68)</td>
<td>(-1.20)</td>
<td>(-2.46)</td>
<td>(-1.01)</td>
<td>(-0.94)</td>
</tr>
<tr>
<td><strong>Anomalies</strong></td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td><strong>Adjusted ( R^2 )</strong></td>
<td>0.26</td>
<td>0.33</td>
<td>0.24</td>
<td>0.14</td>
<td>0.15</td>
</tr>
</tbody>
</table>
### Table 3: Funding Betas Are Explained by Arbitrage Activity

Baseline: $\beta_{j}^{post} = b_{0} + b_{1}DSIR_{j}^{post} + \eta_{j}$

This table reports results from the cross-sectional regressions explaining an anomaly asset’s funding beta using an anomaly-specific measure of arbitrage capital. The first three columns report results for the post-1993 period, and the last three columns report results for the pre-1993 period. The dependent variable is the funding beta of an anomaly asset in the pertaining sample period, when arbitrageur funding shocks are measured by quarterly shocks to the leverage of broker-dealers. Short interest ratio difference, log short interest ratio difference, and days to cover difference are the time-series averages of the difference in the value-weighted (VW) average of each measure between the bottom and top deciles of the anomaly asset. Short interest ratio difference uses 100 times the level difference in the VW average short interest ratio (short interest ÷ shares outstanding) in the bottom and the top deciles. Log short interest ratio difference uses the log difference in the VW average short interest ratio in the bottom and the top deciles. Days to cover difference uses the level difference in the VW average days to cover (short interest ÷ average daily trade volume) in the bottom and the top deciles.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Interest Ratio Difference</td>
<td>4.94 (3.98)</td>
<td>11.33 (5.39)</td>
</tr>
<tr>
<td>Log Short Interest Ratio Difference</td>
<td>14.46 (3.81)</td>
<td>6.37 (5.72)</td>
</tr>
<tr>
<td>Days to Cover Difference</td>
<td>4.11 (3.05)</td>
<td>4.14 (4.22)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.79 (-0.39)</td>
<td>-0.42 (-0.20)</td>
</tr>
<tr>
<td></td>
<td>0.98 (0.48)</td>
<td>-2.00 (-2.45)</td>
</tr>
<tr>
<td></td>
<td>-2.00 (-2.45)</td>
<td>-1.95 (-2.23)</td>
</tr>
<tr>
<td></td>
<td>-1.73 (-2.18)</td>
<td></td>
</tr>
<tr>
<td>Anomalies</td>
<td>34 (0.37)</td>
<td>34 (0.32)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>34 (0.18)</td>
<td>34 (0.43)</td>
</tr>
<tr>
<td></td>
<td>34 (0.39)</td>
<td>34 (0.36)</td>
</tr>
</tbody>
</table>

Note: In the parentheses are OLS t-statistics calculated with heteroskedasticity-consistent standard errors. The average of the arbitrage activity measures across all anomalies are 0.69 ($DSIR$), 0.21 ($DLSIR$), and 0.40 ($DDTC$) in the post-93 period. The average of the arbitrage activity measures across all anomalies are 0.13 ($DSIR$), 0.22 ($DLSIR$), and 0.28 ($DDTC$) in the pre-93 period.
Table 4: Mispricing Turns into a Larger Funding Beta on the Short Side

Baseline: $\beta_{j_{\text{post}}} = b_0 + b_1\text{Short}_j + b_2\alpha_{j\text{pre,CAPM}} + b_3\alpha_{j\text{pre,CAPM}} \times \text{Short}_j + \eta_j$

This table reports results from the cross-sectional regressions explaining an anomaly asset’s post-1993 funding beta net of market exposure using its pre-1993 CAPM alpha. An anomaly asset is either the long or short side of 34 anomalies. The dependent variable is post-1993 funding beta $\beta_{j_{\text{post}}}$, calculated under the assumption that the funding beta net of market exposure is constant during the post-1993 sample: $r_{j,t} - r_{f,t} = a_0 + \beta_{j_{\text{post}}} f_t + \beta_{j_{\text{post,mkt}}} (r_{m,t} - r_{f,t}) + \epsilon_t$, where $r_{f,t}$ and $r_{m,t}$ denote risk-free rate and market return. The independent variable is pre-1993 CAPM alpha: $r_{j,t} - r_{f,t} = \alpha_{j\text{pre,CAPM}} + \beta_{j_{\text{pre,CAPM}}} (r_{m,t} - r_{f,t}) + \epsilon_t$. Arbitrageur funding shocks are measured by quarterly shocks to the leverage of broker-dealers.

<table>
<thead>
<tr>
<th></th>
<th>Long Side</th>
<th>Short Side</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-93 CAPM Alpha</td>
<td>0.74</td>
<td>2.67</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>(1.51)</td>
<td>(8.18)</td>
<td>(1.51)</td>
</tr>
<tr>
<td>Pre-93 CAPM Alpha $\times$ 1(Short)</td>
<td>1.93</td>
<td>6.81</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.26)</td>
<td>(3.07)</td>
</tr>
<tr>
<td>1(Short)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-3.71</td>
<td>3.10</td>
<td>-3.71</td>
</tr>
<tr>
<td></td>
<td>(-1.91)</td>
<td>(1.16)</td>
<td>(-1.97)</td>
</tr>
<tr>
<td>Anomalies</td>
<td>34</td>
<td>34</td>
<td>68</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.06</td>
<td>0.71</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Notes: † This funding beta is net of market exposure. In the parentheses are OLS t-statistics calculated with heteroskedasticity-consistent standard errors.
Table 5: “Intermediary Asset Pricing” of Anomaly Assets

Baseline: $\bar{r}_{j,post} = \lambda_0 + \lambda_1 \beta_{j,post} + \epsilon_j$

This table reports the risk prices of factors and intercepts estimated in the cross-section of anomaly assets. Returns are long-short returns expressed in annualized percentages. Betas are estimated in the time-series regression $r_{j,t} = a_j + \beta_j f_t + \epsilon_{j,t}$ for each anomaly. t-GenReg refers to t-statistic corrected for generated regressors but not for cross-anomaly correlations. That is, to obtain the standard errors accounting for generated regressors, I allow for heteroskedastic residuals $\epsilon_j$ for the mean returns and do the correction derived by Shanken (1992), but under the assumption of $\text{Cov}(\epsilon_j, \epsilon_{j'}) = 0$ for $j' \neq j''$. t-GMM refers to GMM t-statistic that additionally corrects for correlations across anomaly assets. Arbitrageur funding shocks are measured by quarterly shocks to the leverage of broker-dealers.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Arb Funding</td>
<td>Arb Funding</td>
<td>Select 25†</td>
</tr>
<tr>
<td>Arb Funding</td>
<td>0.18</td>
<td>0.20</td>
<td>(t-GenReg) 2.30</td>
</tr>
<tr>
<td>(t-GenReg)</td>
<td>(2.30)</td>
<td>(2.29)</td>
<td>(t-GMM) 1.30</td>
</tr>
<tr>
<td>Market</td>
<td>-9.30</td>
<td>-6.96</td>
<td>(t-GenReg) -2.16</td>
</tr>
<tr>
<td>(t-GenReg)</td>
<td>-2.16</td>
<td>-1.60</td>
<td>(t-GMM) -1.89</td>
</tr>
<tr>
<td>SMB</td>
<td>-0.57</td>
<td>0.19</td>
<td>(t-GenReg) -0.25</td>
</tr>
<tr>
<td>(t-GenReg)</td>
<td>-0.25</td>
<td>0.08</td>
<td>(t-GMM) -0.22</td>
</tr>
<tr>
<td>HML</td>
<td>1.50</td>
<td>2.02</td>
<td>(t-GenReg) 0.53</td>
</tr>
<tr>
<td>(t-GenReg)</td>
<td>0.53</td>
<td>0.72</td>
<td>(t-GMM) 0.52</td>
</tr>
<tr>
<td>MOM</td>
<td>5.25</td>
<td></td>
<td>(t-GenReg) 1.31</td>
</tr>
<tr>
<td>(t-GenReg)</td>
<td>1.31</td>
<td></td>
<td>(t-GMM) 1.33</td>
</tr>
<tr>
<td>CMA</td>
<td>1.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-GenReg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-GMM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMW</td>
<td>2.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-GenReg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-GMM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>3.10</td>
<td>3.14</td>
<td>2.06</td>
</tr>
<tr>
<td>(t-GenReg)</td>
<td>3.15</td>
<td>(4.67)</td>
<td>(3.62)</td>
</tr>
<tr>
<td>(t-GMM)</td>
<td>(3.15)</td>
<td>(2.94)</td>
<td>(3.02)</td>
</tr>
<tr>
<td>Anomalies</td>
<td>34</td>
<td>25</td>
<td>34</td>
</tr>
<tr>
<td>Quarters</td>
<td>88</td>
<td>88</td>
<td>88</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.37</td>
<td>0.29</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Note: † denotes 25 anomaly assets chosen out of 34 by iteratively eliminating anomaly assets that are similar to a linear combination of the other anomaly assets until all anomalies have an $R^2$ of less than 50% when linearly projected to all the other anomalies.
Table 6: Funding Betas Are Generated during Constrained Times

Data-generating process:

\[ r_{jt} = \left( a_{jt}^{\text{uncon}} + \beta_{jt}^{\text{uncon}} f_t \right) 1(t \in \text{Unconstrained}) + \left( a_{jt}^{\text{const}} + \beta_{jt}^{\text{const}} f_t \right) 1(t \in \text{Constrained}) + \epsilon_{jt} \]

This table analyzes the data-generating process in which anomaly assets’ exposures to arbitrageur funding as well as their expected returns change from unconstrained to constrained times within the post-1993 period. The unconstrained and constrained times are proxied by quarters in which the 4-year moving average of arbitrageur funding shock \( f_t \) is above the post-1993 median and below the median, respectively. Panel A reports the mean of the estimated parameters \( (\beta_{jt}^{\text{uncon}}, \beta_{jt}^{\text{const}}, \text{and } a_{jt}^{\text{const}} - a_{jt}^{\text{uncon}}) \) in the cross-section of anomaly assets and jointly tests the hypothesis that the actual parameter values are zero. Here, the residual return \( \epsilon_{jt} \) is assumed to have zero serial correlations. Panel B reports the results from the regression that explains an anomaly asset’s estimated parameters \( (\beta_{jt}^{\text{uncon}}, \beta_{jt}^{\text{const}}, \text{and } a_{jt}^{\text{const}} - a_{jt}^{\text{uncon}}) \), using its pre-1993 mean long-short return \( r_{jt}^{\text{pre}} \). Here, t-OLS is the t-statistic calculated using only the residuals from the cross-sectional regression and accounts for a possible heteroskedasticity of residuals across anomaly assets. t-GMM refers to t-statistic obtained from the GMM estimation procedure and accounts for the effects of generated regressors and cross-anomaly correlations. Arbitrageur funding shocks are measured by quarterly shocks to the leverage of broker-dealers.

<table>
<thead>
<tr>
<th></th>
<th>( \beta_{jt}^{\text{uncon}} )</th>
<th>( \beta_{jt}^{\text{const}} )</th>
<th>( a_{jt}^{\text{const}} - a_{jt}^{\text{uncon}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Joint Hypothesis Test (e.g., ( \beta_{jt}^{\text{const}} = ... = \beta_{jt}^{\text{const}} = 0 ))</td>
<td>0.83</td>
<td>4.65</td>
<td>4.26</td>
</tr>
<tr>
<td></td>
<td>( p ) (jointly zero)</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>B. Cross-sectional Prediction (e.g., ( \beta_{jt}^{\text{const}} = b_0 + b_1 r_{jt}^{\text{pre}} + \eta_{jt} ))</td>
<td>Pre-93 Mean Long-short Return ( r_{jt}^{\text{pre}} )</td>
<td>0.04</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td>(t-OLS)</td>
<td>(0.17)</td>
<td>(3.13)</td>
</tr>
<tr>
<td></td>
<td>(t-GMM)</td>
<td>(0.14)</td>
<td>(2.14)</td>
</tr>
<tr>
<td></td>
<td>Intercept</td>
<td>-0.48</td>
<td>-5.25</td>
</tr>
<tr>
<td></td>
<td>(t-OLS)</td>
<td>(-0.29)</td>
<td>(-1.19)</td>
</tr>
<tr>
<td></td>
<td>(t-GMM)</td>
<td>(-0.25)</td>
<td>(-1.82)</td>
</tr>
<tr>
<td></td>
<td>Anomalies</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>Adjusted ( R^2 )</td>
<td>-0.03</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Table 7: **Cash-flow Exposure Does Not Explain Funding Beta**

Panel A: $\beta_{CF,post}^1 = \ldots = \beta_{CF,post}^J = 0$

Panel B: $\hat{\beta}_{CF,post}^j = b_0 + b_1 \beta_{CF,post}^j + \eta_j$

This table asks if anomaly assets’ cash-flow funding betas explain their exposure to arbitrageur funding shocks in the post-93 period. Panel A tests the joint hypothesis that the cash-flow funding betas are jointly zero. Panel B tests how well the cash-flow funding beta explains the cross-sectional variation in total funding beta. Cash-flow beta of an anomaly is obtained as the beta of a discounted sum of future cash-flow news with respect to the analogous discounted sum of arbitrageur funding shocks. An anomaly’s cash-flow news is calculated by following the procedure of Campbell, Polk, and Vuolteenaho (2010). Arbitrageur funding shocks are measured by quarterly shocks to the leverage of broker-dealers.

<table>
<thead>
<tr>
<th>$K = 1$</th>
<th>$K = 4$</th>
<th>$K = 8$</th>
<th>$K = 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Joint Hypothesis Test (e.g., $\beta_{CF,post}^1 = \ldots = \beta_{CF,post}^J = 0$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross-sectional Average</td>
<td>-0.52</td>
<td>-1.43</td>
<td>-3.10</td>
</tr>
<tr>
<td>$p$(jointly zero)</td>
<td>0.84</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

B. How much of funding beta is explained by CF beta?

<table>
<thead>
<tr>
<th></th>
<th>$K = 1$</th>
<th>$K = 4$</th>
<th>$K = 8$</th>
<th>$K = 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF Funding Beta $\hat{\beta}_{CF}^j$</td>
<td>1.17</td>
<td>0.33</td>
<td>0.07</td>
<td>0.01</td>
</tr>
<tr>
<td>(1.76)</td>
<td>(1.13)</td>
<td>(0.38)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>3.24</td>
<td>3.10</td>
<td>2.84</td>
<td>2.65</td>
</tr>
<tr>
<td>(1.59)</td>
<td>(1.41)</td>
<td>(1.26)</td>
<td>(1.18)</td>
<td></td>
</tr>
<tr>
<td>Anomalies</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.09</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: In the parantheses are OLS t-statistics calculated with heteroskedasticity-consistent standard errors.
Table 8: **Tests Using an Alternative Measure of Shocks to Arbitrage Capital**

### Mispricing to endogenous risk:
\[
\beta_{HF, post}^j = b_0 + b_1 r_{pre}^j + \eta_j
\]

### Endogenous risk to expected return:
\[
r_{post}^j = \lambda_0 + \lambda_1 \beta_{HF, post}^j + \epsilon_j
\]

This table repeats the baseline regressions in Table 2 and Table 5 using the equity market-neutral hedge fund return as the measure of arbitrage capital shocks. The equity market-neutral hedge fund return is obtained by adjusting the market-neutral hedge fund return from Hedge Fund Research (HFR) by removing a small but significant exposure to market excess returns. t-OLS in the first two columns is the t-statistic calculated using only the residuals from the cross-sectional regression and accounts for a possible heteroskedasticity of residuals across anomaly assets. For the last column, t-OLS is the t-GenReg used in Table 5. Specifically, it refers to t-statistic corrected for generated regressors but not for cross-anomaly correlations. That is, to obtain the standard errors accounting for generated regressors, I allow for heteroskedastic residuals \( \epsilon_f \) for the mean returns and do the correction derived by Shanken (1992), but under the assumption of \( \text{Cov}(\epsilon_f, \epsilon_{f'}) = 0 \) for \( f \neq f' \). t-GMM refers to t-statistic obtained from the GMM estimation procedure and accounts for the effects of generated regressors and cross-anomaly correlations. The correlation is reported in percentage (%).

<table>
<thead>
<tr>
<th>Mispricing Turns into Endogenous Risk (Proposition 1)</th>
<th>Endogenous Risk Explains Expected Return (Proposition 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-93 HF Beta ( \beta_{HF, post}^j )</td>
<td>Post-93 HF Beta ( \beta_{HF, post}^j )</td>
</tr>
<tr>
<td>Correlation ( \rho_{HF, post}^j )</td>
<td>Correlation ( \rho_{HF, post}^j )</td>
</tr>
<tr>
<td>Pre-93 Mean Long -short Return ( r_{pre}^{pre} )</td>
<td>0.09</td>
</tr>
<tr>
<td>(t-OLS)</td>
<td>(3.84)</td>
</tr>
<tr>
<td>(t-GMM)</td>
<td>(1.85)</td>
</tr>
<tr>
<td>Post-93 HF Beta ( \beta_{HF, post}^j )</td>
<td></td>
</tr>
<tr>
<td>(t-OLS)</td>
<td></td>
</tr>
<tr>
<td>(t-GMM)</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.06</td>
</tr>
<tr>
<td>(t-OLS)</td>
<td>(-0.38)</td>
</tr>
<tr>
<td>(t-GMM)</td>
<td>(-0.26)</td>
</tr>
<tr>
<td>Anomalies</td>
<td>34</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Figure 1: Arbitrageur Funding Beta and Mean Long-short Return by Sample Period

These figures plot the mean long-short returns and arbitrageur funding betas of 34 equity anomaly assets in the pre-93 (left) and post-93 (right) samples, proxying for pre-arbitrage and post-arbitrage periods. Return is in annualized percentage. Returns roughly line up with funding betas in the post-93 sample, but not in the pre-93 sample. Funding beta is measured by beta with respect to arbitrageur funding shocks measured by quarterly shocks to the leverage of broker-dealers.
Figure 2: **Pre-93 Long-short Return Predicts Post-93 Arbitrageur Funding Beta**

This figure plots the relationship between pre-93 mean long-short return and post-93 funding beta. The pre-93 (1972-1993) and post-93 (1994-2015) samples proxy for pre-arbitrage and post-arbitrage periods, respectively. Return is in annualized percentage. Funding beta is measured by beta with respect to arbitrageur funding shocks measured by quarterly shocks to the leverage of broker-dealers.
Figure 3: **Arbitrageur Funding Beta and Mean Return in the Pooled Sample (1972-2015)**

This figure plots the mean long-short returns and arbitrageur funding betas of 34 equity anomaly assets over the entire sample period of 1972 to 2015. Return is in annualized percentage. Funding beta is measured by beta with respect to arbitrageur funding shocks measured by quarterly shocks to the leverage of broker-dealers.
Figure 4: **Price of Anomaly Asset \( j \) at Time 1**

The figure plots the time-1 price of anomaly asset \( j \), \( p_{j,1} \), as a function of total arbitrage capital \( \mu k_1 \). The price is given by

\[
p_{j,1} = \max\{ v \left( \frac{1}{1 + \bar{r}_j} \right), v \left( \frac{1}{1 + \bar{r}_{j'}} \right) \},
\]

where \( j' = \min\{0, 1 - \sqrt{2\mu k_1} \} \) is the marginal asset determined by the availability of arbitrageur capital \( \mu k_1 \).
Figure 5: **Log Leverage and Arbitrageur Funding Shock**

The figure plots the log of broker-dealer leverage and the arbitrageur funding shock over the period of 1972 to 2015. Broker-dealer leverage is the book leverage of broker-dealers defined as total financial assets net of repo assets divided by the difference between total financial assets and total liabilities. Arbitrageur funding shocks are measured by quarterly shocks to the leverage of broker-dealers. The funding shock is annualized.
Figure 6: **Endogenous Risk Is Larger on the Short Side**

The figure plots the pre-1993 CAPM alphas and post-1993 funding betas of excess returns on long-side and short-side portfolios of anomaly assets. The solid circles are short-side portfolios, and hollow circles are long-side portfolios. The figure shows that mispricing measured by pre-93 CAPM alpha transforms into a larger post-93 funding beta on the short side of the anomalies than on the long side. Funding beta is measured as the beta with respect to arbitrageur funding shocks proxied by quarterly shocks to the leverage of broker-dealers.
Figure 7: Proxies for Constrained and Unconstrained Quarters, Post-93

Each of the three figures plots, for the post-1993 period (1994 to 2015), a series representing the level of arbitrageur funding condition as well as the “unconstrained” and “constrained” quarters defined as the quarters in which the level of arbitrageur funding condition is above and below the median, respectively. A high value of the series indicates a good funding condition for the first two figures and bad funding condition for the last figure. The level of arbitrageur funding condition is proxied by the 4-year moving average of the arbitrageur funding shock (top), the log leverage of broker-dealers with a constant detrending (middle), and quarterly average of month-end VIX obtained from CBOE (bottom).
Figure 8: **Mispricing Turns into Endogenous Risk Only in the Constrained Times**

The left figure plots anomaly assets’ betas with respect to arbitrageur funding condition during the unconstrained quarters of the post-1993 period on the y-axis and the mean long-short returns during the pre-1993 period on the x-axis. The right figure plots the anomaly assets’ betas with respect to the arbitrageur funding conditions during the constrained quarters of the post-1993 period. For each anomaly asset, the betas in the unconstrained and constrained periods are estimated using the time-series regression \( r_{jt} = (\alpha_j^{\text{uncon}} + \beta_j^{\text{uncon}} f_t) 1 (t \in \text{Unconstrained}) + (\alpha_j^{\text{const}} + \beta_j^{\text{const}} f_t) 1 (t \in \text{Constrained}) + \epsilon_t. \) The unconstrained and constrained quarters are defined as the quarters in which the level of arbitrageur funding condition—the 4-year moving average of the arbitrageur funding shock \( f_t \)—is above and below the median, respectively. Arbitrageur funding shock is measured by quarterly shocks to the leverage of broker-dealers.
Figure 9: “Alphas to Betas” and “Intermediary Asset Pricing” Using Equity Market-Neutral Hedge Fund (HF) Return as an Alternative Measure of Arbitrage Capital Shock

The figures show that the main results of this paper are robust to using the equity market-neutral hedge fund (HF) return to measure arbitrage capital shock. The left figure shows that, in the cross-section of anomaly assets, a large pre-1993 mean long-short return predicts a large post-1993 beta with respect to HF return. The right figure shows that post-1993 mean long-short returns roughly line up with post-1993 betas with respect to HF return. HF return is measured by the return on the equity market-neutral hedge fund index provided by Hedge Fund Research (HFR). The market-neutral hedge fund return is adjusted by removing a small but significant exposure to market excess returns.
A Model appendix

A.1 Proofs of lemmas and propositions

Before proving Lemma 1, I establish the following argument made in the body of the text:

Remark 2. The following is true about anomaly assets at $t = 1$:

(i) All anomaly assets “exploited” by arbitrageurs have the same returns.
(ii) There exists a marginal anomaly asset $j^*_1 \in [0, 1]$ such that asset $j$ is exploited if and only if $j \in [j^*_1, 1]$.
(iii) All exploited assets generate return $\bar{r}j^*_1$, the expected return earned by asset $j^*_1$ in the absence of arbitrageurs.

Proof. (i) Suppose otherwise. Then we can find two exploited assets $j'$ and $j''$ such that

$$E_1r_{j',2} < E_1r_{j'',2}$$

But in this case, a risk-neutral arbitrageur can increase its expected portfolio return by reducing its dollar position on $j''$ by $dx_{j'',1}$ and using it to increase its position on $j'$ by $dx_{j',1} = p_{j',1}^{-1}d_{j'',1}$.

To see (ii) and (iii), note that there must be some $j^* \in [0, 1]$ such that the equal expected return earned by all exploited assets is $\bar{r}j^*_1$. It suffices to show that asset $j$ is exploited if and only if $j \in [j^*_1, 1]$. Suppose first that $j \in (j^*_1, 1]$ but is not exploited. Then, since $j$ earns an expected return,

$$E_1r_{j,2} = \bar{r}j > \bar{r}j^*_1$$

so that arbitrageurs are not optimizing. Now suppose $j \in [0, j^*_1)$. Then since its demand by behavioral investors, given expected return $\bar{r}j^*_1$, is

$$B_{j,t} = j^*_1 - j < 0,$$

the market does not clear if arbitrageurs take a long position on the asset.

Proof of Lemma 1 (Equilibrium price at time $t = 1$). Trivially, if $k_1 \leq 0$, behavioral investors price all assets to ensure

$$E_1[r_{j,2}] = \frac{v}{p_{j,1}} = 1 + \bar{r}j,$$
which implies

\[ p_j,1 = \frac{v}{1 + \bar{r}j} \]

Next, suppose \( k_1 \geq 0 \) but arbitrageurs cannot remove all mispricings. Then, by Lemma 1, there exists \( j_1^* \in (0,1) \) such that arbitrageurs exploit assets if and only if \( j \in [j_1^*,1] \) and earn the expected return

\[ E_1 r_{j,2} = \bar{r}j_1^* \]

from them. Then, behavioral investor demand for each asset \( j \in [j_1^*,1] \) is

\[ B_{j,t} = j_1^* - j \]

in dollar position. Thus, for the market to clear, arbitrageur’s demand for the asset needs to be \( x_{j,1} = j_1^* - j \). Integrating this over \([j_1^*,1]\) should equal total arbitrageur capital, so that

\[ \int_{j_1^*}^{1} (j - j_1^*) \, dj = \mu k_1 \]

This gives\(^{46}\)

\[ j_1^* = 1 - \sqrt{2\mu k_1} \]

and

\[ p_j,1 = \frac{v}{1 + \bar{r}j_1^*} \]

On the other hand, unexploited assets are priced by behavioral investors so that for all \( j \in [0,j_1^*] \),

\[ p_j,1 = \frac{v}{1 + \bar{r}j} \]

Finally, suppose arbitrageurs are unconstrained; that is, \( \mu k_1 \geq 1/2 \). Then, all anomaly assets are fully exploited so that

\[ p_j,1 = v \]

for all \( j \in [0,1] \).

**Proof of Lemma 2** *(Anomaly asset’s endogenous risk)*. The negative covariance follows from the fact that

\[ \frac{\partial \Lambda_1}{\partial p_{j,1}} = \frac{\partial \Lambda_1}{\partial k_1} \frac{\partial k_1}{\partial p_{j,1}} = - \text{ or } 0 \quad + \text{ or } 0 \leq 0 \]

\(^{46}\)Note that the other root is ruled out since it is always greater than 1, the largest possible value of \( j_1^* \).
(i) Suppose \( \mu \to 0^+ \). Then, \( k_1 \to 0^+ \) so that \( \psi_1 = 1 + \tau \) and \( p_{j,1} = v/(1 + \tau j) \). Since both are deterministic,

\[
\lim_{\mu \to 0} \text{Cov}_0 \left( p_{j,1}, \Lambda_1 \right) = 0
\]

(ii) Note that

\[
\frac{\partial \text{Cov}_0 (p_{j,1}, \Lambda_1)}{\partial \left( \tau j \right)} = \frac{\partial \text{Cov}_0 (p_{j,1}, \Lambda_1)}{\partial \left( \tau j \right)} \times \frac{1}{\tau} = \frac{1}{\tau} \times \frac{\partial \text{Cov}_0 (p_{j,1}, \Lambda_1)}{\partial j}
\]

Now, since \( \text{Cov}_0 (p_{j,1}, \Lambda_1) = E_0 [\Lambda_1 p_{j,1} - E_0 [\Lambda_1] E_0 [p_{j,1}]] \),

\[
\text{Cov}_0 (p_{j,1}, \Lambda_1) = v \int_{-\infty}^{0} \frac{1 + c}{1 + \tau j} dF (k_1) + v \int_{0}^{k_1(j)} \frac{1 + \tau j^*_1}{1 + \tau j} dF (k_1) + v \int_{k_1(j)}^{\infty} dF (k_1)
\]

\[
- vE_0 [\Lambda_1] \left( \int_{-\infty}^{0} \frac{1}{1 + \tau j} dF (k_1) + v \int_{0}^{k_1(j)} \frac{1}{1 + \tau j} dF (k_1) + v \int_{k_1(j)}^{1/2} \frac{1}{1 + \tau j^*_1} dF (k_1) + v \int_{1/2}^{\infty} dF (k_1) \right),
\]

where \( k_1(j) \) is the value of \( k_1 \) that gives \( j \) as the marginal asset, and \( F \) is the conditional cumulative density function of \( k_1 \). Thus, the derivative of the covariance with respect to \( j \) gives

\[
\frac{\partial \text{Cov}_0 (p_{j,1}, \Lambda_1)}{\partial j} = -v \left( \int_{-\infty}^{0} \frac{(1 + c) \tau}{(1 + \tau j)^2} dF (k_1) + \int_{0}^{k_1(j)} \frac{1 + \tau j^*_1 \tau}{(1 + \tau j)^2} dF (k_1) \right)
\]

\[
+ E_0 [\Lambda_1] v \left( \int_{-\infty}^{0} \frac{\tau}{(1 + \tau j)^2} dF (k_1) + \int_{0}^{k_1(j)} \frac{\tau}{(1 + \tau j)^2} dF (k_1) \right),
\]

where the Leibniz terms cancel out by the fact that \( j^*_1 \left( K_1 (j) \right) = j \). Rearranging terms gives

\[
\frac{\partial \text{Cov}_0 (p_{j,1}, \Lambda_1)}{\partial j} = - \frac{v \tau}{(1 + \tau j)^2} \left( \int_{-\infty}^{k_1(j)} \Lambda_1 dF (k_1) - E_0 [\Lambda_1] \int_{-\infty}^{k_1(j)} dF (k_1) \right)
\]

\[
= - \frac{v \tau}{(1 + \tau j)^2} \left( E_0 [\Lambda_1 | j < j^*_1] - E_0 [\Lambda_1] \right) F (k_1 (j))
\]

\[
< 0,
\]

since \( E_0 [\Lambda_1 | j < j^*_1] > E_0 [\Lambda_1] \).
Lemma 3. (Monotonicity of prices at $t = 0$). For any $j' < j''$ such that $j', j'' \in [0, 1]$,

$$p_{j',0} \geq p_{j'',0}$$

**Proof.** Suppose for a contradiction that $j' < j''$ but $p_{j',0} < p_{j'',0}$. Suppose also that $j''$ is priced by arbitrageurs so that

$$p_{j'',0} = E_0 \left[ \frac{\Lambda_1}{\Lambda_0} p_{j'',1} \right]$$

Since $p_{j',1} \geq p_{j'',1}$ in all states of $t = 1$, it must be that

$$p_{j',0} \geq E_0 \left[ \frac{\Lambda_1}{\Lambda_0} p_{j',1} \right] \geq E_0 \left[ \frac{\Lambda_1}{\Lambda_0} p_{j'',1} \right]$$

which is a contradiction. Now suppose that $j''$ is priced by behavioral investors so that

$$p_{j'',0} = \frac{1}{1 + \bar{r} j''} E_0 \left[ p_{j'',1} \right]$$

Since $p_{j',1} \geq p_{j'',1}$ in all states of $t = 1$, it must be that

$$p_{j',0} \geq \frac{1}{1 + \bar{r} j'} E_0 \left[ p_{j',1} \right] \geq \frac{1}{1 + \bar{r} j''} E_0 \left[ p_{j'',1} \right]$$

which is also a contradiction.

**Proof of Proposition 1** (“Alphas” turn into “betas”). Suppose $j < j'$. Then, by Lemma 2,

$$-\text{Cov}_0 (p_{j,1}, \Lambda_1) > -\text{Cov}_0 (p_{j',1}, \Lambda_1)$$

Furthermore, by Lemma 3 in the Appendix, $p_{j,0} < p_{j',0}$. Thus, $\beta_j = -\text{Cov}_0 (\Lambda_1/\Lambda_0, r_{j,1}) / \text{Var}_0 (\Lambda_1/\Lambda_0) > -\text{Cov}_0 (\Lambda_1/\Lambda_0, r_{j',1}) / \text{Var}_0 (\Lambda_1/\Lambda_0) = \beta_{j'}$.

**Proof of Proposition 2** (Beta is explained by anomaly-specific arbitrage capital). Recall that arbitrageur position on $j$ is $x_{j,1} = j - j_1^*$. Hence, the unconditional expectation of arbitrageur position is $E_0 x_{j,1} = j - E_0 [j_1^*]$. Thus, $\partial \beta_j / \partial E_0 [x_{j,1}] = (\partial \beta_j / \partial j) \times (\partial j / \partial E_0 [x_{j,1}] ) = \partial \beta_j / \partial j > 0$. 

70
A.2 Endogenizing the demand curves of behavioral investors

Suppose that each behavioral investor $i$ believes the state variable underlying the cash flow at $t = 2$ follows
\[ v_{j,t+1} = \frac{1}{\phi_i} v_{j,t} + \epsilon_{j,t+1} \]  
(58)

Asset $j$ has unique set of behavioral investors whose total mass equals one and whose beliefs are distributed according to
\[ F_j(\phi_i) \sim U \left[ 1 + r \left( j - \frac{1}{\theta} \right), 1 + r \left( j + \frac{1}{\theta} \right) \right] \]  
(59)

These behavioral investors, like arbitrageurs, cannot trade on margin. Then, given price $p_{j,t}$ and the expectation that price at $t + 1$ is $p_{j,t+1} = a + v_{j,t+1}$, the net demand for asset $j$ by behavioral investors is
\[ D_{j,t} = \frac{\theta}{2r} \left( \int_{1+\tau(j-\frac{1}{\theta})}^{v_{j,t}/p_{j,t}} di - \int_{v_{j,t}/p_{j,t}}^{1+\tau(j+\frac{1}{\theta})} di \right) \]  
(60)

which implies the demand of
\[ D_{j,t} = \theta \left( \frac{E_{t}r_{j,t+1}}{\bar{r}} - j \right) \]  
(61)

For behavioral investors at $t = 1$, this demand curve is the exact dollar position on $j$ demanded by them, since they expect $p_{j,2} = v_{j,2} = v_{j,1} + \epsilon_{j,2}$. For behavioral investors at $t = 0$, however, price at $t = 1$ is not necessarily expected to be linear in $v_{j,1}$. Still, if they perceive the price at $t = 1$ to be approximately linear in $v_{j,1}$ so that $p_{j,1} \approx a + bv_{j,1}$, then the demand curve is
\[ D_{j,0} = \theta \left( b \frac{E_{t}r_{j,t+1}}{\bar{r}} - j + \frac{b}{\bar{r}} - 1 \right) \]  
(62)

which is analytically identical to (61) for the purpose of this paper's model.
B Empirical appendix

B.1 GMM formulations

Latent mispricing predicts funding beta

Consider the following data-generating process. In the pre-93 period, long-short return is a noisy realization of latent mispricing:

\[ r_{j,t} = \bar{r}_{j}^{pre} + \varepsilon_{j,t} \] (63)

This latent mispricing determines the exposure of an anomaly asset to a funding shock:

\[ \beta_{j}^{post} = b_0 + b_1 \bar{r}_{j}^{pre} + \eta_j \] (64)

where \( \eta_j \) has a cross-sectional mean of zero. This beta then determines the long-short return in the post-93 period:

\[ r_{j,t} = a_j^{post} + \beta_j^{post} f_t + \varepsilon_{j,t} \] (65)

These conditions imply the following 4J moment conditions:

\[ g_{4J \times 1}(b) = \begin{bmatrix} E \left[ (r_{j,t} - \bar{r}_{j}^{pre}) 1(t \in Pre) \right] \\ E \left[ (r_{j,t} - a_j^{post} - \beta_j^{post} f_t) 1(t \in Post) \right] \\ E \left[ (r_{j,t} - a_j^{post} - \beta_j^{post} f_t) f_t 1(t \in Post) \right] \\ E \left[ (\beta_j^{post} - b_0 - b_1 r_{j,t}) 1(t \in Pre) \right] \end{bmatrix}, \] (66)
where the parameter vector is 
\[ b = \begin{bmatrix} \bar{r}_1^{pre} & \ldots & \bar{r}_J^{pre} & a_1^{post} & \ldots & a_J^{post} & \beta_1^{post} & \ldots & \beta_J^{post} & b_0 & b_1 \end{bmatrix} \].

At each \( t \), the errors are,
\[
g_t(b) = \begin{bmatrix} \varepsilon_{1,t}1(t \in Pre) \\
\vdots \\
\varepsilon_{J,t}1(t \in Pre) \\
\varepsilon_{1,t}1(t \in Post) \\
\vdots \\
\varepsilon_{J,t}1(t \in Post) \\
\varepsilon_{1,t}f_11(t \in Post) \\
\vdots \\
(\varepsilon_{1,t} + \eta_1)1(t \in Pre) \\
\vdots \\
(\varepsilon_{J,t} + \eta_J)1(t \in Pre) \end{bmatrix}
\]
\[(67)\]

Note that the errors \( \varepsilon_{j,t} \) represent the error in estimating the true latent mispricing \( \bar{r}^{pre} \) and the errors \( \eta_j \) represent the error in predicting post-93 beta using true \( \bar{r}^{pre} \). Since the errors \( \varepsilon_{j,t} \) are explicitly included in the last set of \( J \) moments, this GMM formulation takes into account the generated regressor problem for \( \bar{r}^{pre}_j \).

Since the last \( J \) moments represent errors in the cross-sectional regression (33), they require an expectation taken in the cross-section of anomaly assets. Hence, I use the following selection matrix to take a cross-sectional expectation:
\[
A_{(3J+2) \times 4J} = \begin{bmatrix} I_{3J \times 3J} & 0_{3J \times J} \\
0_{1 \times 3J} & 1_{1 \times J} \\
0_{1 \times 3J} & \bar{r}'_{1 \times J} \end{bmatrix}
\]
\[(68)\]

where \( \bar{r} \) is a \( J \times 1 \) vector of pre-93 long-short returns. Then, the estimation problem is to choose \( \hat{b} \) to set
\[
Ag_t(\hat{b}) = 0_{(3J+2) \times 1}
\]
\[(69)\]
or, equivalently, to minimize the sum of squared moments:
\[
\hat{b}_{(3J+2) \times 1} = \arg\min \left\{ (Ag_t)'_{1 \times (3J+2)} (Ag_t)_{(3J+2) \times 1} \right\}
\]
\[(70)\]

Under this formulation, the parameter estimates will be identical to those derived from
sequential OLS estimates of (63), (64), and (65). In particular, the last two rows of the selection matrix generate two moment conditions of the OLS estimation of (33). To see this, note that the vector of 1s in the right middle block of the selection matrix gives

\[0 = E_T [\varepsilon_{1,t} + \eta_1] + \ldots + E_T [\varepsilon_{J,t} + \eta_J] \]

\[= E_T \left[ \beta^\text{post}_1 - b_0 - b_1 r_{1,t} \right] + \ldots + E_T \left[ \beta^\text{post}_J - b_0 - b_1 r_{J,t} \right] \]

\[\Leftrightarrow 0 = \sum_{j=1}^{J} \left( \beta^\text{post}_j - b_0 - b_1 E_T [r_{j,t}] \right) \]

\[\Leftrightarrow 0 = E_J \left[ \beta^\text{post}_j - b_0 - b_1 E_T [r_{j,t}] \right] \] (71)

Analogously, the right lower block of the selection matrix implies

\[0 = E_J \left[ \left( \beta^\text{post}_j - b_0 - b_1 E_T [r_{j,t}] \right) E_T [r_{j,t}] \right] \] (72)

This shows that the last two rows of the selection matrix ensures that the cross-sectional regression has the same moments implied by the OLS implementation of it.

The spectral density for the moments is

\[S_{4J \times 4J} = \sum_{\tau=-\infty}^\infty E \left[ g_t(b) g_{t-\tau}(b)' \right], \] (73)

which I estimate assuming no serial correlation:

\[S_T = E_T \left[ g_t(b) g_t(b)' \right] \] (74)

Hansen (1982) shows that

\[\sqrt{T} (\hat{b} - b) \rightarrow N \left[ 0, (Ad)^{-1} ASA' \left( (Ad)^{-1} \right)' \right], \] (75)

where \(d\) is a matrix representing the sensitivity of moments with respect to parameter values:

\[d_{4J \times (3J+2)} = \frac{\partial g_T(b)}{\partial b'} \] (76)

Hence,

\[var(\hat{b}) = \frac{1}{T} (Ad)^{-1} ASA' \left( (Ad)^{-1} \right)' \] (77)
Adding pre-93 beta as another regressor

These conditions imply the following $6J$ moment conditions:

$$g_{6J \times 1}(b) = \begin{bmatrix}
E \left[ (r_{j,t} - r_{j,\text{pre}}^{\text{pre}}) 1(t \in \text{Pre}) \right] \\
E \left[ (r_{j,t} - a_{j,\text{post}} - \beta_{j,\text{post}}^f f_t) 1(t \in \text{Post}) \right] \\
E \left[ (r_{j,t} - a_{j,\text{pre}}^{\text{pre}} - \beta_j^f f_t) f_t 1(t \in \text{Pre}) \right] \\
E \left[ (r_{j,t} - a_{j,\text{post}}^{\text{post}} - \beta_j^f f_t) f_t 1(t \in \text{Post}) \right] \\
E \left[ \left( \frac{T}{T/2-1} (r_{j,t} - \bar{r}_{j,\text{pre}})^2 - \left( \sigma_{j,\text{pre}} \right)^2 \right) 1(t \in \text{Pre}) \right] \\
E \left[ \left( \beta_{j,\text{post}} - b_0 - b_1 r_{j,t} - b_2 \beta_{j,\text{pre}}^f \right) 1(t \in \text{Pre}) \right]
\end{bmatrix}$$

(78)

The selection matrix is

$$A_{(5J+3) \times 6J} = \begin{bmatrix}
I_{5J \times 5J} & 0_{5J \times J} \\
0_{1 \times 5J} & 1_{1 \times J} \\
0_{1 \times 5J} & \tilde{r}_t^{\text{t}}_{1 \times J} \\
0_{1 \times 5J} & \beta_{\text{pre}}^{1 \times J}
\end{bmatrix}$$

(79)

The formulae for spectral density $S$, the matrix $d$, and the covariance of estimates are analogous to those in the previous analysis.

Adding pre-93 volatility as another regressor

These conditions imply the following $6J$ moment conditions:

$$g_{5J \times 1}(b) = \begin{bmatrix}
E \left[ (r_{j,t} - r_{j,\text{pre}}^{\text{pre}}) 1(t \in \text{Pre}) \right] \\
E \left[ (r_{j,t} - a_{j,\text{post}} - \beta_{j,\text{post}}^f f_t) 1(t \in \text{Post}) \right] \\
E \left[ (r_{j,t} - a_{j,\text{pre}}^{\text{pre}} - \beta_j^f f_t) f_t 1(t \in \text{Pre}) \right] \\
E \left[ (r_{j,t} - a_{j,\text{post}}^{\text{post}} - \beta_j^f f_t) f_t 1(t \in \text{Post}) \right] \\
E \left[ \left( \frac{T}{T/2-1} (r_{j,t} - \bar{r}_{j,\text{pre}})^2 - \left( \sigma_{j,\text{pre}} \right)^2 \right) 1(t \in \text{Pre}) \right] \\
E \left[ \left( \beta_{j,\text{post}} - b_0 - b_1 r_{j,t} - b_2 \beta_{j,\text{pre}}^f \right) 1(t \in \text{Pre}) \right]
\end{bmatrix}$$

(80)

The selection matrix is

$$A_{(4J+3) \times 5J} = \begin{bmatrix}
I_{4J \times 4J} & 0_{4J \times J} \\
0_{1 \times 4J} & 1_{1 \times J} \\
0_{1 \times 4J} & \tilde{r}_t^{\text{t}}_{1 \times J} \\
0_{1 \times 4J} & \sigma_{\text{pre}}^{1 \times J}
\end{bmatrix}$$

(81)
The formulae for spectral density $S$, the matrix $d$, and the covariance of estimates are analogous to those in the previous analysis.

**Latent mispricing predicts the rate of increase in funding beta**

I assume the following data-generating process in which an exposure to arbitrageur funding grows at the rate $\beta_{j,1} t^{-1}$, where $t$ is the number of quarters into the post-93 sample (1994Q1 being $t = 1$):

\[
\begin{align*}
\text{Pre-93 return:} & \quad r_{j,t} = r_{j,pre} + \varepsilon_{j,t} \\
\text{Post-93 return:} & \quad r_{j,t} = a_{j}^{\text{post}} + \left( \beta_{j,0}^{\text{post}} + \beta_{j,1}^{\text{post}} \ln(t) \right) f_{t} + \varepsilon_{j,t} \\
\text{Beta determination:} & \quad \beta_{j,1}^{\text{post}} = b_{0} + b_{1} r_{j,pre} + \eta_{j}
\end{align*}
\]

(82)

Here, $\beta_{j,1}^{\text{post}}$ indicates the growth of funding beta due to growth of arbitrageur mass over time ($\mu$ in the model). The moment conditions are now

\[
g_{J \times 1}(b) = \left[ \begin{array}{c}
E \left[ \left( r_{j,t} - r_{j,pre} \right) 1(t \in \text{Pre}) \right] \\
E \left[ \left( r_{j,t} - a_{j}^{\text{post}} - \beta_{j,0}^{\text{post}} f_{t} - \beta_{j,1}^{\text{post}} \ln(t) f_{t} \right) 1(t \in \text{Post}) \right] \\
E \left[ \left( r_{j,t} - a_{j}^{\text{post}} - \beta_{j,0}^{\text{post}} f_{t} - \beta_{j,1}^{\text{post}} \ln(t) f_{t} \right) f_{t} 1(t \in \text{Post}) \right] \\
E \left[ \left( r_{j,t} - a_{j}^{\text{post}} - \beta_{j,0}^{\text{post}} f_{t} - \beta_{j,1}^{\text{post}} \ln(t) f_{t} \right) \ln(t) f_{t} 1(t \in \text{Post}) \right] \\
E \left[ \left( \beta_{j,1}^{\text{post}} - b_{0} - b_{1} r_{j,t} \right) 1(t \in \text{Pre}) \right]
\end{array} \right]
\]

(83)
The parameter vector is

\[ \begin{bmatrix} \bar{r}_{1}^{\text{pre}} \\ \vdots \\ \bar{r}_{J}^{\text{pre}} \\ \bar{a}_{1}^{\text{post}} \\ \vdots \\ \bar{a}_{J}^{\text{post}} \\ \bar{\beta}_{1,0}^{\text{post}} \\ \vdots \\ \bar{\beta}_{J,0}^{\text{post}} \\ \bar{\beta}_{1,1}^{\text{post}} \\ \vdots \\ \bar{\beta}_{J,1}^{\text{post}} \\ b_{0} \\ b_{1} \end{bmatrix} \]  

(84)

The selection matrix is

\[ A_{(4J+2) \times 5J} = \begin{bmatrix} I_{4J \times 4J} & 0_{4J \times J} \\ 0_{1 \times 4J} & 1_{1 \times J} \\ 0_{1 \times 4J} & \bar{r}_{1}^{j} \end{bmatrix} \]  

(85)

The formulae for spectral density \( S \), the matrix \( d \), and the covariance of estimates are analogous to those in the previous analysis.
Latent mispricing predicts funding correlation

The data-generating process here is assumed to be identical to that of the latent mispricing to funding beta regression. The moment conditions are

$$g_{(5J+2) \times 1}(b) = \begin{bmatrix}
E \left[ \left( r_{j,t} - \bar{r}_{j}^{pre} \right) \mathbb{1}(t \in Pre) \right] \\
E \left[ \left( r_{j,t} - \bar{r}_{j}^{post} \right) \mathbb{1}(t \in Post) \right] \\
E \left[ \left( \frac{T/2}{T/2-1} \left( r_{j,t} - \bar{r}_{j}^{post} \right)^2 - \left( \sigma_j^{post} \right)^2 \right) \mathbb{1}(t \in Post) \right] \\
E \left[ \left( \frac{T/2}{T/2-1} \left( f_t - \bar{f}^{post} \right)^2 - \left( \sigma_f^{post} \right)^2 \right) \mathbb{1}(t \in Post) \right] \\
E \left[ \left( \frac{T/2}{T/2-1} \left( r_{j,t} - \bar{r}_{j}^{post} \right) \left( f_t - \bar{f}^{post} \right) - \sigma_{j,f} \right) \mathbb{1}(t \in Post) \right] \\
E \left[ \sigma_{j,f} \left( \sigma_f^{post} \right)^{-1} \left( \sigma_f^{post} \right)^{-1} - b_0 - b_1 r_{j,t} \right] \mathbb{1}(t \in Pre)
\end{bmatrix} \quad (86)$$

where $T/2$ is the number of periods in each subsample. The parameter vector is

$$b_{(4J+4) \times 1} = \begin{bmatrix}
r_{1}^{pre} \\
\vdots \\
r_{J}^{pre} \\
\bar{r}_{1}^{pre} \\
\vdots \\
\bar{r}_{J}^{pre} \\
r_{1}^{post} \\
\vdots \\
r_{J}^{post} \\
\bar{r}_{1}^{post} \\
\vdots \\
\bar{r}_{J}^{post} \\
\sigma_1^{post} \\
\vdots \\
\sigma_J^{post} \\
\bar{f}^{post} \\
\sigma_f^{post} \\
\sigma_{1,f}^{post} \\
\vdots \\
\sigma_{J,f}^{post} \\
b_0 \\
b_1
\end{bmatrix} \quad (87)$$
The selection matrix is

\[
A_{(4J+4) \times (5J+2)} = \begin{bmatrix}
I_{(4J+2) \times (4J+2)} & 0_{(4J+2) \times J} \\
0_{1 \times (4J+2)} & 1_{1 \times J} \\
0_{1 \times (4J+2)} & \beta'_J
\end{bmatrix}
\]  

(88)

“Intermediary asset pricing” of anomaly assets based on endogenous risks

The moment conditions for the cross-sectional test are the conditions for estimating \(\beta\)s in the time series and the conditions for estimating \(\lambda\)s in the cross section:

\[
g(b) = \begin{bmatrix}
E [r_{j,t} - a_j - \beta_j f_t] \\
E [(r_{j,t} - a_j - \beta_j f_t) f_t] \\
E [r_{j,t} - \lambda_0 - \lambda_1 \beta_j]
\end{bmatrix}
\]  

(89)

This vector represents \((2+K)J\) moment conditions. To obtain OLS coefficients, I use the selection matrix

\[
A = \begin{bmatrix}
I_{(J+JK) \times (J+JK)} & 0_{(J+JK) \times J} \\
0_{1 \times (J+JK)} & 1_{1 \times J} \\
0_{K \times (J+JK)} & \beta'_{K \times J}
\end{bmatrix}
\]  

(90)

where \(1_{1 \times J}\) is a vertical vector of ones and \(\beta\) is a \(J\) by \(K\) matrix of \(\beta\)s.

I compute two types of standard errors. I compute Shanken standard errors under the assumption of zero autocorrelations and zero cross-anomaly correlations. Hence, I take the panel of residuals from the time-series regressions for betas,

\[
\epsilon_t = r_t - \beta f_t
\]  

(91)

where \(r_t\) and \(\beta\) are vertical vectors of different anomaly assets’ returns and betas, respectively.

Then, the variance of the price of risk is computed as

\[
Var(\hat{\lambda}) = \frac{1}{T} \left[ \Sigma_f + (\beta' \beta)^{-1} \beta' \Sigma \beta (\beta' \beta)^{-1} \left( 1 + \lambda' \Sigma_f^{-1} \lambda \right) \right]
\]  

(92)

where

\[
\Sigma = \text{diag} \left( \sum_{t=1}^T \epsilon_t \epsilon_t' \right)
\]  

(93)
GMM standard errors are estimated in the standard way, which allows the errors to be correlated in the cross-section.

**Funding betas are formed during constrained times**

Here, I test whether pre-93 mean return predicts post-93 change in beta from unconstrained to constrained times. The moment conditions are,

\[
g_{6 \times 1}(b) = \begin{bmatrix} 
E \left[ (r_{j,t} - r_{j}^{\text{pre}}) 1(t \in \text{Pre}) \right] \\
E \left[ (r_{j,t} - (a_{j,0}^{\text{post}} + \beta_{j,0}^{\text{post}} f_{t}) 1(t \in \text{Post, Unconstrained}) \right] \\
E \left[ (r_{j,t} - (a_{j,0}^{\text{post}} + \beta_{j,0}^{\text{post}} f_{t}) f_{t} 1(t \in \text{Post, Unconstrained}) \right] \\
E \left[ (r_{j,t} - (a_{j,0}^{\text{post}} + \Delta a_{j}^{\text{post}} + (\beta_{j,0}^{\text{post}} + \Delta \beta_{j}^{\text{post}}) f_{t}) f_{t} 1(t \in \text{Post, Constrained}) \right] \\
E \left[ (\Delta \beta^{\text{post}}_{j} - b_{0} - b_{1} r_{j,0}^{\text{pre}}) 1(t \in \text{Pre}) \right] 
\end{bmatrix}
\]  

(94)

To obtain OLS coefficients, I use the selection matrix

\[
A = \begin{bmatrix} 
I_{5J \times 5J} & 0_{5J \times J} \\
0_{1 \times 5J} & 1_{1 \times J} \\
0_{1 \times 5J} & \bar{r}^l_{1 \times J}
\end{bmatrix}
\]  

(95)

Testing if \( \bar{r}_{j}^{\text{pre}} \) predicts \( \beta_{j,0}^{\text{post}} \) or \( \Delta a_{j}^{\text{post}} \) just requires changing the last moment condition.

**B.2 Constructing the anomaly assets**

See the online appendix.